

Modeling the Cabin Capacity Allocation Problem in the Cruise Industry: An Italian Case Study

Giusy Macrina¹^a, Francesca Guerriero¹^b and Luigi Di Puglia Pugliese²^c

¹Dipartimento di Ingegneria Meccanica, Energetica e Gestionale, Università della Calabria, 87036, Rende, Italy

²Istituto di Calcolo e Reti ad Alte Prestazioni, Consiglio Nazionale delle Ricerche, 87036, Rende, Italy

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Abstract: In this paper we present several optimization models for cruise cabin capacity allocation. In particular, we address the problem of managing the booking requests for a set of cabins with different type and price in a cruise ship. We formulate three models, considering several features of the problem such as: the limited number of bookable places on the ship, different planning and operation horizon, the possibility to postpone a departure or to apply special offers. Then, we present an Italian case study and we analyze the impact of different strategies on the revenues achievable by the company.

1 INTRODUCTION

Since 1980, the cruise industry has grown rapidly worldwide, with an impressive annual rate of 8.4%. Between 2009 and 2019, the number of cruise passengers increased from 17.8 millions in 2009 to 30 millions in 2019. In 2017 the cruise sector contributed 114 billion euros to the global economy, counting 28.5 millions of passengers (www.cruising.org). The cruise sector is expected to grow more, in fact, the occupancy rate of cabins is very high compared to other tourism sectors, such as the hotels. It is important to note that this parameter plays a crucial role. In fact, if in other sectors, such as the hotels or the airlines, having an occupancy rate equals to 70% indicates a success, in a cruise ship this rate must be around 95% or more. Thus, a cruise company always tries to complete the booking for a ship, applying discounts or promotions, avoiding empty cabins during the trips. The main reason is that the revenues of a cruise are not only related to the tickets, but also to several services offered on-board, such as excursions, photo books and other entertainment activities. Hence, choosing a strategy for maximizing the overall revenues is a very challenging task. This makes the cruise sector an interesting and profitable area for applying revenue management techniques.

Revenue management methods are useful tools for helping companies in finding profitable policies to allocate their limited resources to different customer segments, in a given planning horizon (Klein et al., 2020). Originating in the airline industry in the 1970s, these techniques have been extended to many industries, we cite for example restaurants (Saito et al., 2019), (Guerriero et al., 2014), railway (Meissner and Strauss, 2010), car rental (Guerriero and Olivito, 2014), (Oliveira et al., 2017). However, the scientific production related to the application of revenue management techniques in the cruise industry is not very extensive (Sturm D., 2018). Only few contributions, addressing real-case applications of revenue management methods in this sector, have been published. The paper of (Ladany and Arbel, 1992) represents one of the first work addressing pricing strategies for passenger cabins on cruise-liners. (Biehn, 2006) emphasized that the cruise ships are not floating hotels, and explained why common hotel revenue management methods cannot be applied to the cruise sector. He focused on several critical factors such as: pricing, multiple capacity limitations, booking periods, the huge number of cabin categories and the trips extensions (i.e., additional services offered to customers for extending the base itinerary). (Mad-dah et al., 2010) proposed a dynamic model to handle multi-dimensional cabin category and lifeboat capacity constraints, and to consider the uncertainty of demand by assuming that customers arrive according to a discrete-time stochastic process. (Li et al., 2014)

^a <https://orcid.org/0000-0001-6762-3622>

^b <https://orcid.org/0000-0002-3887-1317>

^c <https://orcid.org/0000-0002-6895-1457>

modeled a problem which considers pricing and room assignment as well as the potential on-board expenses of customers. (Sturm and Fischer, 2019) extended the work of (Li et al., 2014) taking into account additional issues, such as the interdependence of booking request components, i.e., inseparable group arrivals, and exploiting the possibility to use connected cabins. (Li, 2014) considered the risk of cancellation and focused on the possibility to accept overbooking requests. Hence, he applied a real options approach to formulate a risk decision model for cruise line dynamic overbooking. (Ayvaz-Cavdaroglu et al., 2019) developed a pricing approach focusing on the customer habits and three main features of cruise industry: the long booking period, the restriction on the price variations from week to week, and the effect of promotion expense decisions on the total revenues of the company.

