

# Segmentation Using Histogram and Fuzzy Entropy Principle

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**Abstract:** Segmentation of a composite image which contains two simple subimages is described. The a-priori knowledge about the two simple subimages is that they possess the maximum amount of entropy. The probability density functions(pdf s) of these image pixels are shown to be of the Quasi-gaussian form. Parameters for the pdf are estimated and then the maximum likelihood ratio test is applied to segmentation. An iterative algorithm is employed to improve the segmentation accuracy. Extension of this method to the segmentation of images with arbitrary pdf is discussed. This paper presents a thresholding approach by performing fuzzy partition on a two-dimensional (2-D) histogram based on fuzzy relation and maximum fuzzy entropy principle. The experiments with various gray level and color images have demonstrated that the proposed approach outperforms the 2-D non-fuzzy approach and the one-dimensional(1-D) fuzzy partition approach.

## 1 INTRODUCTION

The standard of evaluating the quality of the image is mostly determined by the subjective of the observer, and there is no general quantitative criterion. Therefore, in the practical application of image enhancement, several algorithms can be selected for the specific application and several enhancement algorithms. Then, how to select a kind of algorithm with good visual effect and small computation It comes out. To this end, only through a number of representative image enhancement algorithms in-depth, systematic study and comparison, in order to find out their corresponding advantages and disadvantages and the best application scene, thus a set of effective application of the image enhancement algorithm guidance rules.

Image enhancement techniques are used to improve an image, where "improve" is sometimes defined objectively (e.g., increase the signal-to-noise ratio), and sometimes subjectively (e.g., make certain features easier to see by modifying the colors or intensities).

This section discusses these image enhancement techniques:

- Intensity Adjustment
- Noise Removal

The functions described in this section apply primarily to intensity images. However, some of these functions can be applied to color images as well. For information about how these functions work with color images, see the reference pages for the individual functions.

Simulation is a virtual representation of the reality. It may also be defined as the process of knowing the characteristics & exhibiting behavior of a particular physical system. Sometimes a learner finds it quite difficult to understand any physical system behavior by just reading it from the written material but once he is able to see the things actually happening on the computer system the things really change. That's why the very important real life techniques of image enhancement such as basic gray level techniques, using arithmetic & logical operations, using spatial filtering and also in the frequency domain various filters like Low Pass Filters, High Pass filters have been simulated on Matlab and studied. The principal objective of Enhancement a Images to process an Image so suitable than the original image for a specific application. Image Enhancement method falls into two broad categories ways: Spatial Domain and Frequency Domain methods.

Image segmentation in image processing and image recognition system has a broad application and prospects. Generally two different images have different pdfs, and the work of determining the difference is statistical in nature. The thresholding method is a significant technique for image processing and pattern recognition, which is considered as the first step in image processing. Many methods are proposed to select the automatic threshold, while most of the single threshold technology can be extended to multiple thresholds, so this paper focuses on the single threshold. The proposed method automatically determines the fuzzy region and the threshold according to the maximum entropy principle, thus to obtain the optimal solution for the 2-D fuzzy entropy and the genetic algorithm[1]. When the gray level is large, the approximation is preferable, the standard deviation is quite small and the average value is not at the edge of the probability density function. This type of images has been identified as images with the maximum amount of entropy. According to the research work above, these simple maximum entropy images can be seen in many real life scenes, such as television images.

## 2 THE PROPOSED METHOD

Complex images are composed of two simple image pixels, the pixel value function is represented by  $f_0(x)$  and  $f_1(x)$ , and  $x$  is the pixel value. Thus, the pdf of the Composite image is:

$$f(x) = \alpha f_0(x) + (1 - \alpha) f_1(x), 0 \leq \alpha \leq 1 \quad (1)$$

Where  $\alpha$  is the mixing ratio and represents two simple sub-images of the relevant type (measured by the pixel). Although it is a point segmentation method, but the overall consideration is also feasible. Thus, only the gray value of the pixel is used in the calculation of the image segmentation. The largest possible inerratic classification for the pixel  $x$  is  $f_0$ , so that the smallest possible error must be satisfied.

$$\frac{f(x)}{f_1(x)} \geq 2 - 2\alpha \quad (2)$$

In fact, when  $\alpha$  and  $f_1(x)$  are unknown to the observer, then the pdf  $f(x)$  of the composite image is the only valid observation data. If the inequality is estimated, then  $\alpha$  and  $f_1(x)$  must be predicted by some practical methods [2].

