Dynamic Modeling and Simulation of a Slurry Mixing and Pumping Process: An Industrial Case

Ridouane Oulhiq[1,2], Khalid Benjelloun[1], Maarouf Saad[3], Yassine Kali[3] and Laurent Deshayes[2]
[1]*AII Laboratory, Ecole Mohammadia d'Ingénieurs, Mohammed V University, Rabat, Morocco*
[2]*Innovation Lab for Operations, Mohammed VI Polytechnic University, Benguerir, Morocco*
[3]*Electrical Engineering Department, École de Technologie Supérieure, Montreal, QC H3C 1K3, Canada*

Keywords: Mixing Tank, Centrifugal Pump, Slurry, Modeling, Simulation, Density Variation, Level Variation.

Abstract: In this paper, a dynamic model of a slurry mixing and pumping process is proposed. The centrifugal pump is modeled based on the hydraulic application, the hydraulic part and the induction motor models, taking into account the pumped slurry density. This paper also proposes a new approach to estimate the parameters of the pump's hydraulic part model based on the pump characteristic curves. Additionally, a dynamic simulation of the system is realized under MATLAB/Simulink environment and the variation effect of the process inputs on the outputs is studied.

1 INTRODUCTION

In many process industries, the mixing and pumping process is a decisive step. It is common to blend different products together to form a mixed slurry for downstream processes. Eventually, the quality of the final product will be derived by how good the mix is and the precision of the inlet flows (Nienow et al., 1997). Considering the widespread of such a process, understanding its dynamics is of great importance. A steady state approach is often used to describe the system. However, a steady state process requires constant properties. For slurry processes, solids' properties such as the granulometry change as a function of time and place as does the solids' density (Miedema, 1996). Since inlet density impacts how easily the slurry is pumped (Blevins and Nixon, 2010), this change in density impacts the flow rate of the slurry. Therefore, a dynamic model is needed, for a better understanding, simulation and control of the system.

A mathematical model, based on first order nonlinear differential equations, is used to describe the mixing process (Deng, 2002). For the slurry pumping process, the centrifugal pump is used. In (Kallesøe et al., 2006), the dynamic model of the centrifugal pump is divided into three sub parts: the pump motor, the hydraulic part and the hydraulic application. The parameters of hydraulic part are calculated based on the physical properties of the pump. The hydraulic application is not detailed and the variation of flow is not discussed. In (Valtr, 2017), the centrifugal pump is modeled based on the hydraulic system without considering the pump's motor. In (Miedema, 1996) a dynamic model for the system pump/pipeline is proposed and simulated. However, the model assumes that the pump drive behaves like a constant torque system. Concerning the pump's motor, the induction motor is used. The model of the induction motor is extensively described in literature. The most popular representation is the so-called qd model based on a series of mathematical transformations (Manekar and Bodkhe, 2013). The idea of a revolving reference frame, dq, is introduced to transform the ac components of the vectors in the stator frame into dc signals in order to simplify calculations (Trzynadlowski, 2000).

In this work, a graphical method is used to estimate the pump's hydraulic part, the head and the load torque parameters. In literature, a numerical method is used to calculate these parameters based on the physical properties of the pump (Isermann, 2007; Kallesøe et al., 2006), which is time consuming and hard to apply in an industrial environment. For the hydraulic application, it depends on the studied system. A generalized method is presented in this work, taking into account the friction factor variation along with the slurry flow rate variation.

Oulhiq, R., Benjelloun, K., Saad, M., Kali, Y. and Deshayes, L.
Dynamic Modeling and Simulation of a Slurry Mixing and Pumping Process: An Industrial Case.
DOI: 10.5220/0007774000270035
In *Proceedings of the 9th International Conference on Simulation and Modeling Methodologies, Technologies and Applications (SIMULTECH 2019)*, pages 27-35
ISBN: 978-989-758-381-0

The outline of this paper is as follows. Section 2 exposes the process description and dynamic analysis of the mixing tank and the centrifugal pump. Section 3 presents the simulation and the performance results. Section 4 concludes the paper.

2 PROCESS DESCRIPTION AND DYNAMICS ANALYSIS

The process studied is a part of an industrial thickening unit as shown in Fig. 1. The slurry arriving to the unit is delivered to a cylindrical mixing tank in which it is kept in agitation in order to avoid the decantation of the solid. The process water is used to adjust the solid rate of the slurry. The mixed slurry is then pumped, using a centrifugal pump driven by an induction motor, to downstream processes.

