A Mechanism for Multi-unit Multi-item Commodity Allocation in Economic Networks

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Abstract: In this work, we introduce a novel resource allocation mechanism that aims to maximise the social welfare of the market in procurement auctions. Specifically, we consider a market setting with multiple units of homogeneous resources. In such settings, buyers submit their resource requests to a limited number of known providers. This limited number of providers might in turn lead to a provider monopoly in the market and a scarcity of the resources. To address this problem, we propose a novel information diffusion-based resource allocation mechanism for resource allocation in procurement auctions. The proposed mechanism focuses on procuring multiple units of homogeneous resources. In this regard, the proposed mechanism incentivises the providers to truthfully diffuse the procurement information to their neighbours. This information diffusion aids the buyers to procure the required amounts of commodities/resources at the minimum possible prices. In addition, the proposed mechanism gives fair chances to the distant providers to fairly participate in the procurement auction. Further, we prove that the proposed mechanism minimises the procurement costs, with no deficits, compared to the Vickrey-Clarke-Groves mechanism. Finally, based on the experiments, we show that the proposed mechanism has comparatively lesser procurement costs.

1 INTRODUCTION

The procurement of multiple units of different types of resources has become a challenging problem. Normally, the procurement of resources in competitive markets takes place through procurement auctions (reverse auctions) (Krishna, 2009). In those procurement auctions, sellers with the lowest offered price are the winners. Further, the procurement cost is computed based on an adopted pricing policy, such as first-price auction, second-price auction, etc.

Generally, designing an optimal resource procurement mechanism depends on finding an optimal winner determination policy and an optimal pricing policy. An optimal winner determination policy (WDP) is usually implemented in auction paradigms for different real-world market settings (Samimi et al., 2016; Weber et al., 1998; Prasad et al., 2016; Zaman and Grosu, 2013; Wu et al., 2018). However, the conventional auction mechanisms (Myerson, 1981; Mishra et al., 2020a; Mishra et al., 2020b), mainly focus on maximising the revenue of the owners of the auction. In classical auctions, the owner (buyer is the owner in the procurement) of the auction is only aware of a limited number of bidding participants (sellers are the participants). This might lead to a monopoly in the market (drop-in competition) and also a shortage of resources. Also, the unbalanced supply or demand in the market might affect the stability of the market (Tobin, 1969). Therefore, there is a need for a procurement mechanism that is capable of controlling the number of participants as per the resource demands. In this regard, an information diffusion-based mechanism become an appropriate choice. Specifically, information diffusion would invite distant sellers to take part in auctions and satisfy the market demands. In the literature, those diffusion aided resource allocation mechanisms are used in several e-commerce platforms to advertise their products to remote buyers (Fieldman and Chaube, 2020; Sepehrian et al., ; Moneypenny and Flinn, 2009). Lately, (Zhao et al., 2018; Li et al., 2017) introduced a set of diffusion-based mechanisms in social networks to sell different resources. In specific, (Zhao et al., 2018; Li et al., 2017) presented diffusion-based mechanisms to reduce the total procurement cost. In this regard, the existing

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work focused on selling a single unit of resources (Li et al., 2018; Li et al., 2017) as well as multi-item single-unit resource allocation (Kawasaki et al., 2019; Takanashi et al., 2019). As a result, the existing diffusion-based mechanisms are being adopted for crowd-sourcing aided bulk data collection (Shen et al., 2019; Zhang et al., 2019).

Briefly, the existing diffusion-based auction mechanisms have the potential to address the challenges in designing procurement auctions. However, those existing mechanisms cannot be directly adopted for the problem we aim to solve in this research, i.e., designing procurement auctions for multi-unit multi-item resource allocation. Because, the existing diffusion-based mechanisms are mainly seller-centric (seller is the owner), but the procurement auctions are buyer-centric. Besides, the existing mechanisms are designed for single-unit or unit-demand resource allocations, with no budget constraints. Therefore, to the best of our knowledge, no known diffusion-based mechanisms are designed for a multi-unit multi-item procurement settings. Owing to this, in this research, we focus on designing a diffusion-based collaborative mechanism for a multi-unit multi-item procurement mechanism. Such that, it not only keeps a check on competition in the market but also encourages cooperative behaviour amongst independent sellers. Such procurement mechanisms could be used in different procurement problems such as the procurement of vehicles (Remli and Rekik, 2013), crops, milk, etc, (Vykhaneswari and Devi, ; Nuthalapati et al., 2020). Also the proposed mechanism is *incentive compatible*, *individually rational* and an optimal payment mechanism for multi-unit multi-item resource allocation in economic networks. To summarise, the contributions of this research are as follows: (1) We propose a novel diffusion mechanism for multi-unit multi-item resource allocation in economic networks; (2) we introduce a practical information propagation mechanism to disclose the private information of the participating sellers; and (3) Then, we introduce a contest function based iterative auction-based group determination strategy.