Contribution and Organization of the Work. In this work we present three mathematical formulations, of incremental complexity, for the cabin capacity allocation problem in the cruise industry. The first model is a basic problem, in which booking requests are accepted until a maximum capacity is reached, then, they are rejected. In the second and third models we consider the possibility to offer customers a postponed departure time at the same price, in case all the cabins of the requested type are all booked for the selected data. The main difference between these last two models is that in the second one, we suppose that all the customers accept the postponement, while in the third one we consider also the possibility of rejection. This specific feature (i.e., postponement of the departure date) has not been considered in the scientific contributions published on the same topic so far. It is worth observing that the proposed models can be used to evaluate booking limits and thus the obtained solutions can support the decision maker in accepting or denying arriving booking requests, when booking limit revenue management policies are implemented.

The behaviour of the proposed models are evaluated empirically on realistic data related to the Italian cruise line: Costa Crociere. In particular, we investigate how different price strategies may influence the achievable revenues. The rest of the paper is organized as follows: in Section 2 we describe the proposed models. In Section 3 we describe the real case study and we discuss on the results obtained by testing the proposed models, on realistic data and considering different pricing schemes. In Section 4 we summarize the conclusions of our work.

2 MATHEMATICAL PROGRAMMING MODELS

In this section we present the proposed models, aimed at allocating the cruise cabin capacity, in such a way to maximize the revenue. In particular, we present three mathematical formulations, we describe the objective functions and constraints of each model and we highlight their most important features and limits.

2.1 First Mathematical Model

Let $k = 1 \dots \bar{K}$ indicate the cabins type available for the booking. In each time period $t = 1 \dots T$ of the booking horizon, a customer may book a cabin of type k . Let $\bar{t} = 1 \dots \bar{T}$ be the operational horizon, i.e. the period of time where the ships embark the passengers. It is worth noting that the boarding/landing operations are not scheduled each day of the operation horizon, hence, let $\Omega \subseteq \bar{T}$ be the subset of days in which a ship embarks the passengers (i.e., boarding/landing operations are allowed). In other words, we may introduce a binary vector H of size \bar{T} which refers to the ports where the ships stop and passengers may get on/off. The generic element $h_{\bar{t}}$ belonging to the vector H is equal to one if during the day \bar{t} the boarding/landings operations are allowed, zero otherwise. Hence, we may set $\Omega = \{\bar{t} : h_{\bar{t}} = 1\}$.

The booking horizon is defined such that the last possible day of booking is the day before the starting of the operational horizon, i.e., the day before the start date of the cruise.

On the other hand, each cabin can be reserved up to F days before departure. Thus, a customer, who wants to depart at time \bar{t} , can book a cabin at any instant of time t belonging the set $L(\bar{t}) = \{t | \bar{t} - F \leq t \leq \bar{t} - 1\}$.

Each cruise trip has a duration of α days. Let $p_{kt}^{\bar{t}}$ be the price at time t for booking a cabin of type k for the departure time \bar{t} , while $d_{kt}^{\bar{t}}$ be the requests received at time t for booking a cabin of type k for the departure time \bar{t} . A ship has a limited number of cabins of type k , indicated as C_k .

Let $x_{kt}^{\bar{t}}$ be an integer decision variable that represents the number of accepted booking requests for cabins of type k arrived at time t , for the period \bar{t} , $\forall \bar{t} \in \Omega; k \in \bar{K}, t \in L(\bar{t})$.

Using the notation introduced above, the first formulation takes the following form.

$$\text{Max} \sum_{k=1}^{\bar{K}} \sum_{\bar{t} \in \Omega} \sum_{t \in L(\bar{t})} p_{kt}^{\bar{t}} x_{kt}^{\bar{t}} \tag{1}$$

$$x_{kt}^j \leq d_{kt}^j \quad \bar{t} \in \Omega, k = 1 \dots \bar{K}, t \in L(\bar{t}) \quad (2)$$

$$\sum_{\tau=\bar{t}-\alpha+1}^{\bar{t}+\alpha-1} \sum_{s=1}^{t-1} x_{ks}^\tau + \sum_{\tau=\bar{t}-\alpha+1}^{\bar{t}+\alpha-1} x_{kt}^\tau \leq C_k \quad \bar{t} \in \Omega, k = 1 \dots \bar{K}, t \in L(\bar{t}) \quad (3)$$

$$x_{kt}^j \geq 0, \text{integer} \quad \bar{t} \in \Omega, k = 1 \dots \bar{K}, t \in L(\bar{t}) \quad (4)$$

The objective function (1) maximizes the total revenue. Constraints (2) guarantee that the number of accepted booking requests does not exceed the demand. Ships capacity constraints are modeled by equations (3), finally; constraints (4) define the variables domain.