2.1 Dynamic Analysis of the Mixing Tank

The dynamic behavior of the mixing tank is described by the following equations (Skogestad, 2008; Hougen et al., 1954):

- Conservation of the total mass in the mixing tank

$$\frac{d(M_{pm}+M_{wm})}{dt} = d_w F_w + d_{ps1} F_{ps1} - d_{ps2} F_{ps2} \quad (1)$$

- Level of the mixing tank:

$$\frac{dL_m}{dt} = \frac{4}{\pi . D_m^2}(F_{ps1} + F_w - F_{ps2}) \quad (2)$$

- Density of the slurry at the outlet of the mixing tank:

$$d_{ps2} = \frac{4}{\pi D_m^2 L_m}(M_{pm} + M_{wm}) \quad (3)$$

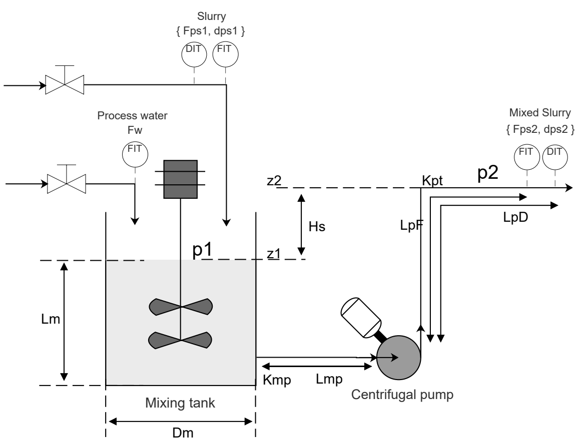

Figure 1: Pump hydraulic application parameters.

Where M_{pm} is the total mass of dry product in the mixing tank (Kg), d_{ps1} is the density of slurry at the inlet of the mixing tank (Kg/m^3), F_{ps1} is the volumetric flow rate of slurry at the inlet of the mixing tank (m^3/s), d_{ps2} is the density of slurry at the outlet of the mixing tank (Kg/m^3), F_{ps2} is the volumetric flow rate of slurry at the outlet of the mixing tank (m^3/s), M_{wm} is the total mass of water in the mixing tank (Kg), d_w is the density of water (Kg/m^3), F_w is volumetric flow rate of water at the inlet of the mixing tank (m^3/s), L_m is the level of slurry in the mixing tank (m) and D_m is the diameter of the mixing tank (m). In what follows, it is assumed that the mixing tank is perfectly mixed. The perfect mixing assumption is valid for low-viscosity liquids that receive an adequate degree of agitation (Seborg et al., 2010).

2.2 Dynamic Analysis of the Centrifugal Pump

The flow of a centrifugal pump is related to two parameters, namely, the pump head and the system head. The intersection of the two head curves is the flow operating point of the system (Bachus and Custodio, 2003). The equation describing this fact is given by the different parts of the pump.

First of all, applying Newton's second law of motion to the fluid in the pipe (Matko et al., 2001; Valtr, 2017) yields to:

$$\frac{dF_{ps2}}{dt} = \frac{gA_p}{L_{mt}} \Delta H \quad (4)$$

where ΔH is the head variation (m), A_p is the pipe section (m^2), L_{mt} is the length of the pipe between the mixing tank and the downstream process (m) and g is the gravity (m/s^2).

Then, based on (4), the equation describing the variation of flow in function of the pump head H_p and the system head H_{sys} is obtained:

$$\frac{dF_{ps2}}{dt} = \frac{gA_p}{L_{mt}}(H_p - H_{sys}) \quad (5)$$

2.2.1 The Hydraulic Application

The system head between p1 and p2 (Fig. 1) (Menon, 2004) is:

$$H_{sys} = H_s + H_l + H_v + \frac{P_2 - P_1}{d_{ps2} g} \quad (6)$$

where H_{sys} is the system head between p1 and p2 (m), H_s is the static head of the system (m), H_l is the total head losses in the system (m), H_v is the velocity head (m), P_1 and P_2 represent the pressure at p1 and p2 respectively (Pa) and d_{ps2} is the density of slurry (kg/m3) (Fig. 1) with:

- $P_1 = P_2 = P_{atm}$ (Atmospheric pressure);
- $H_v = \frac{v_2^2 - v_1^2}{2g}$;
- $H_s = z_2 - z_1$.

v_1 and v_2 are respectively the velocity at p1 and p2 (m/s) where:

- $v_1 \approx 0$;
- $v_2 = \dfrac{4F_{ps2}}{\pi D_p^2}$;

For the total head losses:

$$H_l = H_{lfriction} + H_{llocal} \tag{7}$$

where $H_{lfriction}$ is the friction losses (Pa) and H_{llocal} is the local losses (Pa).

The friction losses are described by the Darcy equation (Green and Perry, 1997):

$$H_{lfriction} = \frac{f v_2^2 L_{mt}}{2gD_p} \tag{8}$$

Then:

$$H_{lfriction} = \frac{fL_{mt}}{2gD_p}\left(\frac{4F_{ps2}}{\pi D_p^2}\right)^2 \tag{9}$$

where D_p is the pipe diameter (m) and f is the Darcy friction factor.

