Coordination Mechanisms with Misinformation

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Abstract: We introduce a novel approach for coordination mechanisms in games, based on the idea of *misinforming* players about the game formulation in order to steer them towards a behaviour that leads to an improved outcome in terms of social welfare. As a use case, we study single-commodity non-atomic congestion games with parallel links and affine cost functions. We propose a simple mechanism that provides to the players the right incentives to adopt a socially optimal behaviour by misinforming them with regards to the latency functions of the links, under various assumptions. We use a metric called the *Price of Misinformation* to quantify the effect of misinformation on social welfare (compared to the optimum of the actual game), and show that our mechanism can minimise this metric, resulting in values that are better than the Price of Anarchy (i.e., the social outcome without any intervention from the designer).

1 INTRODUCTION

In games with selfish and rational players, the outcome may be very inefficient in terms of social welfare. The *Price of Anarchy*, introduced in (Koutsoupias and Papadimitriou, 1999), measures how far the worst *Nash equilibrium* is from the social optimum.

In this paper, we investigate methods to lead players' behaviour to a socially improved outcome. Coordination mechanisms were introduced for this purpose in (Christodoulou et al., 2009; Christodoulou et al., 2014), in which a theoretical framework is proposed where modifications of the game lead to a reduced fraction, compared to the *Price of Anarchy*, of the worst Nash equilibrium in the modified game to the social optimum of the original game. This has been applied to many classes of games such as load balancing and congestion games (Nisan et al., 2007).

A common assumption in game theory is that players have complete information concerning the game of interest, i.e., the set of players, the set of strategies and the payoffs for each player is common knowledge among players. In (Varsos et al., 2021), this assumption is dropped by studying situations where each player may have a different (and thus incorrect) perception about the game being played, while being unaware that she has only a subjective view of the interaction. To that direction, (Varsos et al., 2021) agglomerate the objective interaction (called actual game) and the subjective views, introducing the concept of *misinformation games* for normal-form games. A key characteristic of misinformation games is that each player's choices are dictated by her view, whereas her payoff is provided by the actual game. This leads to the introduction of a new equilibrium concept, the *natural misinformed equilibrium*, which is the set of strategic choices where no player wants to deviate in her view.

Moreover, a metric, the *Price of Misinformation (PoM)*, was introduced to measure the impact of misinformation in games, compared to the socially optimum situation (Varsos et al., 2021). Clearly, misinformation could lead players to strategic choices that are different from the ones they would make in the absence of misinformation. This includes choices that are actually beneficial (from the perspective of social welfare) for the players. Inspired by this observation, we combine misinformation games and coordination mechanisms to address the following question:

Is it possible for the designer of a game to misinform players regarding the game parameters, in order to provide incentives for a better (or even optimal) behaviour in terms of social welfare?

We positively answer this question, and provide a novel way for applying coordination mechanisms using the concept of misinformation, thereby establish-

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ing a connection between the classical coordination mechanisms and misinformation games.

As in classical coordination mechanisms, where the designer modifies the game in order to minimize the ratio between the worst Nash equilibrium of the modified game and the social optimum of the original game, we propose a similar approach where the designer misinforms players. Next, we compare the worst natural misinformed equilibrium (i.e., the worst result of misinformation) with the social optimum of the actual game, that is Price of Misinformation. Interestingly, it could be less than the Price of Anarchy, resulting in an improved behaviour (from the perspective of social welfare) of the players, compared to the scenario without misinformation.

A key difference between classical coordination mechanisms and coordination mechanisms with misinformation is that in the first case the designer can influence the actual interaction as a whole, whereas in the latter the designer determines the subjective views of the players, i.e., the misinformed views. Thus, in the first case the designer modifies the actual game specification, whereas in the latter the designer changes players' (subjective) information, but has no power over the actual game specification, In this paper, we consider the setting where the designer cannot impose a different game specification, but can misinform players about the actual set up.