The rest of this paper is organised as follows: we first discuss the model and the different key definitions in modelling the proposed mechanism in Section 2. Section 3 presents the proposed *(DMMP)* mechanism. In Section 4, the properties of the proposed *DMMP* mechanism is presented. Section 5 discusses the experimental results. Finally, the paper is concluded in Section 6.

2 THE MODEL

We consider an economic network with a single buyer denoted as b having multi-unit resource request of k types of non-substitute-able heterogeneous resources in set K, denoted as $Q_b = \{q_{b,1}, q_{b,2}, \ldots, q_{b,k}\}$, $k \in K$, where Q_b is termed as the resource package or simply the package. This procurement request is submitted directly or through diffusion to the set N of n independent sellers denoted as $N = \{s_1, \ldots, s_n\}$. In this context, an economic network is represented as a directed acyclic graph $G \equiv (V, E)$, where $V = N \cup \{b\} = \{s_1, \ldots, s_n\} \cup b$ representing the set of all the nodes (including the buyer and all the sellers reachable to buyer b), whereas E represents the set of edges between these nodes representing the neighbourhood relationship. For any node $i, j \in V (i \neq j)$, if there is a directed edge from i to j, then it means j is the direct successor of i, and i is the direct predecessor of j and the edge is represented as $e_{ij} = 1$, else $e_{ij} = 0$, $e_{ij} \in E$. Further, for all the nodes $i \in V$, its set of immediate children is termed as neighbours and denoted as $Ng_i \subseteq V$, s.t., for $j \in Ng_i$ there exists an edge $e_{ij} \in E$ between node i and j. Further, a set of direct successor for node $i \in G$ is termed as neighbours denoted as Ng_i, whereas direct predecessor is termed as parent node denoted as P^i. In this regard, if there is a path between two nodes $i, j \in V (i \neq j)$, the distance between i, j is denoted as $dist(i, j)$, and if no path exists, then $dist(i, j) = \infty$. Also, we represent set of all the predecessors as Ng_i^{all}, s.t. $Ng_i^{all} = \{j \in V : 0 < dist(i, j) < \infty\}$. Similarly, set of all the predecessors $P_i^{all} = \{j \in V : 0 < dist(j, i) < \infty\}$. Also let $d_i > 0$, represent the depth for node $\forall i \in N$, representing the shortest path from buyer b to seller i. In our setting, initially, buyer b has no prior information about all the sellers in the market.

In such a market setting for multi-unit multi-item procurement auctions, it would be ideal for the buyer to be reachable to maximum possible sellers, so as to procure the resources at the minimum possible price. However, initially, buyer b can only submit its procurement request to its neighbour nodes $j \in Ng_b$. Also, submitting its procurement request to distant potential sellers would incur extra cost on the buyer's budget. Therefore, to avoid this extra cost, we propose a diffusion-based mechanism, that encourages each seller to invite their respective neighbours to participate in the procurement. In specific, firstly, buyer b submit its procurement request to set of neighbours in Ng_b. Then, all the seller $j \in Ng_b$ would diffuse the information to their respective neighbour set Ng_j. The subgraph formed with root node j is termed as *local economic network* of the node $j \in V$ denoted as $G^j \equiv$

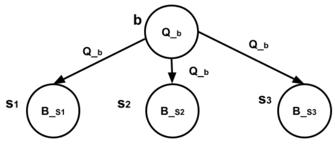

Figure 1: Local economic networks.

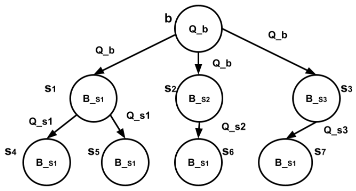
Figure 2: Global economic networks with three local networks.

$(V_j, E_j) \equiv (Ng_j, E_j)$. In this regard, independent *local economic network*s are connected to form a complete economic network denoted as $G = G^1 \cup \cdots \cup G^k$. This whole economic network G resembles a rooted tree, wherein the buyer b is the root node and a set N of potential sellers are leaf nodes.

For instance, Figure 1 depicts an example of economic network. In this regard, upon receiving a resource request from the buyer b, b's immediate neighbours along with their *local economic network*s are combined to form an economic network as depicted in Figure 2.