The Darcy friction factor depends on the pipe properties and on the nature of the pumped fluid (Menon, 2004; El-Emam et al., 2003).

Moreover, for the local losses :

$$H_{llocal} = \frac{1}{g} v_2^2 \sum_i^n K_i \tag{10}$$

where K_i are are the local loss coefficients.

Then:

$$H_{llocal} = \frac{1}{g}(K_{mp} + K_{pt})\left(\frac{4F_{ps2}}{\pi D_p^2}\right)^2 \tag{11}$$

where K_{mp} is the local loss coefficient caused by the sudden contraction between the mixing tank and the pipe and K_{pt} is the local loss coefficient caused by the change in pipe geometry between the pump and the thickener.

After simplifying (6), the relation of the system head is obtained:

$$H_{sys} = \alpha + \beta F_{ps2}^2 + \gamma f F_{ps2}^2 \tag{12}$$

where :

$\alpha = z_2 - L_m$

$\beta = \frac{1}{2g}\left(\frac{4}{\pi . D_p^2}\right)^2 [1 + 2(K_{mp} + K_{pt})]$

$\gamma = \frac{1}{2g}\left(\frac{4}{\pi . D_p^2}\right)^2 \frac{L_{mt}}{D_p}$

In classic pumping systems design, the friction factor f is fixed at the beginning according to the flow operating point (Wright and Gerhart, 2009; Chiasson, 2016). However, the friction factor depends on the fluid velocity and properties. Thus, the friction factor changes as the flow changes and it must be updated in each iteration of the dynamic model simulation.

2.2.2 The Hydraulic Part

The pump head H_p (m) is a function of flow F_{ps2} and shaft speed w_p (rad/s). The equation describing this fact is (Kallesøe et al., 2006; Kallesøe et al., 2004):

$$H_p = a_h F_{ps2}^2 + b_h F_{ps2} . \omega_p + c_h \omega_p^2 \tag{13}$$

where a_h, b_h and c_h are constant parameters fixed from the physical properties of the pump.

However, obtaining these physical properties of the pump is time consuming and maybe impossible in an industrial environment. In this work, these parameters are determined using the H-Q curves of the used pump.

The pump torque (or the load torque) T_p (N.m) is described by the equation (Kallesøe et al., 2006; Kallesøe et al., 2004):

$$T_p = -d_{ps2} a_t F_{ps2}^2 + d_{ps2} b_t F_{ps2} \omega_p + c_t \omega_p^2 \tag{14}$$

where a_t, b_t and c_t are constants found from the physical properties of the pump.

In this work, these parameters are determined using the H-Q curves of the used pump.

2.2.3 The Induction Motor

The equation describing the mechanical part of the pump is (Chan and Shi, 2011; Trzynadlowski, 2000):

$$\frac{d\omega_p}{dt} = \frac{1}{J_m + J_p}(T_e - T_p) - \frac{C_f}{J_m + J_p}\omega_p \tag{15}$$

where J_m is the moment of inertia of the pump mechanical parts (Kg m^2), J_p is the moment of inertia of the fluid inside the pump impeller (Kg m^2), C_f is the friction losses coefficient of pump induction motor (Kg m^2/s) and T_e is the torque produced by the pump induction motor (N m).

The torque T_e is calculated based on the electromechanical and the electrical parts of the induction motor (Chan and Shi, 2011; Trzynadlowski, 2000).

2.3 Mixing and Pumping Unit Global Model

From the process parts dynamic analysis, the overall model of the system is described by Fig. 2 and the equations:

$$\frac{dL_m}{dt} = \frac{4}{\pi D_m^2}(F_{ps1} + F_w - F_{ps2}) \tag{16}$$

$$\frac{d(d_{ps2})}{dt} = \frac{4}{\pi D_m^2 L_m}\left[(d_w - d_{ps2})F_w + (d_{ps1} - d_{ps2})F_{ps1}\right] \tag{17}$$

$$\frac{dF_{ps2}}{dt} = \frac{gA_p}{L_{mt}}\left[(a_h - \beta - \gamma f)F_{ps2}^2 + b_h\omega_p F_{ps2} + c_h\omega_p^2 - \alpha\right] \tag{18}$$

$$\frac{d\omega_p}{dt} = \frac{1}{J_m + J_p}(T_e + d_{ps2}a_t F_{ps2}^2 - d_{ps2}b_t F_{ps2}\omega_p - c_t\omega_p^2) - \frac{C_f}{J_m + J_p}\omega_p \tag{19}$$

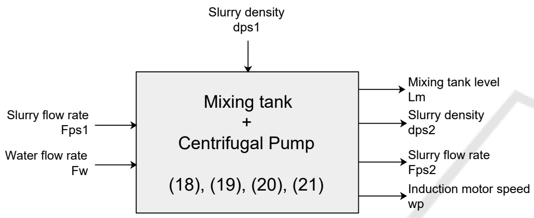

Figure 2: The mixing unit bloc diagram.