We consider the problem under assumptions about the number of misinformed views that the designer can spawn. In particular, the designer has bounded capabilities with regards to the number of different misinformed views that can be spawned (see Section 6).

As a real-life situation, consider the case where two companies compete with each other in a market. We can see one of them (say D) as the designer, and the other (say P) as a group of divisions $(P_1,\ldots,P_2)$, representing players. The designer D can use a coordination mechanism with misinformation in order to benefit from the (in-)efficiency of the rival company, by misleading the divisions of P into actions that are not optimal for P. Although this is a rather simplistic scenario, in general, coordination mechanisms with misinformation can be used in (among others) bargaining, negotiation and conflict scenarios.

We apply the above ideas for single-commodity non-atomic congestion games with n parallel links and affine cost functions. We first adapt the concept of misinformation to the class of non-atomic congestion games (Section 4), and design a polynomial-time algorithm for computing a pure Nash equilibrium in a network (Section 5). Moreover, we describe a mechanism for designing misinformation games with an optimal Price of Misinformation (and thus better social outcomes) under various assumptions (Section 6).

2 RELATED WORK

The idea of designing mechanisms to improve coordination in multi-player systems with selfish players is not new. One approach is to introduce taxes ((Fleischer et al., 2004), (Fotakis and Spirakis, 2008), (Caragiannis et al., 2010), etc.); this has been applied in congestion games, where players pay a toll for every edge they use. In (Cole et al., 2003) it was shown that there exist taxes that reduce the *Price of Anarchy* to 1. However, there are two issues for this approach: i) taxes may be very high, and ii) if taxes are part of the cost, then the *Price of Anarchy* is not improved. Similarly, (Lavi and Swamy, 2007), (Seregina et al., 2017), (Turrini, 2013) used rewards to improve coordination. In (Monderer and Tennenholtz, 2003) the game is extended by adding new strategies and enhanced monetary policies for the players, such that all Nash equilibria of the new game involve strategies of the original game exclusively. In that case, the *Price of Anarchy* is decreased.

In this paper, we will use the concept of misinformation games (Varsos et al., 2021). This concept lies in the area of games with misperceptions (see Chapter 12 in (Luce and Raiffa, 1957)), which studies games where players have a subjective knowledge regarding game specifications. Related ideas include: i) hypergames ((Bennett, 1980; Sasaki and Kijima, 2012; Bakker et al., 2021) etc.), ii) games with unawareness/awareness ((Copic and Galeotti, 2006; Halpern and Rêgo, 2014; Schipper, 2017; Feinberg, 2020) etc.), and iii) other works ((Esponda and Pouzo, 2016; Ozbay, 2007) etc.) where players may have misperception. To the best of our knowledge, there has been no study in this area regarding non-atomic congestion games, and also none of these has been applied to improve the social outcome of games.

Another approach dealing with unawareness is that of incomplete information that is described by Bayesian games. They were introduced by (Harsanyi, 1967) and have received a lot of attention by many scholars ((Székely and Rizzo, 2007; Zamir, 2009; Myerson, 2004), etc.). The main idea of this approach is that players' beliefs with regards to game specifications are represented by probability distributions.

There are several key differences between misinformation and Bayesian games. Firstly, in Bayesian games, players are aware of the incompleteness of their information (as represented by the probability distributions), whereas in misinformation games players are unaware of the fact that their game specification is wrong. Therefore, they will not consider mitigating strategies "just in case" they have a wrong specification. Moreover, the payoffs they get are dictated by the actual game and may be radically different from the ones specified in their own game.

Moreover, in the Bayesian approach the concept of non-common knowledge is dealt indirectly through the assignment of distributions over actions and beliefs, as opposed to misinformation games, where we can directly deal with non-common knowledge games, using subjective views. Consequently, the two aforementioned approaches have different equilibria concepts, namely Bayesian Nash equilibrium and natural misinformed equilibrium, respectively.