Define four adjacent mean  $g(x, y)$  around pixel  $f(x, y)$ :

$$g(x, y) = \left[ \frac{1}{4} [f(x, y+1) + f(x, y-1) + f(x+1, y) + f(x-1, y)] + 0.5 \right] \quad (3)$$

The 2-D histogram is an array of  $f(x, y)$ ,  $g(x, y)$  functions relative to the number of occurrences. It can be seen as the two quantities  $X$  and  $Y$ , where  $X$  is the gray level,  $Y$  is the average gray level, and  $X = Y = \{0, 1, 2, \dots, L-1\}$ . The image points have the same intensity, but different 2-D histogram of which different spatial features may be distinguished in second dimensions (local average gray level).

Block B and block W are each defined by (3), and four fuzzy quantities  $BrightX$ ,  $DarkX$ ,  $BrightY$ , and  $DarkY$  are defined with the S function and the corresponding Z function.

$$BrightX = \sum_{x \in X} \frac{\mu BrightX(x)}{x} = \sum_{x \in X} \frac{S(x, a, b, c)}{x} \quad (4)$$

$$DarkX = \sum_{x \in X} \frac{\mu Dark(x)}{x} = \sum_{x \in X} \frac{Z(x, a, b, c)}{x} \quad (5)$$

$$BrightY = \sum_{y \in Y} \frac{\mu BrightY(y)}{y} = \sum_{y \in Y} \frac{S(y, a, b, c)}{y} \quad (6)$$

$$DarkY = \sum_{y \in Y} \frac{\mu DarkY(y)}{y} = \sum_{y \in Y} \frac{Z(y, a, b, c)}{y} \quad (7)$$

Here,  $Z(x, a, b, c) = 1 - S(x, a, b, c)$ . The fuzzy relation  $Bright$  is a subset of the full analytic space  $X \times Y$ , with

$$Bright = BrightX \times BrightY \subset X \times Y \quad (8)$$

$$\begin{aligned} \mu Bright(x, y) &= \mu BrightX \times BrightY(x, y) = \\ &= \min(\mu BrightX(x), \mu BrightY(y)) \end{aligned} \quad (9)$$

Similarly, there are:

$$Dark = DarkX \times DarkY \subset X \times Y \quad (10)$$

$$\begin{aligned} \mu Dark(x, y) &= \mu DarkX \times DarkY(x, y) = \\ &= \min(\mu DarkX(x), \mu DarkY(y)) \end{aligned} \quad (11)$$

Use the  $\mu_A(x_i)$  function to define  $A$  as a fuzzy set of elements  $x_i, i=1, \dots, N$ ,  $P(x_i)$  is the probability of occurrence of  $A$ . The maximum entropy of the element  $A$  is defined by equation (12) [3].

$$H_{fuzzy}(A) = -\sum_{i=1}^N \mu_A(x_i) P(x_i) \log P(x_i) \quad (12)$$

The global entropy of the image is defined as:

$$H(image) = H(Block_B) + H(Block_W) \quad (13)$$

The dark block B as shown in Figure 1 can be divided into a non-fuzzy region  $R_B$  and a fuzzy region  $R_1$ :

$$Block_B = R_B \cup R_1 \quad (14)$$

$$R_B = \{(x, y) \mid \mu Dark(x, y) = 1, (x, y) \in Block_B\} \quad (15)$$

$$R_1 = \{(x, y) \mid \mu Dark(x, y) < 1, (x, y) \in Block_B\} \quad (16)$$

Similarly, the bright block  $Block_w$  consists of a non-fuzzy region  $R_w$  and a fuzzy region  $R_2$ , as shown in Figure 1 (b).

$$Block_w = R_w \cup R_2 \quad (17)$$

$$R_w = \{(x, y) \mid \mu Dark(x, y) = 1, (x, y) \in Block_w\} \quad (18)$$

$$R_2 = \{(x, y) \mid \mu Dark(x, y) < 1, (x, y) \in Block_w\} \quad (19)$$

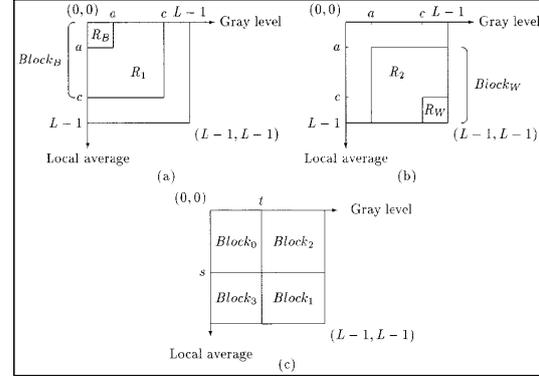


Figure 1: Image blocks.