Further, it should be noted that the resource package request Q_b from buyer b denotes the minimum resource demand, such that buyer b aims to procure at least Q_b package of resources from a single or a group of sellers in an economic network through a diffusion mechanism. In this context, the objective of the buyer is to procure at-least Q_b at minimum possible price from least number of sellers. Because, with the increase in number of sellers, would lead to increase in transaction overhead, also referred as chaining cost in communication domain (Kayal and Liebeherr, 2019). Each seller $i \in N$ has two private values, i.e., per-unit valuation of the resource (bid-density) $v_i \geq 0$ and the maximum quantity of resource the seller is available for selling denoted as $A_i = \{a_{i,1}, a_{i,2}, \ldots, a_{i,k}\}$. Also, $\forall j \in N$, let set of all the sellers except seller j denoted as N_{-j}, s.t., $N_{-j} \equiv N \setminus j$. For example, in Figure 2, for the seller s_1, $Ng_{s_1} \equiv \{s_4, s_5\}$, and $Ng_{s_1}^{all} = \{s_4, s_5, s_8, s_9, s_{11}, s_{13}, s_{14}\}$.

Further, in any multi-item market setting, combinatorial auctions that allow bidders to bid on combinations (bundles or packages) of items make business sense when there are bundles of items that have a combined valuation to bidders higher than the sum of their individual valuations. Such items are said to be complementary. In addition to that, we assume that each seller has incentive in selling their maximum possible resources to avoid wastage of their remaining resources. Therefore, it is practical to assume that sellers give discounted price over bulk procurement. To simplify the valuation, we consider two types of valuation, namely, valuation for bulk purchases and single item purchases. Therefore, $\forall i \in N$, its truthful type is represented as $\theta_i = (bv_i, sv_i, A_i, Ng_i)$, where bv_i, sv_i, A_i and Ng_i are the per unit valuation for bulk purchase (i.e, all the offered resources), the per unit valuation for single item purchase, offered quantity of resource and set of neighbours, respectively. Further, the *type profile* of all the sellers is denoted as $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_n)$. Let $\boldsymbol{\theta}_{-i}$ be the *type profile* for sellers except i, s.t $\boldsymbol{\theta} = (\boldsymbol{\theta}_{-i}, \theta_i)$. Also, let Θ_i be the *type space* for seller i, s.t., $\Theta = (\Theta_i, \ldots, \Theta_n) = (\Theta_{-i}, \Theta)$ be the the *type profile space* for all sellers. Also, we consider a strategic setting, wherein sellers might not report their true type to maximise their utility, this reported *type* of the seller $i \in N$ is denoted as $\theta'_i \equiv (bv'_i, sv'_i, A'_i, Ng'_i)$, where bv'_i, sv'_i, A'_i and Ng'_i are the reported per unit valuation for bulk purchase (i.e, all the offered resources), the per unit valuation for single item purchase, offered quantity of resource and set of neighbours, respectively. Also, let $\emptyset$ be the default reported *type*, when $i \in N$ had not received the information $I_{p^i} \equiv (Q_i, b)$ from its parent seller or seller $i \in N$ do not want to participate in the procurement, where Q_i denotes the minimum resource package requested by seller i, whereas b resembles the procurement information from buyer b. In this context, we assume that, if a seller $i \in N$ is not invited, then the mechanism will not observe any action from that seller, called feasible *type profile* denoted as $F(\boldsymbol{\theta}')$, s.t. $F(\boldsymbol{\theta}') \subseteq \boldsymbol{\theta}'$. Further, the diffusion mechanism for feasible *type profile* is defined as follows:

Definition 1. *A diffusion mechanism M in the economic network is denoted by an allocation policy $\pi = (\pi_1, \pi_2, \ldots, \pi_n)$ and a payment policy $pay = (pay_1, pay_2, \ldots, pay_n)$, where $\pi_i : \boldsymbol{\theta} \to \{0,1\}$, $\pi : \boldsymbol{\theta} \to \mathbb{R}$ and pay_i denotes the payment received by the seller $i \in N$ from buyer b.*

Given the *type profile* $\boldsymbol{\theta}' = (\theta'_1, \ldots, \theta'_{(n)}) \in F(\theta')$, the payment policy $\boldsymbol{pay}(\boldsymbol{\theta}') = (pay_1(\boldsymbol{\theta}'), \ldots, pay_{(n)}(\boldsymbol{\theta}'))$ represents the amount of money each seller would be given at the end of the resource procurement. For seller $i \in N$, if $pay_i(\boldsymbol{\theta}') \geq 0$, then it receives $pay_i(\boldsymbol{\theta}')$ from the buyer and if $pay_i(\boldsymbol{\theta}') \leq 0$, then it will pay to the buyer. In this regard, the allocation policy $\pi(\boldsymbol{\theta}') = (\pi_1(\boldsymbol{\theta}'), \ldots, \pi_n(\boldsymbol{\theta}'))$ represents the resource allocation. We have, $\pi_i = 1$ if seller i is among the winning seller, else $\pi_i = 0$.

