The Cardinal TOPSIS via Grey Relational Grade

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Abstract: Quite a lot of soft computing models have over the past few years been developed. They are developed to meet different purposes and needs. These models, however, may not satisfy the technique for order preference by similarity to ideal solution (TOPSIS). Aware of this phenomenon, the present study applied globalization grey relational grade of the grey system to convert subjective weighting in the computing process of technique for order preference by similarity to ideal solution into objective weighting. Data analysis demonstrated that applying grey relational grade to cardinal technique for order preference by similarity to ideal solution was not only rational but also could transfer ordinal answer into cardinal answer.

1 INTRODUCTION

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), originally developed by Hwang and Yoon in 1981, is a multi-criteria decision analysis method. The fundamental assumption of TOPSIS is that the criteria are either monotonically increasing or decreasing. It is based on the concept that the positive ideal solution is composed of the best score in all criteria, given, for instance, benefit as the maximal value and cost as the minimum value. In contrast, the negative ideal solution is composed of the worst score in all criteria, given, for instance, benefit as the minimum value and cost as the maximum value (Wen and You, 2018). According to TOPSIS, a set of alternatives is measured in terms of the Euclidean norm to compare their closeness to the positive ideal solution. TOPSIS is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution. Such a method, according to Pi, can prevent an alternative from being both the shortest distance to the positive ideal solution and the negative ideal solution on the one hand, and being both the longest distance from the positive ideal solution and the negative ideal solution on the other hand (Pi, 2005).

A close observation of TOPSIS reveals that one step of the TOPSIS utilizes subjective weighting for analysis. Different weighting inevitably generates different result. There has been research which either focused on soft computing and environment area such as grey entropy-TOPSIS method (Liu et al., 2014), combined TOPSIS with grey relation to decide the weighting of TOPSIS in contractor selection (Zavadskas et al., 2010), compared fuzzy AHP and fuzzy TOPSIS for road pavement maintenance prioritization (Ouma et al., 2015), used the AHP and TOPSIS approaches under fuzzy environment (Shahab, 2016) or applied fuzzy TOPSIS-TODIM hybrid method for green supplier selection (Khamseh and Mahmoodi, 2014). (Qian et al., 2009) was the only one study applying the grey relation grade method to TOPSIS. The present study, which was based on the research method mentioned above, used cardinal grey relational grade to convert subjective weighting into objective weighting (Wen, 2013). Section Two of this study discussed the mathematical model related to the Technique for the Order Preference by Similarity to Ideal Solution. The third section investigated and ranked four kinds of drinking water in Changhua County as to their quality by using this soft computing method. The last part of this paper provided research findings and proposed suggestions for forthcoming research.

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2 MATHEMATICS MODEL

The analysis procedure of TOPSIS and grey relational grade is described step by step (Wen, 2016).

2.1 TOPSIS

1. Input the project data

$$D=\begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \cdots & x_{mn} \end{bmatrix} \quad (1)$$

2. Normalize the data in equation (1)

$$r_{ij}=\frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \quad (2)$$

then show normalization matrix in equation (3)

$$R=\begin{bmatrix} r_{11} & r_{12} & r_{13} & \cdots & r_{1n} \\ r_{21} & r_{22} & r_{23} & \cdots & r_{2n} \\ r_{31} & r_{32} & r_{33} & \cdots & r_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & r_{m3} & \cdots & r_{mn} \end{bmatrix} \quad (3)$$

3. Decide the weighting of $\omega_i=[\omega_1,\omega_2,\omega_3,\cdots,\omega_n]$

4. Calculate the weighting decision matrix

$$V=\omega_i\times R=\begin{bmatrix} \omega_1 r_{11} & \omega_2 r_{12} & \omega_3 r_{13} & \cdots & \omega_n r_{1n} \\ \omega_1 r_{21} & \omega_2 r_{22} & \omega_3 r_{23} & \cdots & \omega_n r_{2n} \\ \omega_1 r_{31} & \omega_2 r_{32} & \omega_3 r_{33} & \cdots & \omega_n r_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega_1 r_{m1} & \omega_2 r_{m2} & \omega_3 r_{m3} & \cdots & \omega_n r_{mn} \end{bmatrix} =\begin{bmatrix} v_{11} & v_{12} & v_{13} & \cdots & v_{1n} \\ v_{21} & v_{22} & v_{23} & \cdots & v_{2n} \\ v_{31} & v_{32} & v_{33} & \cdots & v_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & v_{m3} & \cdots & v_{mn} \end{bmatrix} \quad (4)$$