It is worth noting that this model can be easily decomposed in \bar{K} smaller sub-problems, simpler to be solved. In fact, since the requests arrive for single type of cabins and they are not related each others, the total number of cabins of each type k may be considered as a single ship. Hence, for modelling the sub-problem, we may modify the quantities defined for the previous model as follows: let d_{kt}^j the booking requests of cabins received at time t for the time departure \bar{t} , and p_t^j be the price for booking a cabin at time t for the time departure \bar{t} . C is the maximum capacity of the ship. Let x_t^j be an integer decision variable that represents the number of accepted booking requests for cabins arrived at time t , starting from the period \bar{t} , $\forall \bar{t} \in \Omega; k \in \bar{K}, t \in L(\bar{t})$. We named this requests regular requests. A sub-problem for a ship composed of cabins of type k , which maintains the same objective and constraints of the previous one, can be modelled as follows:

$$\text{Max} \quad \sum_{\bar{t} \in \Omega} \sum_{t \in L(\bar{t})} p_t^j x_t^j \quad (5)$$

$$x_t^j \leq d_{kt}^j \quad \bar{t} \in \Omega, t \in L(\bar{t}) \quad (6)$$

$$\sum_{\tau=\bar{t}-\alpha+1}^{\bar{t}+\alpha-1} \sum_{s=1}^{t-1} x_s^\tau + \sum_{\tau=\bar{t}-\alpha+1}^{\bar{t}+\alpha-1} x_t^\tau \leq C \quad \bar{t} \in \Omega, t \in L(\bar{t}) \quad (7)$$

$$x_t^j \geq 0, \text{integer} \quad \bar{t} \in \Omega, t \in L(\bar{t}) \quad (8)$$

2.2 Second Mathematical Model

The model proposed in Section 2.1 is a basic version in which the booking requests for cabins of type k are accepted until the maximum number of available cabins in the ship, for the departure time \bar{t} , is reached. The other requests are rejected. However, adopting this strategy could be not profitable, since the possibility to offer alternative travel options to the customer is not taken into account. Sometime, some customers could be interested in booking a cruise, but their requests cannot be accepted due to the unavailability of cabins at departure time \bar{t} . Hence, an interesting strategy is to propose to customers who try to book a cabin of type k for the departure time \bar{t} , another

cabin of the same type k but in another departure time indicated as α , in the same port, maintaining the same price proposed at time t . In fact, since α is the duration of the trip, the same tour of a cruise ship starts each α days, in the same port.

To model this possibility, we need to introduce a new set of variables y_{kt}^j , $\bar{t} \in \Omega, k = 1 \dots \bar{K}, t = 1 \dots L(\bar{t})$, that represent the number of accepted requests of cabins of type k for the departure time $(\bar{t} - \alpha)$ that will be scheduled at departure time \bar{t} . We referred to this type of requests as promo requests. The related problem can be represented mathematically as follows.

$$\text{Max} \quad \sum_{k=1}^{\bar{K}} \sum_{\bar{t} \in \Omega} \sum_{t \in L(\bar{t})} p_t^j (x_{kt}^j + y_{kt}^{\bar{t}+\alpha}) \quad (9)$$

$$x_{kt}^j \leq d_{kt}^j \quad \bar{t} \in \Omega, k = 1 \dots \bar{K}, t \in L(\bar{t}) \quad (10)$$

$$\sum_{\tau=\bar{t}-\alpha+1}^{\bar{t}+\alpha-1} \sum_{s=1}^{t-1} (x_{ks}^\tau + y_{ks}^\tau) + \sum_{\tau=\bar{t}-\alpha+1}^{\bar{t}+\alpha-1} (x_{kt}^\tau + y_{kt}^\tau) \leq C_k \quad \bar{t} \in \Omega, k = 1 \dots \bar{K}, t \in L(\bar{t}) \quad (11)$$

$$y_{kt}^j \leq d_{kt}^{\bar{t}-\alpha} - x_{kt}^{\bar{t}-\alpha} \quad \bar{t} \in \Omega, k = 1 \dots \bar{K}, t \in L(\bar{t}) \quad (12)$$

$$x_{kt}^j \geq 0, \text{integer} \quad \bar{t} \in \Omega, k = 1 \dots \bar{K}, t \in L(\bar{t}) \quad (13)$$

$$y_{kt}^j \geq 0, \text{integer} \quad \bar{t} \in \Omega, k = 1 \dots \bar{K}, t \in L(\bar{t}) \quad (14)$$