In this context, for the diffusion mechanism $M =$

$(\pi, \boldsymbol{pay})$, we assume that there is no cost for a seller to spread the procurement information to its neighbours. Thus, for seller $i \in N$ of type profile θ'_i, given a feasible *type profile* $\boldsymbol{\theta}' \in F(\theta)$ of all sellers, then the utility of seller i's is defined as the payment received minus the expected payment based on its true valuation. In this context, we say that a diffusion mechanism is individually rational if the utility of every seller involved is non-negative as long as it performs its actions truthfully, i.e., reports the valuation truthfully no matter how many neighbours it invites to join the mechanism and how many resources it is willing to sell, as defined in Theorem 1. It should be noted that the definition does not rely on diffusion and disclosed quantity of resources as we do not want to force the seller to invite others and sell all its resources to guarantee him a non-negative gain. Further, if all the sellers are willing to report their valuations truthfully for the reported quantities of resource, we say the mechanism satisfies the property of incentive compatibility. However, in this mechanism, sellers also need to invite their neighbours. Thus, we want to incentivise sellers not only to report their truthful bids but also to invite all their neighbours, as defined in Theorem 2.

In the next section, we present a novel diffusion mechanism for multi-unit homogeneous resource allocation wherein a seller collaborates within their *local economic network*. Then, finally, the resource is served by a group of sellers having the minimum valuation for the minimum requested resource. Also, all the related sellers who contributed to inviting the winner are given incentives for diffusion.

3 DIFFUSION BASED MULTI-UNIT MULTI-ITEM PROCUREMENT

In this section, we introduce a novel procurement mechanism for decentralised multi-unit multi-item resource allocation. This proposed mechanism, i.e., Diffusion based Multi-Unit Multi-Item Procurement (*DMMP*) is based on information diffusion technique. In specific, *DMMP* mechanism adopts a reverse-auction paradigm (Krishna, 2009), wherein each buyer/seller submits a multi-unit multi-item of resource request to their neighbours sequentially. In this context, a seller $i \in N$ and its respective neighbours Ng_i is called *local economic network*.

Firstly, buyer b submits its minimum package of resource request Q_b to all the sellers in set Ng_b. Then, seller $\forall j \in Ng_b$ are encouraged through incentives to diffuse this information within their respective *local economic network*. Finally, package Q_b is allocated to a single or a group of sellers and its corresponding payment is computed. Besides, all the intermediate sellers between the buyer and the winner are given incentives for information diffusion. In specific, in this novel *DMMP* mechanism, two independent procurements are carried out, namely *local procurement* and *global procurement*. The *local procurement* takes place within the *local economic network*s, whereas *global procurement* takes place between sellers and the buyer. In this regard, once the seller agrees to become the part of the global network by reporting its type θ', then the *DMMP* mechanism computes diffusion information for that seller. In addition, mechanism computes bids for local and global procurement sequentially on behalf of the sellers. In this context, it should be noted that the set of neighbours remains the same throughout in local and global procurement. Also, we assume that, any seller can participate in the *global procurement* , only if participates in *local procurement*.

Briefly, whole mechanism can virtually [1] be divided into three major stages, that is, (1) information propagation, (2) group determination, and (3) winner determination, discussed in the following subsections.

3.1 Information Propagation

In this subsection, we present the information diffusion stage in the *DMMP* mechanism. In specific, upon receiving $Q_b \equiv \{q_{b,1}, q_{b,2}, \ldots, q_{b,k}\}$ from b, all the sellers $i \in Ng_b$ diffuse the information message $I_i = (Req_i, b)$ to their neighbours Ng_i, wherein, $Req_i = \{req_{i,1}, req_{i,2}, \ldots, req_{i,k}\}$ is minimum local package request for Ng_i, i.e., minimum quantity required by its neighbours to participate in the *local procurement*, computed by the mechanism using Equation 1 $\forall k \in K$ and b represent the buyer b's information.

$$req_{i,k} = \begin{cases} q_{i,k}, & \text{if } i = b. \\ req_{P^i,k} - a'_{i,k}, & \text{if } req_{p^i,k} > a'_{i,k}. \\ 0, & \text{if } q_{p^i} \leq q'_i. \end{cases} \quad (1)$$

In this way, mechanism would model the diffusion information I_i for all the sellers sequentially based on the reported type θ'_i $\forall i \in N$. Note that, from Equation 1, diffusion information reveals only the difference of the resources request for each of the sellers. This difference-mechanism is designed to preserve the privacy of the sellers. Also it promotes participation of the sellers and maintain the competition in the overallprocurement process. In real-world setting, assum-