Another approach to cope with incomplete information in non-atomic routing games was introduced in the works of (Meir and Parkes, 2015a; Meir and Parkes, 2015b; Meir and Parkes, 2018), where the authors propose a model with player-specific costs. In these studies, the authors suggest that players play a modified game with cost functions potentially different than the actual game, without considering either misinformation or coordination mechanisms.

3 SINGLE COMMODITY NON-ATOMIC CONGESTION GAMES

We define single-commodity non-atomic congestion games using an approach similar to that of (Meir and Parkes, 2015b):

Definition 1. *A single-commodity non-atomic congestion game is a tuple* $\Gamma = \langle G, l, s, t, r \rangle$*, where:*

- $G = (V, E)$ *is a directed graph,*
- l *is the set of non-decreasing, continuous, and non-negative latency cost functions with* $l_e(x) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ *(one for each edge* $e \in E$*), where* x *is the flow,*
- $s \in V$ *is the source,*
- $t \in V$ *is the destination,*
- r *is the total mass of flow.*

We consider that any player controls an infinitesimal amount of flow r.

Let P be the set of total paths from s to t, then we define as $f(r) \in [0, r]^{|P| \times 1}$ a feasible flow of the players routing r units of flow on the paths. We define as $f_p(r)$ the flow of players that follows the path $p \in P$. The flow of the players on the edge $e \in E$ is $f_e(r)$.

The cost of following a path p is $C_p(f_p(r)) = \sum_{e \in p} l_e(f_e(r))$ and the total Social Cost of the flow $f(r)$ is $SC(f(r)) = \sum_{e \in E} f_e(r) l_e(f_e(r))$. The socially optimal flow is the feasible flow $f(r)$ such that $SC(f(r))$ is minimum. We define a *pure Nash equilibrium* (also known as *Wardrop equilibrium*) in non-atomic congestion games as follows:

Definition 2 (Pure Nash equilibrium (Nash, 1951)/Wardrop equilibrium (Wardrop, 1952))**.** *A pure Nash equilibrium is a feasible flow* $f^*(r)$ *such that for any* $p, \hat{p} \in P$,

$$C_p(f_p^*(r)) \leq C_{\hat{p}}(f_{\hat{p}}^*(r)),$$

in other words, a flow is an equilibrium if no player has any incentive to deviate from her path.

Any non-atomic game has at least one equilibrium (Schmeidler, 1973; Rosenthal, 1973).

Next, we present the specific class of single-commodity non-atomic congestion games with n parallel links from s to t with $r = 1$. We consider that the cost function of an edge/link $k \in \{1, 2, \ldots, n\}$ is an affine function, that is, $l_k(x_k) = a_k x_k + b_k$, with $a_k, b_k > 0$, and $x_k \in [0, 1]$ is the flow of link k, with $\sum_{k=1}^{n} x_k = 1$. Without loss of generality, we assume that the links are sorted in an increasing order with respect to b_k, or in other words that if $k \leq p$, then $b_k \leq b_p$. Let $x = (x_1, \ldots, x_n)$ be an allocation of the flow on the links. Here the cost on an edge k is $l_k(x_k)$, thus the total Social Cost of this allocation is $\sum_{k=1}^{n} x_k l_k(x_k)$. We denote this game as Γ, see Figure 1.

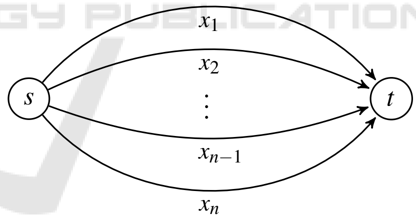

Figure 1: Single-commodity non-atomic congestion game with n parallel links.

In the case of non-atomic congestion games with n parallel links, Definition 2 provides that an allocation $x^* = (x_1^*, \ldots, x_n^*)$ of the flow on the links is a pure Nash equilibrium if and only if, $\forall\, i \in \{1, \ldots, n\}$, and $\forall\, j \in \{1, \ldots, n\}$, $l_i(x_i^*) \leq l_j(x_j^*)$. Consequently, $x^* = (x_1^*, \ldots, x_n^*)$ is a pure Nash equilibrium if and only if, $l_i(x_i^*) = l_j(x_j^*)$ for all $i, j \in \{1, \ldots, n\}$ with $x_i^*, x_j^* > 0$. Moreover, all Nash equilibria result in the same Social Cost (Chapter 18 of (Nisan et al., 2007)).