The objective function (9) maximizes the total revenue, obtained by accepting the regular and the promo requests. Constraints (10) are the same as the conditions (2). Equations (11) model the capacity constraints, taking into account both the regular requests and the promo ones. Constraints (12) guarantee that the number of accepted promo requests for the departure time \bar{t} and the cabin type k , does not exceed the demand, considering also the regular requests. Constraints (13) and (14) define the domain of variables.

As for the model (1)– (4) presented in Section 2.1, we can decompose this model in \bar{K} sub-problems, one for each type of cabin k . With this purpose, we modify the parameters and the variables by removing the index related to the type of cabin k , maintaining the same objective function and constraints. Hence, for each type of cabin, we model and solve the following optimization problem, which is easier to be solved than the mathematical formulation (9) – (14).

$$\text{Max} \quad \sum_{\bar{t} \in \Omega} \sum_{t \in L(\bar{t})} p_t^j (x_t^j + y_t^{\bar{t}+\alpha}) \quad (15)$$

$$x_t^j \leq d_t^j \quad \bar{t} \in \Omega, t \in L(\bar{t}) \quad (16)$$

$$\sum_{\tau=\bar{t}-\alpha+1}^{\bar{t}+\alpha-1} \sum_{s=1}^{t-1} (x_s^\tau + y_s^\tau) + \sum_{\tau=\bar{t}-\alpha+1}^{\bar{t}+\alpha-1} (x_t^\tau + y_t^\tau) \leq C$$

$$\sum_{\tau=\bar{t}-\alpha+1}^{\bar{t}+\alpha-1} (x_{\tau}^{\bar{t}} + y_{\tau}^{\bar{t}}) \leq C \quad \bar{t} \in \Omega, t \in L(\bar{t}) \quad (17)$$

$$y_{\tau}^{\bar{t}} \leq d_{\tau}^{\bar{t}-\alpha} - x_{\tau}^{\bar{t}-\alpha} \quad \bar{t} \in \Omega, t \in L(\bar{t}) \quad (18)$$

$$x_{\tau}^{\bar{t}} \geq 0, integer \quad \bar{t} \in \Omega, t \in L(\bar{t}) \quad (19)$$

$$y_{\tau}^{\bar{t}} \geq 0, integer \quad \bar{t} \in \Omega, t \in L(\bar{t}) \quad (20)$$

2.3 Third Mathematical Model

The main limit of the model proposed in Section 2.2 is that it assumes a customer will accept to book a cabin even if the departure will be postponed of α days. Actually, a customer could not accept the departure postponement, hence, taking into account the possibility of rejection is a critical issue. To model this possibility, we introduce the customer's probability of acceptance. Let $a_{kt}^{\bar{t}}$ be a binary parameter that will be equal to one if the probability of acceptance is larger than P , zero otherwise for each $k = 1, \dots, \bar{K}$, $t = 1 \dots L(\bar{t})$, $\bar{t} \in \Omega$.

Hence, we formulate the problem as follows:

$$\text{Max} \quad \sum_{k=1}^{\bar{K}} \sum_{\bar{t} \in \Omega} \sum_{t \in L(\bar{t})} p_{kt}^{\bar{t}} (x_{kt}^{\bar{t}} + a_{kt}^{\bar{t}} y_{kt}^{\bar{t}+\alpha}) \quad (21)$$

$$x_{kt}^{\bar{t}} \leq d_{kt}^{\bar{t}} \quad \bar{t} \in \Omega, k = 1 \dots \bar{K}, t \in L(\bar{t}) \quad (22)$$