[1] stages are interdependent and run simultaneously

ing that seller would reveal the true resource requirements from its ancestors is unrealistic. This is unrealistic because any greedy seller would be interested in selling its resource first and then inviting other sellers. Therefore, this difference-mechanism is in accordance with the rational behaviour of the sellers. Further, to avoid information diffusion for infinitely larger economic networks, diffusion phase will continue until enough resources are available in the overall economic network. In this context we set the maximum available resources is reached at-least A_{max} or T_{max} time is reached. The value of A_{max} is computed as $A_{max} = Q_b \times \delta$, where, δ represents the maximum resource factor. This value is decided such that, there should be enough resource available in the global economic network to maintain the competition. Also, too much availability of the resource would lead to wastage of resources as well as bidder drop problem (Baranwal and Vidyarthi, 2015). Therefore δ should be chosen such that it maintains the trade-off between supply and demand in the global market. In this regard, A_{max} is computed such as, for $k = 3$, then for $Q_b \equiv (q_{1,b}, q_{2,b}, q_{3,b}) \equiv (5,7,8)$, then for $\delta = 2$ $A_{max} \equiv Q_b \times \delta \equiv (5,7,8) \times 2 \equiv (10,14,16)$. In addition to that, in order to restrict the infinitely information diffusion, we set the maximum time T_{max}, this is there to stop the algorithm if A_{max} is never reached. This maximum time step is fixed at the beginning of the mechanism. So once algorithm has A_{max} or T_{max} is reached, the mechanism moves to the next stage, i.e, group determination discussed in the next subsection. Algorithm 1 gives the full pseudocode for information propagation stage.

This algorithm takes buyer b's information i.e., location and request Q_b along with the whole network information $G \equiv (V,E)$, δ and T_{max} as input. Besides, it has temporary (arbitrary) neighbour list which is a queue used for traversing the neighbour list. After that, the algorithm en-queues the neighbours of buyer list. Then the algorithm runs until maximum T_{max} or A_{max} volume of resources available, and dequeue the top element from the arbitrary neighbours' list and diffuses the information to its neighbours. Finally, it enqueues, sequentially, all its neighbours to the neighbour list and arbitrary list and update the value of the level of each seller and computes the maximum depth d_{max}. In the next subsection, we will present the second stage of the *DMMP* mechanism i.e., Group Formation.

3.2 Group Determination

In this subsection, we present the second stage of the novel *DMMP* mechanism, i.e., the group determination stage. After receiving the information messages I_i from ancestor node P^i, on behalf of seller $i \in N$, the proposed mechanism starts to fill the requested package Req_i. In fact, the proposed mechanism does this based on an iterative auction, sequentially for all the local networks from bottom to top. In specific, all the seller at level $j \in N$ submit their bids $bid_i^1 \equiv (bv_i^+, sv_i^+, Q_i)$ to their parent node P^j, where bv_i^+, sv_i^+ and Q_i are the updated valuations and the combined available bids. Then, finally, after forming groups all the seller's bids can now be submitted directly to buyer b. In this context, it should be noted that, in any economic market, any greedy seller would prefer to sell its resource first then would try to sell others resource. Therefore, we assume that seller would consume the resource from its neighbours, only when its available resource is less than the resource request from its respective parent. In specific, if $a'_{i,r} < req_{p^i,r}$, then the mechanism would aid the seller i to collaborate with its neighbour(s) and submit the bid the combined resource in the *local procurement*. Similarly, if $a'_{i,k} < q_{b,k}$, then only the mechanism would reveal the combined resource in its submitted bid bid_i^2. Then, at the end of *local procure-*

Algorithm 1: Information Propagation.

```
Input: G ≡ (V,E), Q_b, δ, T_max
Result: d_max
A_max ← Q_b × δ ; sn ← b ;
A_total, h_max ← 0 ;
req_sn,k ← q_b,k ; ∀k ∈ K ;
Req_sn = {req_sn,k : ∀k ∈ K} ;
I_sn ≡ (Req_sn; ∅) ;
neighbour_list[b][][∅];
  neighbour_list_arbitrary[b][] ← R_b ;
while t = T_max or A_total ≥ A_max do
    for neighbour_list_arbitrary[sn] ≠ ∅ do
        node ←
          DEQUEUE(neighbour_list_arbitrary[sn])
          node ← []diffuseI_sn ≡ (Q_sn; b) ;
        seller_list; seller_list_arbitrary ← node
          ;
    end
    sn ← DEQUEUE(seller_list_arbitrary) /*
      changing the seed node */ ;
    compute Req_sn using Equation 1 */ ;
    A_total + = Q_sn ;
    if Ng_sn ≠ ∅ then
        if sn ∉ neighbour_list then
            d_max + = 1
        end
        neighbour_list[sn];
          neighbour_list_arbitrary[sn] ← Ng_sn ;
        t + + ;
    end
end
```

ment i.e., after receiving reported type $bid_i^1, \forall j \in Ng_i$, set of local winners L^i is elected by performing iterative auction. In specific, winning seller in each iteration is decided based on customised contest success function (CSF) (Skaperdas, 1996). A CSF determines each sellers probability of winning the local auction in terms of other sellers bidding package. In specific, based on CSF, a local winner in each iteration is determined from the local network with seed node $i \in N$, $lw \in Ng_i$ is elected as $lw \equiv min(csf_j)$, $\forall j \in Ng_i$, whereas csf_j is computed using Equation 2.