The following four kinds of entropy can be calculated as follows:

$$H_{fuzzy}(R_1) = -\sum_{(x,y) \in R_1} \mu Dark(x, y) \frac{n_{xy}}{\sum_{(x,y) \in R_1} n_{xy}} \log \frac{n_{xy}}{\sum_{(x,y) \in R_1} n_{xy}} \quad (20)$$

$$H_{nonfuzzy}(R_B) = -\sum_{(x,y) \in R_B} \frac{n_{xy}}{\sum_{(x,y) \in R_B} n_{xy}} \log \frac{n_{xy}}{\sum_{(x,y) \in R_B} n_{xy}} \quad (21)$$

$$H_{fuzzy}(R_2) = -\sum_{(x,y) \in R_2} \mu Bright(x, y) \frac{n_{xy}}{\sum_{(x,y) \in R_2} n_{xy}} \log \frac{n_{xy}}{\sum_{(x,y) \in R_2} n_{xy}} \quad (22)$$

$$H_{nonfuzzy}(R_w) = -\sum_{(x,y) \in R_w} \frac{n_{xy}}{\sum_{(x,y) \in R_w} n_{xy}} \log \frac{n_{xy}}{\sum_{(x,y) \in R_w} n_{xy}} \quad (23)$$

$n_{xy}$  is the number of occurrences of  $(x, y)$  in the 2-D histogram. The membership function  $\mu Bright(x, y)$  and  $\mu Dark(x, y)$  are defined by the equations (9) and (10), respectively. It should be noted that the calculations of  $n_{xy} / \sum n_{xy}$  in the four regions are independent.

### 3 IMAGE SEGMENTATION PROCESS

Consider that the images are mixed by two maximum entropy images [4] and both satisfy the pdf that is the Gaussian distribution:

$$f_0(x) = \frac{1}{\sigma_0 \sqrt{2\pi}} \exp \left[ -\frac{(x - \mu_0)^2}{2\sigma_0^2} \right] \quad (24)$$

$$f_1(x) = \frac{1}{\sigma_1 \sqrt{2\pi}} \exp \left[ -\frac{(x - \mu_1)^2}{2\sigma_1^2} \right] \quad (25)$$

In order to estimate  $f_1(x)$  from the mixed pdfs, some well-known classical measures such as matrix method are applicable. One alternative is to assume that  $\alpha$  is 0 and  $\mu_1$  and  $\sigma_1$  in the mixed image  $f(x)$  are constant in terms of sample mean and sample deviations. The mixed image contains a maximum entropy sub-image, which is much smaller than the mixed image in size, for example,  $\alpha$  is much less than 1%, otherwise the probability of segmentation error would be very large [5]. This iterative algorithm will be discussed below, extended to the case of a large  $\alpha$ . The probability density curve is shown in Figure 2.

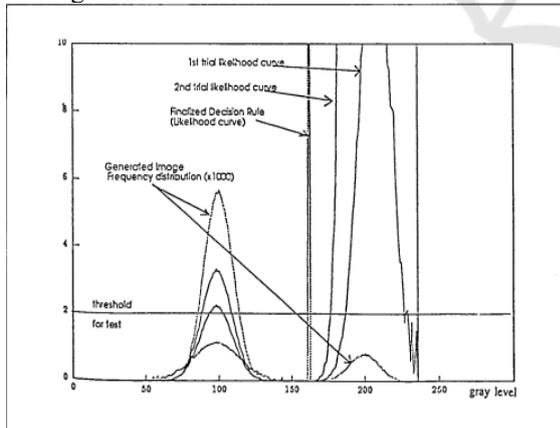


Figure 2: Probability density.

Finding the optimal a, b, c is an optimization problem, which can be solved by heuristic search, genetic algorithm [6], stew fire simulation, etc. This paper will use genetic algorithm to find the optimal solution, and the process is shown in Figure 3. The

2-D histogram of the image is calculated first, then the fuzzy internal functions on the 2-D histogram are calculated, followed by the fuzzy entropy and finally the result is obtained.