$$\sum_{\tau=\bar{t}-\alpha+1}^{\bar{t}+\alpha-1} \sum_{s=1}^{t-1} (x_{ks}^{\bar{t}} + a_{ks}^{\bar{t}} y_{ks}^{\bar{t}}) + \sum_{\tau=\bar{t}-\alpha+1}^{\bar{t}+\alpha-1} (x_{kt}^{\bar{t}} + a_{kt}^{\bar{t}} y_{kt}^{\bar{t}}) \leq C_k \quad \bar{t} \in \Omega, k = 1, \dots, \bar{K}, t \in L(\bar{t}) \quad (23)$$

$$a_{kt}^{\bar{t}} y_{kt}^{\bar{t}} \leq d_{kt}^{\bar{t}-\alpha} - x_{kt}^{\bar{t}-\alpha} \quad \bar{t} \in \Omega, k = 1, \dots, \bar{K}, t \in L(\bar{t}) \quad (24)$$

$$x_{kt}^{\bar{t}} \geq 0, integer \quad \bar{t} \in \Omega, k = 1, \dots, \bar{K}, t \in L(\bar{t}) \quad (25)$$

$$y_{kt}^{\bar{t}} \geq 0, integer \quad \bar{t} \in \Omega, k = 1, \dots, \bar{K}, t \in L(\bar{t}) \quad (26)$$

The objective function (21) maximizes the revenue, while constraints (22) – (26) are similar to (10) – (14). This model, as the previous ones, can be decomposed in easier sub-problems. However, the main limit of this model is that the demand is considered an aggregate data. Hence, it supposes that all the customers behave similarly.

3 COMPUTATIONAL RESULTS

In this section we describe our computational study and analyze the obtained results. In particular, we firstly describe the case study features. Then, we discuss on the experimental results obtained by applying the different models presented in Section 2, by highlighting the impact of the use of these models in terms of achievable revenue.

3.1 The Italian Case Study

In this section, we present the main features of the realistic case study used in the computational phase. We have considered the Italian cruise line: Costa Crociere. Thus the cabin capacities, the cabin types and the the tariff segments are derived from the web-site of the considered cruise line (<https://www.costacruises.com/>), whereas the cabin rates are obtained from the data published by the on-line travel agency Logitravel (www.logitravel.it).

Some of the available data have been slightly modified in order to fit with the models assumptions. In particular, we consider only three categories of cabins, i.e., low that includes the intern cabins, medium that contains both ocean and balcony, and high that refers to the suite. Thus, capacities and prices have been appropriately defined (see Tables 1–3).

We have chosen the itinerary "Marseille - Barcelone - Palma de Maiorca - Cagliari - Civitavecchia (Rome)" depicted in Figure 1. It is worth observing, that not all the cities are embarkation ports. In fact, it is possible to start the cruise only from Savona, Cagliari and Rome.



Figure 1: Cruise itinerary.

The ship which performs this trip, named "Costa Diadema" (see Figure 2), is one of the largest in the fleet of Costa Crociere. In fact, it can host 4947 passengers and 1253 crew members, and also has an outside promenade extending over 500 m. There are several on-board activities in Costa Diadema: 4D cinemas, lively bars, gourmet restaurants and the spa area.



Figure 2: Costa Diadema (www.costacruises.com).

Costa Diadema has 1862 cabins of three types: Inside (i.e., the most economic), Balcony and Ocean view, Suite and Samsara. We will refer to these classes as: low, medium and high, respectively. Table 1 summarizes the capacities, i.e., the number of available places, of each type of cabin.

Table 1: Capacity of the cabins classified by type.

Type	Low	Medium	High
Capacity	701	1086	75

The cruise starts its journey in June and repeats its tour until the end of September. It is 7-days long and it visits the same port each week in the same day. The first city, visited on Monday, is Savona, then Marseille in Tuesday and so on. We have considered data referring to 2018, thus the first possible departure was on June the 2nd. For our study, we have considered two weeks of service, thus 14 days when it is possible to embark on the cruise and 21 days of service.

Prices. The cabins rates vary depending on the departure date and the type of cabins, Table 2 reports the prices related to the period 02/06 – 30/06 of the year 2018.

Table 2: Prices of the cabins for the year 2018 (www.logitravel.it).