4 MISINFORMATION NON-ATOMIC CONGESTION GAMES

Misinformation games were defined in (Varsos et al., 2021) for the general class of normal-form games. Under the definition of (Varsos et al., 2021), a misinformation game of N players consists of $N+1$ game specifications ($mG = \langle G^0, G^1, \ldots, G^{N+1} \rangle$). The first ($G^0$) is the so-called *actual game*, which corresponds to the game that the players actually play, whereas each one of games G^i corresponds to the subjective view of the game from the perspective of player *i*, that is the game that player *i* believes that is being played.

Note that this definition allows all types of misinformation to occur (including, e.g., cases where the players are misinformed regarding the available strategies or players). However, in (Varsos et al., 2021) a special form was considered, called *canonical* misinformation games, where subjective views differ only in the values of the elements of the payoff matrices. It was shown that all non-canonical misinformation games can be transformed into canonical ones, therefore we can, without loss of generality, restrict our attention to canonical games.

In this Section, we use an analogous approach to define misinformation games for the case of non-atomic congestion games, where each player has a subjective view about the game she plays, which may be different from the others. Formally:

Definition 3 (Misinformation game)**.** *A misinformation non-atomic congestion game $m\Gamma$ with θ splitting is an $(N+1)$-tuple $m\Gamma = \langle \Gamma^0, \Gamma^1, \ldots, \Gamma^N \rangle$, where N is the number of different views of the game that different players may assume, $\Gamma^0 = \langle G, l, s, t, r \rangle$ is the actual game, $\Gamma^j = \langle G, l^j, s, t, r^j \rangle$ are the different subjective game specifications assumed by the players, of which each player assumes only one, and $\theta = \langle \theta^1, \ldots, \theta^N \rangle$, where θ^i is the portion of players that experience view Γ^i.*

Here, we assume that the total mass of flow across all Γ^j (for $j > 0$) is equal to the respective mass in Γ^0, $r^j = r$. Further, it must hold that $\sum_{i \in [N]} \theta^i = 1$. Thus, the players have the correct view of the graph and the flow at hand, although they may assume different cost functions. In this case, we call the misinformation single commodity non-atomic congestion games as *canonical*, similar to canonical misinformation games, as defined in (Varsos et al., 2021).

Definition 4 (Misinformed equilibrium strategy)**.** *A misinformed strategy is a flow for portion θ^j, that θ^j with subjective view Γ^j follows in a pure Nash equilibrium strategy of its game view Γ^j.*

Definition 5 (Natural Misinformed equilibrium)**.** *A natural misinformed equilibrium is a flow f such that each portion θ^j plays a misinformed equilibrium strategy according to its game-specific view j.*

Since any non-atomic congestion game has at least one Nash equilibrium, then any misinformation game of a non-atomic congestion game as defined above has at least one natural misinformed equilibrium.

Having at hand the formal definition of the natural misinformed equilibrium, we measure the deterioration/leverage in efficiency of a non-atomic congestion game due to misinformation. For that, we adapt the concept of Price of Misinformation (*PoM*), that was defined in (Varsos et al., 2021), in the case of non-atomic congestion games. Formally:

Definition 6 (Price of Misinformation)**.** *Given a misinformation congestion game, the* Price of Misinformation (PoM) *is defined as*

$$PoM = \frac{\max\limits_{f_{NME} \in NME} SC(f_{NME})}{SC(f_{opt})}, \tag{1}$$

where f_{opt} is the flow that minimizes the Social Cost in the actual game Γ^0 and the nominator is the worst (maximum) value of the Social Cost of the set NME as computed with regards to the actual game.