$$csf_i = \sum_{k \in K} \frac{(q_{i,k})^{\sigma} * v'_i}{\sum_{j \neq i}^{N} (q_{j,k})^{\sigma} x v'_j} \quad (2)$$

where, $0 < \sigma \leq 1$ represents the noise parameter in the contest, interpreted as the marginal increase in probability with the increase in valuation (Shen et al., 2019). Towards this end, seller i would update its total available resources Q_i using Equation 3 $\forall k \in K$

$$q_{i,k} = a'_{i,k} + \sum_{l \in L_i} q_{j,k} \quad (3)$$

where, L_i is set of local winner from all the local iterative auctions. In this regard, mechanism conducts a iterative auction based matching for all the seed node belonging to same height h sequentially with continuous iteration. In each round of auction for seed node $i \in N$, mechanism elicits a single winner. The iteration continues until minimum package Q_i is filled or maximum waiting time W_d has been reached, where d denotes the depth of the seed node. In addition, in each iteration XOR bids (Leyton-Brown et al., 2000) are submitted, i.e. each local seller can win only once. This will reduce the complexity of the iterative auction by reducing the number of combinations in each iteration.

Further, at the end of each iteration for local auction for seed node $i \in N$, mechanism computes the per unit valuation for each of the winning sellers in L_i using Equation 4 at which each of the local winner $j \in L_i$ would get its payment, if seller i is the winner in the procurement.

$$v_l = \begin{cases} min(v'_j) \forall j \in Ng'_i \setminus l, & \text{if } Ng_i \neq \{null\}. \\ max(v'_i, v'_j), & \text{if } |Ng_i| = 1. \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

where, $v'_j = bv'_j$, if all the resources in the offered package Q_j is allocated, else $v'_j = sv'_j$. Intuitively the above equation depicts that, if there are more than one neighbours, i.e., $|Ng_i| > 1$, then the valuation of the local winner is computed based on the *VCG* mechanism (Krishna, 2009). However, if there is only one local neighbour, i.e., $|Ng_k| = 1$, then the valuation is the maximum valuation among seller i and $j \in Ng_i$; otherwise its set to be zero. Similarly, the valuation for the seed seller is updated based on the valuation of the local winner using Equation 5.

$$v_i = \begin{cases} v'_i, & \text{if } Ng_i = \{null\}. \\ \frac{v'_i \times (q'_{i,k} - \sum_{\forall l \in L_i} q'_{l,k}) + v_{l \in L} \times \sum_{\forall l \in L_i} q'_{l,k}}{q_{i,k}}, & \text{otherwise.} \end{cases} \quad (5)$$

Then, after end of the iterative auction at depth d, mechanism moves upward at depth $d-1$. In specific, at depth d, $\forall i \in N$, where $d_i = d$, seller i submit their bids to parent $P^i \in N$. In this way, local groups are formed within the *local economic networks* in a decentralised manner. In this regard, maximum time-step w_d for which iterative auction will continue at depth d is computed as $w_d = ((d_{max} - d) + 1) * (T_{max}/d)$. Finally, after completion of auctions in local networks, a new global economic network with updated valuation and available resources is formed, represented as $G' \subseteq G$.

In the next subsection, we will present the third and the final stage of the *DMMP* mechanism i.e., winner determination and payment distribution.

3.3 Winner Determination

In this section, we present the third stage i.e., winner determination, wherein, payment of the winning seller(s) are computed. In addition to that, rewards for all the sellers who has contributed by information diffusion are computed. Specifically, the mechanism determines a single winner w which can fill all the requested resources in the package Q_b, s.t. $w \equiv min(csf_i)$, $\forall i \in G'$, whereas csf_i is computed using Equation 2. Also, all the sellers in the path $path^{bw}$ from buyer b to the winning seller w are rewarded for diffusing the information to winner w. This path $path^{bw}$ is called as *winning path* and represented as $path^{bw} \equiv \{b, \ldots, w\}$. Let, $v^*_D = min_{i \in D} v'_i$ be the minimum reported valuation in the subset $D \subseteq N$ and the corresponding seller is represented as w^*_D, and then $v'_w = v*^*_N$ whereas $w = w^*_N$. Similarly, $v^*_{D \setminus i}$ denotes the minimum valuation and $w^*_{D \setminus i}$when seller $i \in D$ does not participate. Also to simplify the notations, let $v^*_{N* \setminus i} = v^*_{-i}$.