5. Calculate the positive ideal solution A^+ and ideal negative solution A^-

$$A^+=\max.\{v_i^+\}=(v_1^+,v_2^+,v_3^+,\cdots,v_m^+)$$
$$A^-=\min.\{v_i^-\}=(v_1^-,v_2^-,v_3^-,\cdots,v_m^-) \quad (5)$$

6. Calculate the positive ideal distance S_i^+ and negative ideal distance S_i^-

$$S_i^+=\sqrt{\sum_{j=1}^{n}(v_{ij}-v_j^+)^2} \quad S_i^-=\sqrt{\sum_{j=1}^{n}(v_{ij}-v_j^-)^2} \quad (6)$$

7. Calculate the relative approaching of ideal distance C_j, then, weighting can be found.

$$C_j=\frac{S_i^-}{S_i^+ + S_i^-},\ i=1,2,3,\cdots,n,\ j=1,2,3,\cdots,n \quad (7)$$

2.2 Objective Weighting Analysis

Step 3 of TOPSIS is observed to be subjectively postulated. As for the existent data, a mathematical method can be used to convert subjective weighting into objective weighting. The present paper, which applied grey relational grade to TOPSIS, could yield objective result. The basic concept of grey relational grade is shown below. Five kinds of globalization grey relational grade have been developed in the past few years(Wen, 2016). This study referred to the mathematical method proposed by Liu.

$$\Gamma_{ij}=\frac{1}{1+\sum_{k=2}^{n-1}[\Delta_{ij}(k)]+\frac{1}{2}\times\Delta_{ik}(n)} \quad (8)$$

where: $i=1,2,3,\cdots,m,\ \ k=1,2,3,\cdots,n,\ j\in I$

i. x_i : Reference sequence,

x_j : Inspected sequences

ii. $\Delta_{ij}=||x_i(k)-x_j(k)||$

According to Saaty, the eigenvector method can be used to rank the sequence, and then choose an optimal one.

1. Base on the original sequences

$$\begin{aligned} x_1&=(x_1(1),x_1(2),x_1(3),\cdots,x_1(k)) \\ x_2&=(x_2(1),x_2(2),x_2(3),\cdots,x_2(k)) \\ x_3&=(x_3(1),x_3(2),x_3(3),\cdots,x_3(k)) \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ x_m&=(x_m(1),x_m(2),x_m(3),\cdots,x_m(k)) \end{aligned} \quad (9)$$

2. Constructing the relative weighting matrix $[R]_{m\times m}$, by using the cardinal globalization grey relational grade method to find the grey relational grade, which is called grey relational matrix.

$$R_{m\times m}=\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \dots & \Gamma_{1m} \\ \Gamma_{21} & \Gamma_{22} & \dots & \Gamma_{2m} \\ \vdots & \vdots & \ddots & \Gamma_{11} \\ \Gamma_{m1} & \Gamma_{m2} & \dots & \Gamma_{mm} \end{bmatrix} \quad (10)$$

3. Find the eigenvalue for the relative weighting matrix $[R]_{m\times m}$: $AR=\lambda R$

4. Use eigenvector method to find the weighting for each target $P^{-1}RP=diag\{\lambda_1,\lambda_2,\lambda_3,\ldots\lambda_n\}$

5. The maximum λ corresponding eigenvector is the weighting for the sequence.

3 REAL EXAMPLE

Four kinds of running water, including tap water, Funyuan spring water, Puli spring water and Hungmaojing water, served as the objects for investigation. The original data was based on the test conducted by the Environmental Protection Bureau of Changhua County, and was modified for scaling. The analysis steps are shown below.