We can show the following:

Proposition 1. *For every misinformation non-atomic congestion game, we have that:*

$$1 \leq PoM \leq \left(r \cdot \max_{p \in P} C_p(r)\right)/opt \tag{2}$$

Proof. In the worst case, all flow is routed through the most costly routes, which leads to a Social Cost of $r \cdot \max_{p \in P} C_p(r)$. Thus, $PoM \leq \left(r \cdot \max_{p \in P} C_p(r)\right)/opt$. Also, $PoM \geq 1$ by definition. □

Note that when $\Gamma^0 = \Gamma^j \; \forall \; j$, then *PoM* coincides with the *Price of Anarchy (PoA)*. Using the definition of *PoA* and (1), we can link these formulas as follows:

$$PoM = PoA \cdot \left(\frac{\max\limits_{f_{NME} \in NME} SC(f_{NME})}{\max\limits_{f_{NE} \in NE} SC(f_{NE})} \right), \tag{3}$$

where NE is the set of all Nash equilibria of Γ^0.

Interesting results can be derived by comparing the worst Nash equilibrium of Γ^0 (or *PoA* of Γ^0) with the worst natural misinformed equilibrium of $m\Gamma$ (or *PoM* of $m\Gamma$). If $PoM < PoA$, then misinformation has a beneficial effect on the social outcome, as players are inclined (due to their misinformation) to choose socially better strategies. Otherwise ($PoM > PoA$), misinformation leads to a worse outcome from the perspective of social welfare. Next, we provide an illustrative example of the above concepts.

Example 1. *We consider the non-atomic congestion game as depicted in Figure 2a (known as Pigou network (Pigou, 1933)), with latency functions* $l_1(x) = \varepsilon x + 1$, $l_2(x) = x + \varepsilon$, $r = 1$ *and* $x \in [0,1]$. *It is clear that selfish players in a pure Nash equilibrium will all choose route* r_2, *resulting to a Social Cost equal to* $1 + \varepsilon \approx 1$, *for arbitrarily small* $\varepsilon > 0$. *On the other hand, the social optimum is achieved by allocating the flow as follows:* $\approx 1/2$ *through route* r_1 *and* $\approx 1/2$ *through route* r_2*; so,* $SC(f_{opt}) \approx 3/4$, *and* $PoA \approx 4/3$.

Now, consider the actual game Γ^0 *as depicted in Figure 2a and the game* Γ^1 *as depicted in Figure 2b. Also, assume the misinformation game* $m\Gamma = \langle \Gamma^0, \Gamma^1, \Gamma^2 \rangle$ *with* θ *splitting, where* $\Gamma^0 = \langle G, l, s, t, r \rangle$, $\Gamma^1 = \langle G, l^1, s, t, r^1 \rangle$, $\Gamma^2 = \langle G, l^2, s, t, r^2 \rangle$, *and* $\theta = \langle \theta^1, \theta^2 \rangle$. *Further,* $\theta^1 = 2/3$ *of the players have the view* Γ^1, *and the rest the view* Γ^2. *In this example,* $l^2 = l^0 = l$, $l_1^1(x) = \varepsilon x + 1$, *and* $l_2^1(x) = x + 1$. *The equilibrium for* Γ^1, Γ^2 *is to choose the route* r_1, r_2 *respectively, leading to* $SC_{NME}(f_{NME}) \approx 7/9$ *in the* $m\Gamma$*; so* $PoM \approx 28/27 < PoA$. *Thus, the social outcome is improved, despite selfishness and misinformation.*

A metric similar to *PoM* was introduced in (Meir and Parkes, 2015b), called the *Biased Price of Anarchy*, which measures the ratio of the equilibrium under biases in knowledge compared to the optimal outcome. In this concept, all players play a game with modified costs and, thus, (possibly) different than the actual costs. In our concept, all players play a game according to the misinformation that they assume that is the same for anyone resulting to different outcomes, so in general the two concepts *PoM* and *Biased Price of Anarchy* are not equal. The following example clarifies the differences between the two studies:

Example 2. *Consider now the game as provided in Figure 2a, where* $1/2$ *of the players knows the latency functions:* $l_1(x) = x$ *and* $l_2(x) = 1/4$, *whereas the rest knows* $l_1(x) = x + 1$ *and* $l_2(x) = 1$, *for routes* r_1 *and* r_2 *respectively. The equilibrium point provided by (Meir and Parkes, 2015a) is that the flow is splitted into two equal halfs, each routed through different edge.*

On the other hand, the misinformation non-atomic game that is produced takes the form $m\Gamma = \langle \Gamma^0, \Gamma^1, \Gamma^2 \rangle$ *with* θ *splitting,* $\theta = \langle 1/2, 1/2 \rangle$. *The flow in* Γ^1 *has Nash equilibrium such that* $1/4$ *is routed through* r_1, *while* $3/4$ *is routed through* r_2. *Similarly, in* Γ^2, *the Nash equilibrium is such that all the flow is routed through* r_2. *Therefore, the resulting natural misinformed equilibrium is* $1/8$ *routed through* r_1, *and* $7/8$ *routed through* r_2.

5 THE WATERFILLING ALGORITHM

Here, we provide an algorithm that computes a *pure Nash equilibrium* in a single-commodity non-atomic congestion game with n parallel links, and affine latency functions, inspired by the waterfilling theorem of Information theory (Cover and Thomas, 2006; Fasoulakis et al., 2019). To our best knowledge, there is not a similar approach in the literature.

One of the fundamental problems in wireless communications is how to allocate a budget of power in a constant number of different quality (different noise levels) and independent wireless communication channels in order to maximize the sum of the transmission rate. The optimal solution of this problem is given by the *waterfilling theorem*. Namely, the algorithm fills with water (power) the channels in a way that minimizes the maximum level of water, where the level of water is the maximum value of power plus noise in the channels that are used. By the end of the algorithm, the noise plus the water in the channels that are used is the same (see Figure 3)

Interestingly, this idea can be used to find a *Nash equilibrium* flow allocation in a single-commodity non-atomic congestion game with n parallel links and affine latency functions.

Algorithm 1: Waterfilling approach algorithm for computing a pure Nash equilibrium in single-commodity non-atomic congestion games with n parallel links and affine latency functions.

Input: n parallel links with affine latency functions $a_k x_k + b_k$, with $a_k, b_k > 0$, $\forall\, k \in \{1, 2, \ldots, n\}$.
Output: A pure Nash equilibrium allocation.

Sort links in an increasing order based on b_k.
for $1 \leq i \leq n$ **do**
  Solve the Linear program (Algorithm 2), for $t = b_{i+1}$ and $j = i$. If it returns a feasible solution x^*, then STOP and return x^*.

By the definition of Nash equilibrium, we know that there is at least one Nash equilibrium in which all links that are used have the same latency, $v = a_i x_i + b_i$, and any link k that is not used has a latency no less than v, or in other words $a_k \cdot 0 + b_k \geq v$. However, we do not know *a priori* the value of v, but we do know that the possible values are between the intervals of $[b_1, b_2], (b_2, b_3], \ldots, (b_{n-1}, b_n], (b_n, +\infty)$, since b_is are sorted in an increasing order. Note that if the optimal threshold is in interval $[b_{k-1}, b_k]$, then in the Nash equilibrium allocation we will have exactly $k-1$ links. We exhaustively search the optimal values

(a) Players without misinformation. (b) Misinformed players.

Figure 2: Non-atomic congestion game with 2 parallel links.

in an increasing order for any possible interval, see Algorithm 1. For any interval we solve a linear program to check if there is an allocation with the Nash equilibrium properties, if there is such an allocation we return it. Since, we know the existence of such an equilibrium, our algorithm always returns a feasible solution at the end. It is easy to see that the total computational time of the algorithm is polynomial.

Algorithm 2: Linear program.