Definition 2. *feasible global neighbours ($N*$) is a set of all sellers for buyer b having $req_b \leq q'_k$, s.t $k \in N$ and $N* \subseteq N$*

Definition 3. *A critical seller set (C), is a set of all the sellers in winning path $path^{bw}$ including the local winner of the winning seller, i.e., $C \equiv \{c_1, \ldots, c_h, L_{c_h}\}$, where $i \in path^{bw}$, $w = w^*_N$. This critical seller C set is an ordered set, s.t., $d_{c_1} \supset d_{c_2} \supset, \ldots, \supset d_{c_h} \supset d_{L_{c_h}}$,* *where d_{c_i} denotes the depth of the node c_i from buyer b.*

In this regard, the allocation policy for diffusing mechanism *DMMP* is computed using Equation 6.

$$\pi_i(\theta) = \begin{cases} 1 & \text{if } i \in C, v'_i = v^*_{-(i+1)} \ . \\ 0, & \text{if } i \notin C. \end{cases} \quad (6)$$

Intuitively, in *DMMP* mechanism, the first *critical seller* who has the least per unit valuation when critical seller $i+1$ is removed from the economic network is the winner. Then the bid density $bd_i \ \forall i \in C$ is computed using Equation 7., bid density represents the value at which payment for a seller will be calculated

$$bd_i = \begin{cases} v^*_{-i} + \varepsilon_i & \text{if } i = w. \\ \varepsilon_i, & \text{if } i \in C_{-w}. \\ v'_{L_i}, & \text{if } i = L_w. \\ 0, & \text{Otherwise.} \end{cases} \quad (7)$$

where, ε_i represents the reward factor for seller $i \in C$ for diffusing the information to its neighbours, which is computed using Equation 8

$$\varepsilon_i = \frac{v^*_{-(\alpha_{i+1}, w^*_{\alpha_i})} - v^*_{-\alpha_i}}{|C-1|} \quad (8)$$

where, $\alpha_i = 1$, if $i \in C$ and $i \in N^*$, else $\alpha_i = 1$ if $i \in C$ and $i \notin N^*$ and $P^i \in N^*$, represent the closest ancestor node in N^* if $i \notin N^*$

According to above bid density calculation policy, winning seller's bid density is sum of *vcg* payment v^*_{-w}, i.e., winner is paid the second lowest per unit valuation. In addition, winning seller is rewarded γ_w for diffusing the information to its neighbour. On the other hand, all the other critical sellers are rewarded γ_i for information diffusion. Intuitively, reward is the decrease in payment for buyer b for seller $i \in C$ diffusion action. In particular, it is change in payment for buyer b when seller $i+1$ along with the seller w^*_{-i} (when seller i do not participate) do not participate and when seller $i \in C$ do not participate in the procurement. Finally, bid density of local winner is same as computed during its *local procurement* based on *VCG* mechanism.

To the end, based on the valuation computed using Equation 7 and 8, payment pay_i is computed for all the critical sellers in set C.

$$pay_i = \begin{cases} bd_i \times (q'_i - q'_{L_i}), & \text{if } L_i \neq null \text{ and } i \in C. \\ bd_i \times q'_i, & \text{otherwise.} \end{cases} \quad (9)$$

Further, the total payment given by the buyer b to all the sellers in *winning path* $path^{bw}$ is computed as $pay = \sum_{i \in C} pay_i$

In this way, winners and their respective payment are computed, also rewards for all the sellers in winning path is computed for information diffusion. In the next section we would discuss the properties of the proposed *DMMP* mechanism.

4 PROPERTIES OF *DMMP*

In this section, we prove that *DMMP* mechanism is individual rational (*IR*) (Theorem 1) and incentive compatible (*IC*) (Theorem 2). The proofs for both the Theorems are in the appendix.

Theorem 1. *A diffusion mechanism $M = (\pi, \boldsymbol{pay})$ is IR, if $u_i(\theta_i, (\theta_i, \boldsymbol{\theta}'_{-i})) \geq u_i(\theta_i, (\theta'_i, \boldsymbol{\theta}''_{-i}))$, $\forall i \in N$, all $\theta'_i \in \Theta_i$, where $(\theta'_i, \theta''_{-i}) \in F(\theta'_i, \theta'_{-i})$. In this theorem we will prove that DMMP is individually rational.*

Theorem 2. *A diffusion mechanism $M = (\pi, \boldsymbol{pay})$ is IC, if $u_i(\theta_i, (\theta_i, \boldsymbol{\theta}'_{-i})) \geq u_i(\theta_i, (\theta'_i, \boldsymbol{\theta}''_{-i}))$, $\forall i \in N$, all $\theta'_i \in \Theta_i$, where $(\theta'_i, \theta''_{-i}) \in F(\theta'_i, \theta'_{-i})$. In this theorem we will prove that DMMP is incentive compatible.*

5 EXPERIMENTAL RESULTS

In this section, we present the experimental results performed to evaluate the performance of the novel *DMMP* mechanism based on two conventional mechanisms. In this regard, we compare the performance of the following three mechanisms: (1) **Simple Procurement**: This is the classical procurement, wherein the buyer procures only from its neighbours through iterative first price reverse auctions; (2) **Fixed Reward**: In this mechanism, sellers are given a fixed reward ω for the diffusion of information, whereas payment is based on a second price auction: and (3) **DMMP**: In this mechanism, the buyer is not aware of all the sellers in the economic network until its neighbour directly or indirectly diffuses the procurement information; whereas, payment and rewards are computed using *DMMP*.