The model takes into consideration the density and the friction factor variations and assume that the mixing tank is perfectly mixed.

The model inputs and outputs shown in Fig. 2 are as follows: The model manipulated inputs are the slurry flow rate (F_{ps1}) and the water flow rate (F_w); The model disturbance input is the slurry density (d_{ps1}); The model outputs are the mixing tank level (L_m), the slurry density (d_{ps2}), the slurry flow rate (F_{ps2}) and the induction motor speed (w_p).

3 SIMULATION

3.1 Simulation Parameters

The simulation parameters are given in Table 1. Where p is the pole number of the pump induction motor, L_{mn} is the mutual inductance of the pump induction motor (H), L_s and L_r are the stator and the rotor inductances (H), R_s and R_r are the stator and the rotor resistances (Ω), E is The voltage of the power supply (V), θ is the Initial phase angle of the power supply (rad), ω is the supply angular frequency (rad/s) and R_i is the internal resistance of the power supply (Ω). To determine the parameters a_h, b_h and c_h in (13), the pump characteristic curve is used. Three different operating points are chosen (Fig. 3) in order

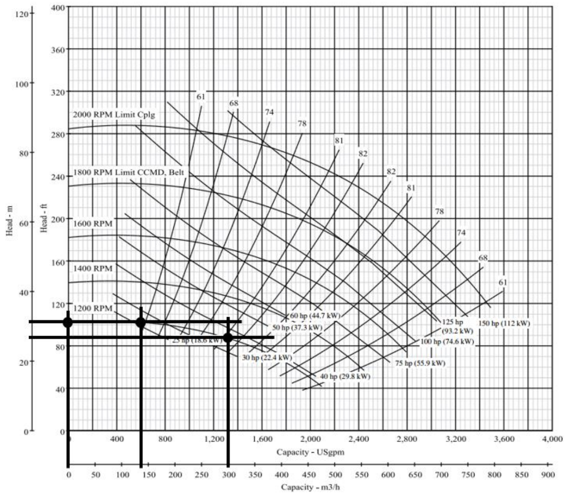

Figure 3: The method used to determine a_p, b_p and c_p.

to get three equations with the three variables. The solution of the system gives :

$$\begin{cases} a_h = -1.04 \times 10^{-4} \\ b_h = \ \ 1.21 \times 10^{-6} \\ c_h = \ \ 2.15 \times 10^{-5} \end{cases} \tag{20}$$

To fix the parameters a_t, b_t and c_t in (14), equation (21) is used (Isermann, 2007) :

$$P_p = \omega_p T_p \tag{21}$$

where P_p is the power required by the pump.

The characteristic curve of P_p is given in the H-Q curve of the pump. Using this characteristic, three different points are chosen (Fig. 4). Each point is associated with a shaft speed, flow and power. Then, using (21), the torque T_p is determined for each couple (shaft speed, flow). Based on (14), three equations with the variables a_t, b_t and c_t, are obtained. The solution of the system gives :

$$\begin{cases} a_t = -20.23 \\ b_t = -1.56 \times 10^{-4} \\ c_t = \ \ 7.98 \times 10^{-3} \end{cases} \tag{22}$$

3.2 Simulation Results

The simulation is done using the Matlab/Simulink software. The simulation blocks are created based on Level-2 Matlab s-functions. After the simulation of the different blocks of the process, the simulation of the whole process is done as depicted in Fig. 5. The chosen initial parameters are: Mixing tank slurry density : 1600 Kg/m^3; Mixing tank level : 6 m; Inlet slurry flow rate : 300 m^3/h; Inlet slurry density : 1600 Kg/m^3; Inlet water flow rate : 0 m^3/h

Table 1: Simulation parameters.

Parameter	Value	Parameter	Value
D_m (m)	13	D_p (m)	0.4
L_{mt} (m)	21	K_{mp}	0.5
K_{pt}	0.3	g (m/s^2)	9.81
ν (m^2/s)	0.015	ε (m)	0.07
z_2 (m)	11	J_m (Kg m^2)	0.4
J_p (Kg m^2)	0.4	C_f (Kg m^2/s)	0.062
p	6	L_{mn} (m)	0.041
L_s (H)	0.0425	L_r (H)	0.0418
R_s (Ω)	0.288	R_r (Ω)	0.158
E (V)	220	θ (rad)	0
ω (rad/s)	$2\pi\times50$	R_i (Ω)	0.05

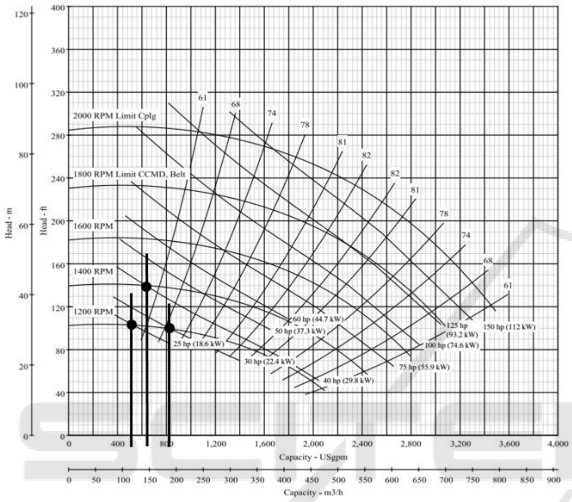

Figure 4: The method used to determine a_t, b_t and c_t.