Input: A positive threshold t and an index j.
Output: x.

minimize v

$$s.t.\ \ x_i = \frac{v-b_i}{a_i},\ \text{for } i \leq j.$$
$$\textstyle\sum_{i\in[n]} x_i = r.$$
$$x_i \geq 0, v \geq 0 \text{ and } v \leq t.$$

6 COORDINATION MECHANISMS WITH MISINFORMATION

Now we focus on single-commodity misinformation non-atomic congestion games, where the actual game has n parallel links and affine latency functions. We restate the main question of the paper, that is how we can use misinformation to improve the performance of single-commodity non-atomic congestion games with n parallel links in terms of Social Cost.

Specifically, note that if we properly change the coefficients of the latency functions of the misinformed games then the flow according to the worst natural misinformed equilibrium will change. To that direction we choose to increase the coefficients of the latency function from a_k, b_k to $\hat{a}_k^j, \hat{b}_k^j$ (one for each different subjective view Γ^j respectively). Further, we assume that the designer has the constraint that he can provide a limited number of misinformed views.

We will show that it is always possible to find a unique natural misinformed equilibrium that coincides with the optimal allocation, in terms of social welfare, by appropriately changing the coefficients. The constructed misinformation game $m\Gamma = \langle \Gamma^0, \Gamma^1, \ldots, \Gamma^N \rangle$, with θ splitting, described in the next Subsection, has the following properties:

i) $\Gamma^0 = \Gamma$ (the case of n parallel links),

ii) $\Gamma^0 = \langle G, l, s, t, 1 \rangle$, $\Gamma^j = \langle G, l^j, s, t, 1 \rangle$, where $l_k(x_k) = a_k x_k + b_k$, $l_k^j(x_k) = \hat{a}_k^j x_k + \hat{b}_k^j$.

iii) $\theta = \langle \theta^1, \ldots, \theta^N \rangle$.

Next, we provide a methodology to construct a misinformation game so that the unique natural misinformed equilibrium in the misinformation game is an optimal allocation of the actual game. Forthwith, we give an algorithm that takes as input a single-commodity non-atomic congestion game Γ^0, the optimal allocation in Γ^0, and an arbitrary break down (partition) of the links that are used in the optimal flow. Let $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ be the optimal solution, which we can easily find in polynomial time as a minimization of a convex function, and an abstract partition $(k_1, \ldots, k_m)$ of the allocation x^* over the parallel links that are used; m is the number of misinformed views that the designer provides to the players. E.g., if $n = 3$ with $x_i^* > 0\ \forall\, i$, then $k_1 = \{1, 2\}$, $k_3 = \{3\}$ is a possible partition. With Algorithm 3 we construct a misinformation game $m\Gamma$, where players perform optimally in terms of Social Cost.

To produce $m\Gamma$ in Algorithm 3, we call Algorithm 4 to compute the coefficients for the latency functions for each Γ^i separately. Afterwards, $m\Gamma$ is entailed easily.

At the beginning of Algorithm 4, we initialize v by setting it equal to the maximum of the costs of the latency functions over the links that are used in the allocation y^*. Then, we increase the b_i for the unused links in y^* in order to make them no less than the cost v. For any link i that is used in the allocation y^* we can increase the b_i in such a way that the cost of this link with the allocation y_i^* is equal to the cost v. This procedure can be done in polynomial-time by solving a system of linear inequalities. For any y^* it is easy to see that Algorithm 1 gives a unique pure Nash equilibrium in the modified game for the players that have this view. Taking the natural misinformed equilibrium

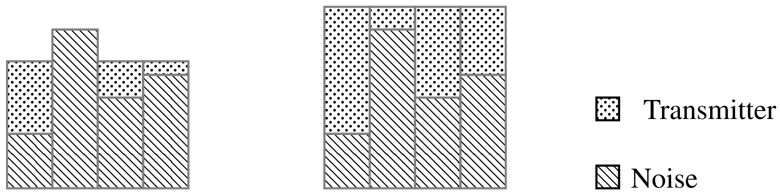

Figure 3: Two possible cases of waterfilling with different budget of power.