Type	02/06	09/06	16/06	23/06	30/06
Intern	€669	€669	€669	€749	€749
Ocean	€839	€869	€869	€919	€929
Balcony	€949	€949	€949	€998	€1,018
Suite	€1,194	€1,194	€1,194	€1,243	€1,263

In our experimental study, we assume that the price is influenced by the time of booking, the departure date and the availability of the cabins in the ship. Hence, for representing the variation of price, we follow the idea considered by (Joshi, 2004). At first, we evaluated a maximum and a minimum price for each cabin type, denoted as p_{max} and p_{min} , respectively and summarized in Table 3.

Table 3: Maximum and minimum prices for each type of cabin.

Type	Low	Medium	High
p_{min}	€669	€839	€1,194
p_{max}	€749	€1,018	€1,263

Then, the first strategy we consider for incentivizing customers to buy, is related to the time remaining before the departure. Hence, we apply the lowest fare if the requests arrive very early, i.e., when the ship is almost empty. On the contrary, the fares will be increased if the booking requests arrive close to the departure time, i.e., when only few cabins are avail-

able and the customers are willing to pay the most extensive price for booking a cabin. To calculate the corresponding fare p_{ask} (i.e., the price offered to the customers), we use the next linear equation (27) proposed by (Joshi, 2004):

$$p_{ask}^a = p_{max} - t_r j \quad (27)$$

where t_r is the remaining time that is the time left for the cruise to start and j is a normalizing constant defined in such way that p_{ask}^a will be p_{min} , when t_r is equal to the maximum number of days before the departure in which the booking is possible.

The second strategy is related to the number of available cabins. Hence, we calculate the price as follows:

$$p_{ask}^b = p_{max} - c_r k \quad (28)$$

where c_r is the number of remaining cabins and k is a normalizing constant such that p_{ask}^b will be p_{min} , when c_r is equal to the maximum number of available cabins.

The last strategy is a hybrid approach, which takes into account both time of booking and the available cabins. The p_{ask}^c is calculated as follows:

$$p_{ask}^c = p_{max} - (t_r j) - (c_r k) \quad (29)$$

where j and k are normalizing constants such that p_{ask}^c is p_{max} , when t_r is equal to the booking period and c_r is close to zero.

Demands. To calculate the demand, we consider a linear demand function (Cohen et al., 2015) denoted by:

$$Q(p) = A - Bp \quad (30)$$

We use the price elasticity of demand to calculate the parameters A and B. The demand curve is high elastic for luxury goods and we can state that the higher the price the higher the elasticity. We use a linear regression and calculate the curve demand through Microsoft Excel. The equations are depicted in Tab 4, where y represents $Q(p)$ and x is p .

Table 4: Demands equations for each type of cabin.

Type	elasticity	equation
low	7	$y = -0.2857x + 221.43$
medium	15	$y = -0.7778x + 765.22$
high	20	$y = -0.3347x + 420$

3.2 Results

We now describe the results obtained by solving the three mathematical models proposed in Section 2. We

use the Excel solver to find the solutions and we analyse the results discussing the obtained revenues as well as the occupancy of the cabins, considering the 21 days of service.

First Mathematical Model. We firstly analyse the results obtained for the basic mathematical model represented by equations (1) – (4). We use the decomposed formulation (5)– (8), hence, we solve three sub-problems, one for each type of cabin. We consider four scenarios, by varying the prices p : 1) $p =$ fixed price, 2) $p = p_{ask}^a$, 3) $p = p_{ask}^b$ and 4) $p = p_{ask}^c$.

The first scenario is the most improbable, however, we want to analyse the case in which the price is fixed, equal for each day of the time horizon.

Looking at results in Table 5, which summarizes the revenues for each type of cabin as well as the total revenue for the 21 days of service, it is evident that the medium” cabins are the most profitable.

Table 5: Cruise revenue solving the first mathematical model with fixed price.

type	low	medium	high	total
revenue (€)	892,386.00	1,880,706.00	20,980.00	2,976,072.00

This result, considering this setting, is obvious because of the higher number of available medium cabins. Looking at Figure 3 we can observe that overall, the highest numbers of reservations is concentrated in the central departure dates.

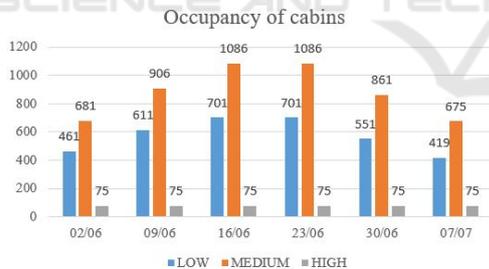


Figure 3: Occupancy of cabins, using the first mathematical model and a fixed price.