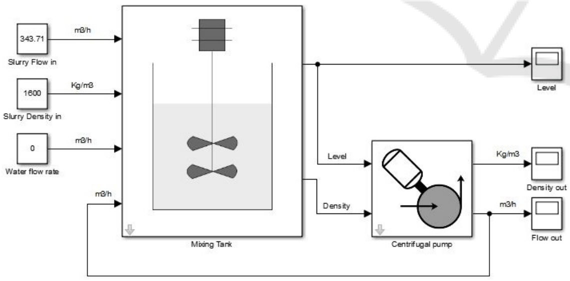

Figure 5: The mixing and pumping unit simulation using Matlab/Simulink.

3.2.1 The System Operating Point

After the simulation is launched, the outlet flow of the pump stabilizes at 306 m^3/h, the induction motor speed stabilizes at 826 rpm and the load torque at 293 N m (6). In order to validate the operating point of the system, the pump and the system head curves are plotted (Fig. 7). The intersection of the two curves gives the same flow operating point 285 m^3/h. Then the pump load torque is plotted in function of the induction motor speed for a flow of 306 m^3/h (Fig. 8) .

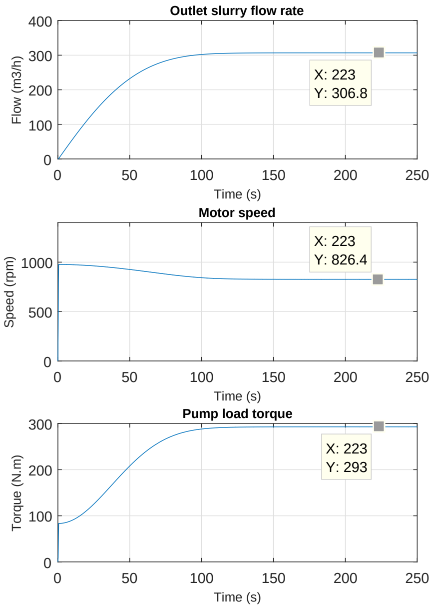

Figure 6: The operation point of the system.

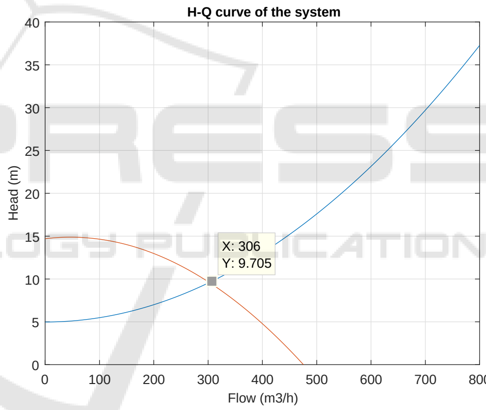

Figure 7: The H-Q curve of the system.

For a speed of 826 rpm the curve gives the same load torque 293 N m.

3.2.2 The System Response to Large Variations

Effect of Inlet Slurry Flow Rate Large Variation. Keeping the inlet slurry density and the water flow rate constants, the inlet slurry flow rate is varied. Fig. 9 shows the effect of the inlet slurry flow rate large variation on the outputs. An increase in the inlet slurry flow rate results in an increase in the mixing tank level. Which results in an increase in the outlet slurry flow rate and a decrease in the motor speed. This is explained by a change in the static head of the system along with a change in the load torque of the pump.

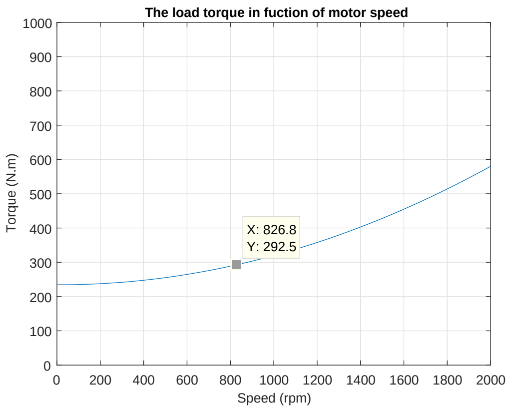

Figure 8: The Load torque in function of motor speed.