Algorithm 3: Coordination mechanism algorithm for an abstract partition $(k_1,\ldots,k_m)$ of n.

Input : An actual game $\Gamma^0 = \langle G, l^j, s, t, 1\rangle$
An optimal allocation $x^* = (x_1^*,\ldots,x_n^*)$
A partition over the links, $(k_1,\ldots,k_m)$.
Output: A misinformation game $m\Gamma$.
A splitting θ.

while $1 \le i \le m$ **do**
New allocation y^*:
$$y_j^* = \begin{cases} \frac{x_j^*}{\sum_{t\in\{k_i\}} x_t^*}, & j \in k_i \\ 0, & \text{elsewhere} \end{cases}$$
Apply Algorithm 4 for y^* to construct the latency functions of Γ^i.
$\theta^i = \sum_{j\in\{k_i\}} x_j^*$.
$m\Gamma \leftarrow \langle\Gamma^0,\Gamma^1,\ldots,\Gamma^m\rangle$.
$\theta \leftarrow \langle\theta^1,\ldots,\theta^m\rangle$.

Algorithm 4: Coordination mechanism algorithm.

Input : Coefficients a_i, b_i of latency functions for any link i. The allocation y^*.
Output: New coefficients of latency functions.

Put $v = \max_{i:y_i^*>0}\{a_i y_i^* + b_i\}$.

Find $\hat{b}_i$, $\forall\, i$,

$$\begin{aligned} s.t.\ \ & \hat{b}_i \ge v,\ \forall\, i \text{ such that } y_i^* = 0, \\ & a_i y_i^* = v - \hat{b}_i,\ \forall\, i \text{ such that } y_i^* > 0, \\ & \hat{b}_i \ge b_i,\ \forall\, i. \end{aligned}$$

return $\hat{b}$.

we construct the allocation x^*, which is the optimal allocation of the actual game, hence $PoM = 1$.

A similar mechanism can be used when the designer can influence only some of the players, i.e., when she can construct a mechanism where the θ^1 portion gets misinformed, whereas the rest use the actual game. In this scenario, the resulting misinformation game would be of the form $m\Gamma = \langle\Gamma^0,\Gamma^1,\Gamma^2\rangle$, where $\Gamma^2 = \Gamma^0$, for the splitting $\theta = \langle\theta^1,\theta^2\rangle$.

To do so, we reconsider the optimal allocation for the θ^1 fragment of the flow, taking into account the fact that there is a fixed part of the players θ^2 who will route according to Γ^2, which has the same latency functions as Γ^0. We reconsider the coefficients of the latency functions as they experience the additional cost of the Nash equilibrium flow of the unmisinformed fragment θ^2. Afterwards, we implement our mechanism and get the desired $m\Gamma$. Note that, as we can affect only some of the players, the rest would route (maybe sub-optimally) according to the actual specifications, so it may happen that $PoM > 1$.

7 CONCLUSIONS

In this paper, we explored the use of *misinformation* as a novel method for coordination mechanisms. We applied this idea in single-commodity non-atomic congestion games with parallel links and affine latency functions. Our goal was to steer players' behaviour towards the socially optimum allocation, by misinforming them with regards to the latency functions of the network. Towards this, we provided two polynomial-time algorithms. The first finds a Nash equilibrium flow allocation in a single-commodity non-atomic congestion game with n parallel links and affine cost functions. The second takes as input an abstract partition over the links that are used in the optimal allocation, and creates a misinformation game whose subjective games follow the required specification, and, thus, its natural misinformed equilibrium is the optimum allocation in the actual game. Further, we redefine misinformation games (originally proposed for normal-form games (Varsos et al., 2021)) for non-atomic congestion games and recast all relevant game-theoretic concepts for the new setting.

A future step is to design a mechanism for serial-parallel networks and general latency functions.

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