1. Table 1 shows the measurement results.

2. Table 2 shows the normalization of four kinds of drinking water by using equation (2).

3. Table 3, which is based on the data in Table 2, demonstrates the weighting of four kinds of water by using the grey relational grade.

4. Table 4, which is based on the data in Table 3, shows the normalization matrix of four kinds of water by using equation (4).

5. Table 5 depicts the calculation of the positive ideal solution and ideal negative solution by using equation (6).

6. Table 6 demonstrates the calculation of the relative approaching of ideal distance of four kinds of water by using equation (7).

Table 1: The modified data of four kinds of drinking water.

Item/source	A	B	C	D
1. Turbidity(10 times)	7.0	17.0	3.0	1.0
2. pH	8.0	7.5	6.8	6.7
3.Chlorine (100mg/10l)	54.0	12.0	6.0	61.0
4. Sulfates (100mg/10l)	8.5	2.1	2.0	18.2
5. Free chlorine	1.0	23.0	0.5	7.0
6. Total hardness (100mg/10l)	24.2	10.4	6.8	33.4
7. Iron content (times 10)	4.0	2.0	1.0	14.0
8. Total number of viable cells (1,000mg/100l)	13	74.8	6.24	7.3

Table 2: The normalization results.

Item/source	A	B	C	D
1. Turbidity(10 times)	0.0018	0.0025	0.0165	0.0002
2. pH	0.0021	0.0011	0.0374	0.0012
3.Chlorine (100mg/10l)	0.0139	0.0018	0.0330	0.0111
4. Sulfates (100mg/10l)	0.0022	0.0003	0.0110	0.0033
5. Free chlorine	0.0003	0.0034	0.0028	0.0013
6. Total hardness (100mg/10l)	0.0062	0.0015	0.0374	0.0061
7. Iron content (times 10)	0.0010	0.0003	0.0055	0.0025
8. Total number of viable cells (1,000mg/100l)	0.0034	0.0111	0.0343	0.0013

Table 3: The weighting from Liu's grey relational grade.

Water source	A	B	C	D
Weighting	0.6941	0.1226	0.1705	0.6885

Table 4: The normalization decision matrix after add the weighting.

Item/source	A	B	C	D
1. Turbidity(10 times)	0.0013	0.0003	0.0028	0.0001
2. pH	0.0014	0.0001	0.0064	0.0008
3.Chlorine (100*mg*/10*l*)	0.0097	0.0002	0.0056	0.0076
4. Sulfates (100*mg*/10*l*)	0.0015	0.0000	0.0019	0.0023
5. Free chlorine	0.0002	0.0004	0.0005	0.0009
6. Total hardness (100*mg*/10*l*)	0.0043	0.0002	0.0064	0.0042
7. Iron content (times 10)	0.0007	0.0000	0.0009	0.0017
8. Total number of viable cell (1,000mg/100*l*)	0.0023	0.0014	0.0059	0.0009

Table 5: The positive ideal solution and ideal negative solution of four kinds of drinking water.

Water source	A	B	C	D
S_i^+	0.0214	0.0031	0.0099	0.0164
S_i^-	0.0108	0.0014	0.0115	0.0090

Table 6: The relative approaching of ideal distance of four kinds of drinking water.

Weighting/water	A	B	C	D
Ideal distance	0.3354	0.3111	0.5374	0.3543
Rank	3	4	1	2

*A: Tap water(Changhua). B: Funyuan spring water. C: Puli spring water. D: Hungmaojing water

4 CONCLUSIONS

The main purpose of the TOPSIS is to obtain the sorting of all alternatives. Despite the effort of this research to avoid the difficulty in comparing the distance between two directions in the weighting steps, it was subjective to a certain degree. The research result could therefore be uncertain. One of the major contributions of this study was using the grey relational grade to convert subjective weighting into objective weighting for further cardinal. The present research, which referred to and analysis four kinds of running water, effectively verified and supported this soft computing method; The results obtained from the cardinal TOPSIS were consistent with the real situation.

Forthcoming study is suggested to implement other cardinal weighting methods such as grey cluster analysis and GM(h,N) to make analysis more reliable.

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