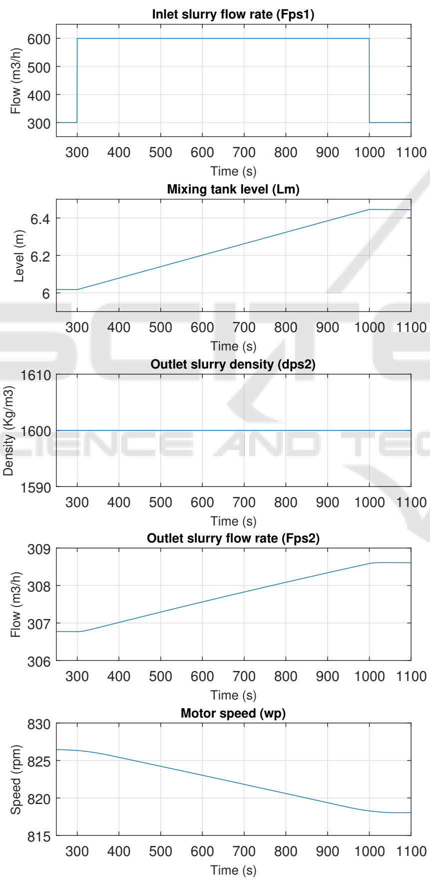

Figure 9: The effect of inlet slurry flow rate large variation.

Effect of Inlet Slurry Density Large Variation. In this case, the inlet density is varied, keeping the other inputs constants. Fig. 10 shows the effect of the inlet slurry density variation on the system outputs. It is clear that an increase in the inlet slurry density increases the outlet density. Since the higher is the density, the greater is the resistance to flow (Blevins

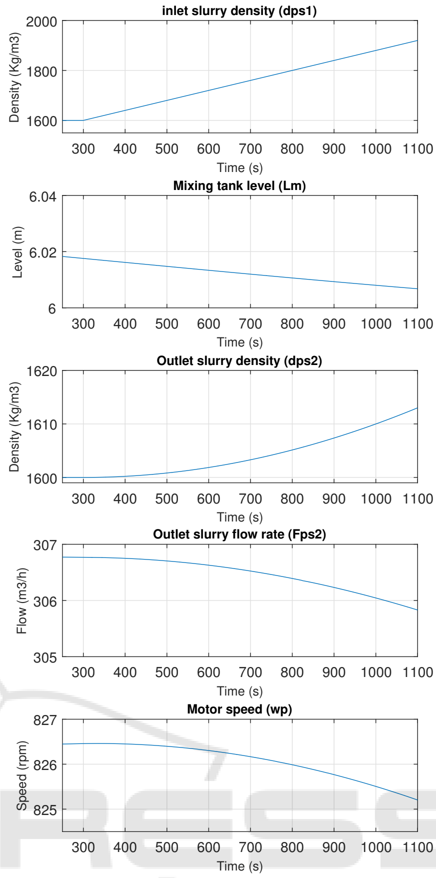

Figure 10: The effect of inlet slurry density large variation.

and Nixon, 2010), this increase in the outlet slurry density increases the pump load torque. The rise in load torque decreases the motor speed then the outlet slurry flow rate decreases. Thus, a change in slurry density impacts inversely the outlet slurry flow rate.

Effect of Inlet Water Flow Large Variation. In order to study the effect of the inlet water flow variation, water flow rate is varied while the other inputs remain constant. The Fig. 11 shows the effect of varying the inlet water flow rate on the system outputs. By adding water, the level starts to increase, the density decreases as $d_w < d_{ps1}$ and the outlet slurry flow rate starts to increase as a result of level and density changes (cf. 3.2.2 and 3.2.2).

3.2.3 The System Response to Small Variations

Effect of Inlet Slurry Flow Rate Small Variation. In order to study the response of the system to inputs' small variations, the inlet slurry flow rate (F_{ps1}) is increased by 3% of the initial value (Fig. 12). By increasing F_{ps1}, the mixing tank level and the outlet slurry flow rate increase but with a slight evolution,

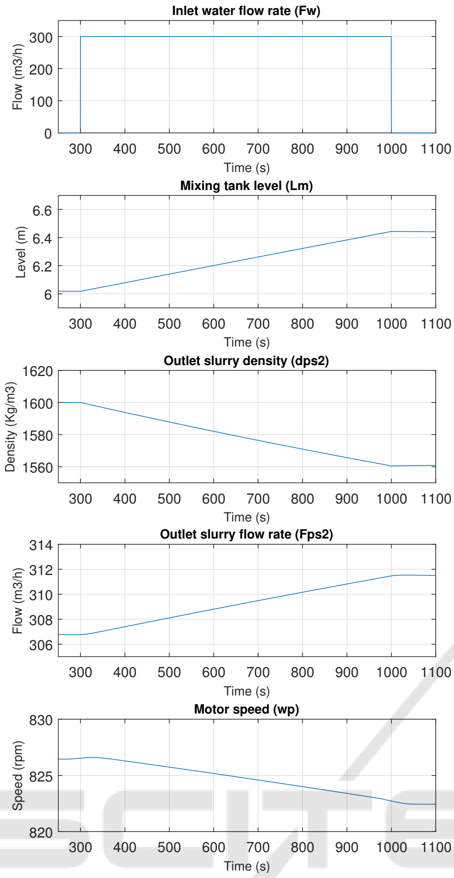

Figure 11: The effect of water flow rate large variation.