Then, we analyse the results obtained by varying the prices. Table 6 reports the parameters used in the computational experiments

Table 6: Parameters setting.

Type	Low	Medium	High
p_{min} (€)	669	839	1,194
p_{max} (€)	749	1,018	1,263
capacity	701	1086	75
j	2.150	3.210	3.500
k	0.002	0.020	0.320

We summarize the results in Table 7 which de-

picts the obtained revenues, for each price strategy and each type of cabins. Overall, the total revenues are less than that obtained with the fixed price, this numbers confirm that making a sales forecast using a fixed price leads to erroneous results, since using a fixed price is an impracticable strategy. Looking at Table 7 we may observe that the most profitable strategy is obtained by using p_{ask}^c . Hence, considering the price as a function of both booking time and available cabins is the most profitable approach.

Table 7: Cruise revenue solving the first model varying price strategy.

type	low	medium	high	total
revenue p_{ask}^a	603,730.00	993,900.00	182,460.00	1,780,063.00
(€) p_{ask}^b	566,647.00	1,008,569.00	182,138.00	1,757,354.00
p_{ask}^c	600,678.00	1,071,410.00	130,649.00	1,802,737.00

Figure 4 depicts the comparison of the cabins occupancy, for each departure date, varying p_{ask} . It is evident that p_{ask}^c is not only the price that allows to reach higher values of revenues, but also higher value of occupancy.

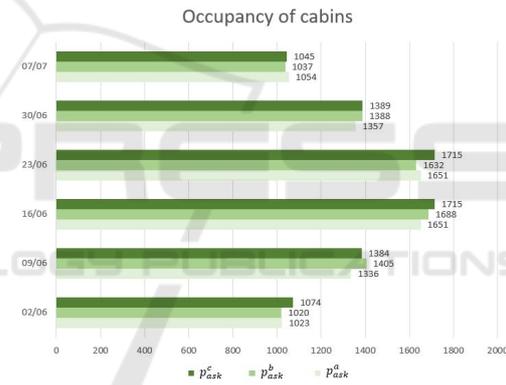


Figure 4: Occupancy of cabins, using the first mathematical model and varying p_{ask} .

Second and Third Mathematical Models. As for the first mathematical model, we fix the price and solve also the second mathematical model presented in Section 2 (i.e., (9) – (14)), that considers the possibility to postpone the departure time of one week. In this setting, we suppose that all the customers accept the postponement. We recall that the third mathematical model is an extension of the second one, in which the probability of acceptance is considered. Hence, we need to take into account the possibility that a customer accepts or not this postponement. For our computational study, we suppose that the probability of acceptance is about 70%. Table 8 summarizes the results for both the models considering a fixed price. From Table 8, it is evident that both the models provide the same revenue and the most profitable type cabins is once again the medium one.

Table 8: Cruise revenue solving the second and third mathematical models with fixed price.

	type model	low	medium	high	total
revenue	II	610,370.00	1,297,326.00	179,100.00	2,086,796.00
(€)	III	610,370.00	1,297,326.00	179,100.00	2,086,796.00

Figures 5 and 6 show the occupancy of cabins for the solutions obtained using the second and third models, respectively. Even if we obtain the same revenues, looking at Figures 5 and 6 we may see some differences in the configurations. In particular, focusing on the first departure date, using the second model the number of medium type cabins is higher than that used by solving the third model. On the contrary, the number of low cabins is lower.

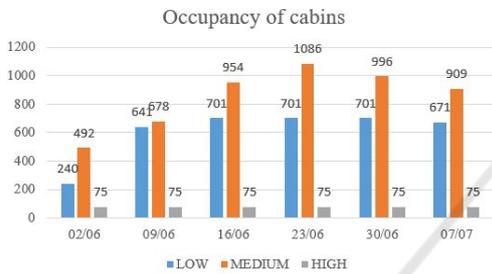


Figure 5: Occupancy of cabins, using the second mathematical model and a fixed price.