In our experimental setting, we consider the procurement of five different types of resources i.e., $k = 5$ and set the maximum neighbours for each node as 4, i.e. $0 < |Ng_i| \leq 4$, $\forall i \in V$, and for fixed reward, ω is set to root of the per-unit valuation of the corresponding node. In this regard, we randomly generate economic networks and set available resource package for each seller. Besides, we set that $|Ng_b| = 4$, s.t., buyer has at least four sellers directly reachable which can collectively fill the requested package Q_b, i.e., $Q_b \subseteq \{A_i : \forall i \in Ng_b\}$. Further, values for Q_b

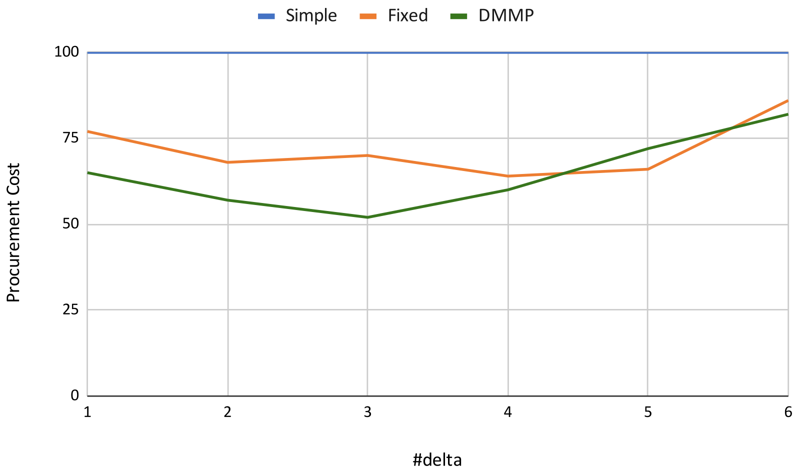

Figure 3: Impact of value of δ on the total procurement cost.

and $A_i \forall i \in N$, for all the five types of resources are sampled from a random generator which takes values $[200, 1000]$ units. Further, both the per unit valuations, i.e., sv_i and bv_i $\forall i \in N$ is also drawn from a random generator which takes values $[20, 50]$ s.t. $sv_i > bv_i$. In this setting, for the economic network, we run all mechanism for 20 times. Then we evaluate the results to show the merits of adopting *DMMP* mechanism for multi-unit multi-item resource procurement based on the cost of the procurement for the buyer. Besides, in our experimental setting, we intend to analyse the impact of change in the value of δ on the cost of procurement. Therefore, we designed *six* different experimental settings concerning the value of δ, such as, $\delta = (1, 6)$. Finally, all the mechanism are implemented in *Python 3* and the experiments are performed on *Intel Xeon 3.6GHz 6* core processor with *32 GB RAM*.

From Figure 3, it can be observed the procurement cost is minimum for *DMMP* mechanism as compared to the other two mechanisms. An interesting observation here is, initially, procurement cost decreases with the increase in value of δ. However, later the procurement cost increases with the increase in value of δ. For instance, at $\delta = 3$, procurement cost is the least, but at $\delta = 5$ procurement cost rises. This is possibly because of the increase in the number of nodes in the winning path, which leads to an increase in the total rewards distributed. Overall, the experimental results highlight that the novel *DMMP* mechanism outperforms the other two mechanisms and demonstrate its efficiency for procurement of multi-unit multi-item resources through economic networks.

6 CONCLUSION

In this research, we present a novel information diffusion-based resource allocation mechanism in economic networks. In specific, we consider the economic networks where independent buyers submit their multi-unit multi-item resource requests to multiple independent sellers. In this regard, the proposed mechanism aids the buyers to procure the required resources from a group of distant sellers with the minimum possible prices. Besides, the proposed mechanism encourages the independent sellers to share their available resources amongst each other. Also, the proposed *DMMP* mechanism guarantees that every seller receives an incentive to reveal their truthful type and invite all their neighbouring sellers to participate in the procurement. In this context, rewards are given to all the sellers in the winning path for diffusing information. Most importantly, those rewards do not increase the buyer's payment. In fact, the buyer's payment is even improved as compared to the *VCG* mechanism. As for future work, we plan to focus on multiple buyers with multi-unit heterogeneous resource combinatorial auctions in economic networks.

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