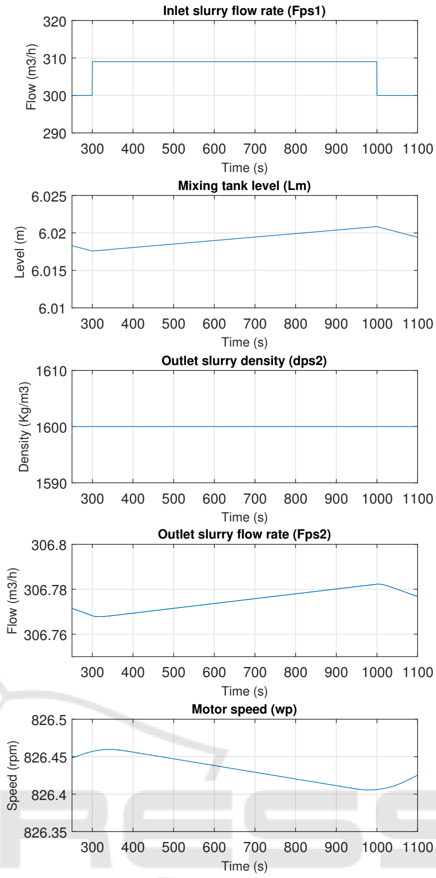

Figure 12: The effect of inlet slurry flow rate small variation.

without impacting the other outputs. However, this increase of level will surely have an impact on the static head of the system in a long-term and subsequently an important impact on the outputs (3.2.2).

Effect of Inlet Slurry Density Small Variation. Increasing the inlet slurry density by 3% of the initial value, the outlet slurry density, the outlet slurry flow rate and the mixing tank level vary slightly (Fig. 13). However, this confirms the indirect impact of the inlet density on the level and on the outlet slurry flow rate even for small variations.

Effect of Inlet Water Flow Small Variation. Fig. 14 shows the effect of the inlet water flow rate small variation on the system outputs. By adding water, the level starts to increase, the density decreases as $d_w < d_{ps1}$ and the outlet slurry flow rate starts to increase as a result of level and density changes. Thus, even a slight variation impacts the outputs.

4 CONCLUSION

The study puts forward a dynamic model and a simulation for a slurry mixing and pumping process. The dynamic model of the centrifugal pump takes into account its different parts: the hydraulic application, the hydraulic part and the induction motor. In addition to that, the proposed model takes the slurry density into consideration. Therefore, the applicability of the model is extended to other slurry types.

In order to determine the parameters of the pump head and load torque equations, a practical method is used based on the pump's graphical experiments schemes. Regarding the simulation of the process, it is conducted using level 2 Matlab s-fucntions and simulink environment.

According to the developed model and the simulation results, it is clear that both the mixing tank level and the slurry density impact the outlet slurry flow rate. With regard to the slurry density effect, it directly impacts the pump torque, then the pump torque

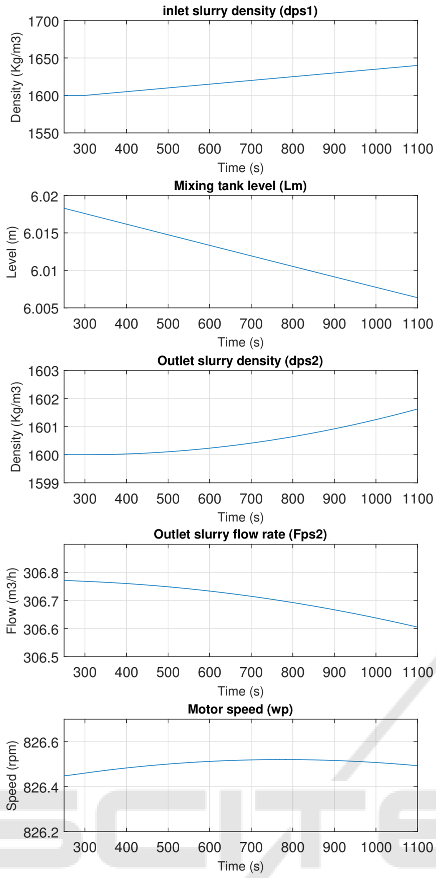

Figure 13: The effect of inlet slurry density small variation.