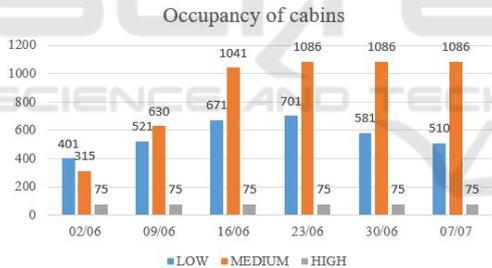


Figure 6: Occupancy of cabins, using the third model and a fixed price.

As for the first model, we analyse the results obtained by varying the prices. Table 9 reports the results obtained for the second and third mathematical model, respectively. Looking at Table 9 and focusing on low type cabin, the third model finds more effective solutions. As a matter of fact, overall the revenues are higher than those obtained with the second model. The revenues achieved for the medium and high types of cabins are similar, with the only exception of the medium type when considering p_{ask}^b , in that case, the third model finds more profitable solutions than the second one.

Figures 7 and 8 depict the overall occupancy of the cabins for the second and third mathematical model, respectively, by varying p_{ask} . Comparing these fig-

Table 9: Cruise revenue solving the second and third mathematical models varying price strategy.

		second			
revenue	type	low	medium	high	total
(€)	p_{ask}^a	579,735.00	943,484.00	183,237.00	1,706,456.00
	p_{ask}^b	546,678.00	998,892.00	182,220.00	1,727,790.00
	p_{ask}^c	559,054.00	882,467.00	182,974.00	1,624,495.00
		third			
(€)	p_{ask}^a	603,854.00	943,484.00	182,460.00	1,729,798.00
	p_{ask}^b	603,727.00	1,166,082.00	182,163.00	1,951,972.00
	p_{ask}^c	608,690.00	800,025.00	182,203.00	1,590,918.00

ures, it is easy to see that using p_{ask}^b , the third model finds more balanced solutions also in terms of occupancy of the cabins.

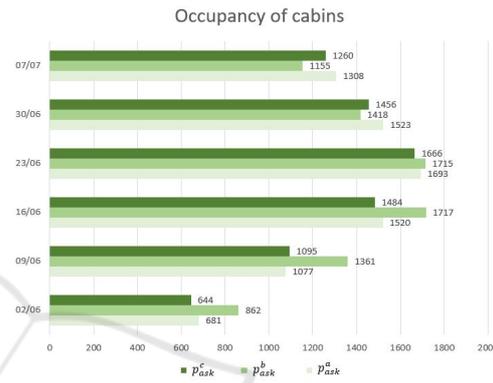


Figure 7: Occupancy of cabins, using the second mathematical model and varying p_{ask} .

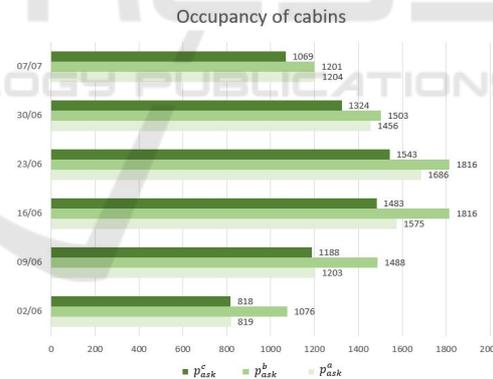


Figure 8: Occupancy of cabins, using the third mathematical model and varying p_{ask} .

4 CONCLUSIONS

In this work we presented three optimization models for the cabin allocation problem in the cruise industry. We assess the performance of the proposed models by considering realistic data, derived from an Italian cruise line (i.e., Costa Crociere). In the computational experiments we investigate the impact of different pricing schemes on the total revenues the

company can achieve. Some extensions to our work are possible. It could be interesting to develop sophisticated revenue management strategies to support cruises business, by optimizing the cabins allocation and boosting revenues growth. In particular, it could be interesting to define policies with upgrading, that allow to sell superior-type cabins to a lower price if some booking requests for cabins of a certain lower type cannot be accepted, because of the capacity constraints. In this case, the main decision is to accept the risk of selling a superior cabin at lower price, given that unknown but, probably, more profitable demand will arrive in the future. The development of buy-up policies represents another important topic for future investigation. It is important to note that, the implementation of revenue management policies requires the efficient solution of the cabin capacity allocation problem, studied in this paper. Thus it could be also interesting to deeply investigate the mathematical structure of the models proposed and to exploit the related features to improve the solution approaches.

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