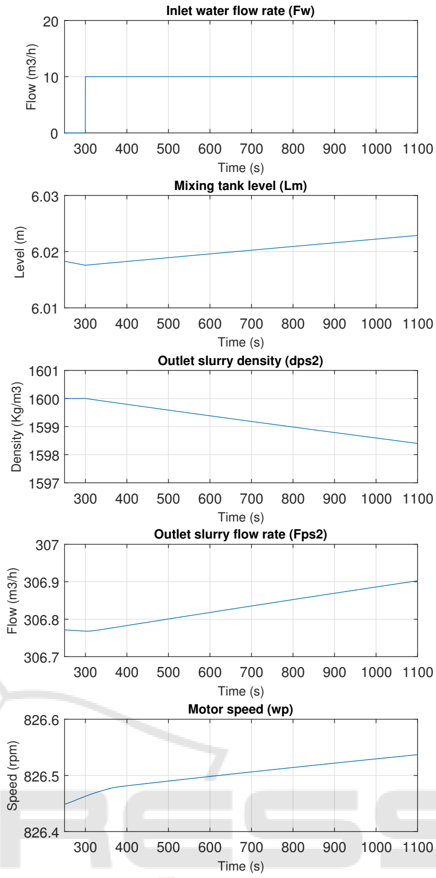

Figure 14: The effect of water flow rate small variation.

affects the induction motor speed, then a change of the induction motor speed changes the outlet flow rate. That highlights the importance of these parameters in the control strategy design.

Further research will be pursued to model the non perfect mixture inside the mixing tank and to propose new methods to control pumping slurry flow rate in presence of slurry density variations.

REFERENCES

Bachus, L. and Custodio, A. (2003). *Know and understand centrifugal pumps*. Elsevier.

Blevins, T. L. and Nixon, M. (2010). *Control Loop Foundation: Batch and Continuous Processes*. ISA.

Chan, T. F. and Shi, K. (2011). *Applied intelligent control of induction motor drives*. John Wiley & Sons.

Chiasson, A. D. (2016). *Geothermal heat pump and heat engine systems: Theory and practice*. John Wiley & Sons.

Deng, S. Y. (2002). Nonlinear & linear mimo control of an industrial mixing process. Master's thesis, McGill University.

El-Emam, N., Kamel, A., El-Shafei, M., and El-Batrawy, A. (2003). New equation calculates friction factor for turbulent flow on non-newtonian fluids. *Oil & gas journal*, 101(36):74–74.

Green, D. W. and Perry, R. H. (1997). *Perry's Chemical Engineers' Handbook/edición Don W. Green y Robert H. Perry*. Number C 660.28 P47 2008.

Hougen, O. A., Watson, K. M., and Ragatz, R. A. (1954). *Chemical Process Principles: Material and Energy Balances*, volume 1. John Wiley & Sons.

Isermann, R. (2007). *Mechatronic systems: fundamentals*. Springer Science & Business Media.

Kallesøe, C. S., Cocquempot, V., and Izadi-Zamanabadi, R. (2006). Model based fault detection in a centrifugal pump application. *IEEE Transactions on Control Systems Technology*, 14(2):204–215.

Kallesøe, C. S., Izaili-Zamanabadi, R., Rasmussen, H., and Cocquempot, V. (2004). Model based fault diagnosis in a centrifugal pump application using structural analysis. In *Proceedings of the 2004 IEEE International Conference on Control Applications, 2004.*, volume 2, pages 1229–1235 Vol.2.

Manekar, G. and Bodkhe, S. B. (2013). Modeling Methods of Three Phase Induction Motor. *National Conference on Innovative Paradigms in Engineering & Technology*, pages 16–20.

Matko, D., Geiger, G., and Werner, T. (2001). Modelling of the pipeline as a lumped parameter system. *AUTOMATIKA-ZAGREB-*, 42(3):177–188.

Menon, S. (2004). *Piping calculations manual*. McGraw Hill Professional.

Miedema, S. (1996). Modeling and simulation of the dynamic behavior of a pump/pipeline system. In *17th Annual Meeting & Technical Conference of the Western Dredging Association. New Orleans*.

Nienow, A. W., Edwards, M. F., and Harnby, N. (1997). *Mixing in the process industries*. Butterworth-Heinemann.

Seborg, D. E., Mellichamp, D. A., Edgar, T. F., and Doyle III, F. J. (2010). *Process dynamics and control*. John Wiley & Sons.

Skogestad, S. (2008). *Chemical and Energy Process Engineering*. CRC Press.

Trzynadlowski, A. M. (2000). *Control of induction motors*. Elsevier.

Valtr, J. (2017). Mass Flow Estimation and Control in Pump Driven Hydronic Systems. Master's thesis, Czech Technical University, Czech Republic.

Wright, T. and Gerhart, P. (2009). *Fluid machinery: application, selection, and design*. CRC press.