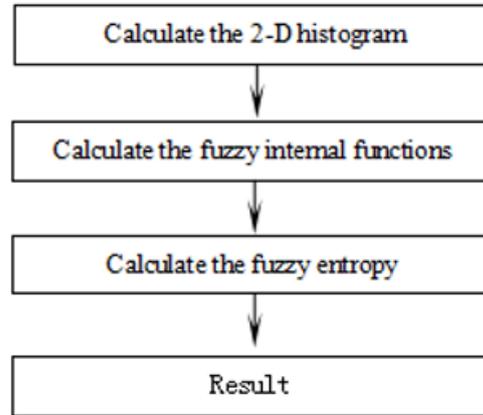


Figure 3: Optimal solution flow.

### 4 ANALYSIS OF RESULTS

Most grayscale horizontal image thresholding can be extended to color images to directly process the various components of the color space and then combine the results to obtain the final image in one way or another. Finally, process the color image in each RGB color space respectively, and then merge the three results to a new RGB color image [7].

Therefore, a simulation system based on SUN SPARC workstation is built to investigate the performance of the iterative algorithm. For different alternative image parameters, such as mean, standard deviation and mixing ratio, this system will be artificial synthesis. The iterations are then run on the same machine and the iterative process is investigated under different synthetic parameters [8]. The simulation results show that if the segmentation error is not large in the first run, then the pdfs of all pixels is classified into different  $f_1(x)$  or  $f_0(x)$ . Repeat this process to reduce the total classification error until the end of the operation.

For Figure 4 (a), the threshold obtained by the 1-D maximum fuzzy entropy method is 127, and the entropy vectors obtained by the 2-D non-fuzzy method and the 2-D fuzzy method are (119,159) and (112,112) as shown in Figure 4 (b), Figure 4 (c) and Figure 4 (d). Figure 4 (d) is clear than Figure 4 (b) and Figure 4 (c) in detail, since the sky and the tower are better segmented [9].



(a) Original image



(b) 1-D fuzzy entropy result



(c) 2-D non-fuzzy entropy result



(d) 2-D fuzzy entropy result

Figure 4: Example 1 of image segmentation.

In Figure 5 (b), the three thresholds of RGB are 102, 113 and 112; for Figure 5 (c), the RGB thresholds are (82, 82), (81, 75), and (66, 69); for Figure 5 (d), the RGB threshold vectors are (81,81), (100,100) and (154,154). Figure 5 (d) is the only one that extracts the blue sky and the tractor from the heaven and the earth, so the partition is better than Figure 5 (b) and Figure 5 (c). The upper right corner of Figure 5 (d) is misclassified because of the threshold value which also appears in Figure 5 (c). In summary, Figure 5 (d) gives the best results.



(a) Original image



(b) 1-D fuzzy entropy result



(c) 2-D non-fuzzy entropy result



(d) 2-D fuzzy entropy result

Figure 5: Example 2 of image segmentation.

In Figure 6 (b), the RGB thresholds are 252, 211, 164; for Figure 6 (c), the threshold vectors are (138, 128), (164, 196), and (180, 182); for Figure 6 (d), the threshold vectors are (215,215), (196,196), (171,171). In Figure 6 (d), the eyes, nose and mouth are extracted very well, the color of the clothes and hair are also different. The color of the clothes, the details of the hair and face are the same in Figure 6 (b). For Figure 6 (c), the clothes are misclassified to the background, so they can not stand out of the background<sup>[10]</sup>. This experiment is conducted on the Lenovo G450 platform based on vc ++ 6.0, and the computing time is shown in Table 1.

Table 1: Computing time.

Image <sup>⊖</sup>	Resolution <sup>⊖</sup>	1-D fuzzy <sup>⊖</sup>	2-D non-fuzzy <sup>⊖</sup>	2-D fuzzy <sup>⊖</sup>
TOWER <sup>⊖</sup>	512×768 <sup>⊖</sup>	3 s <sup>⊖</sup>	4 s <sup>⊖</sup>	13 s <sup>⊖</sup>
Comfield <sup>⊖</sup>	256×256 <sup>⊖</sup>	8 s <sup>⊖</sup>	3 s <sup>⊖</sup>	14 s <sup>⊖</sup>
Girl <sup>⊖</sup>	256×256 <sup>⊖</sup>	7 s <sup>⊖</sup>	3 s <sup>⊖</sup>	14 s <sup>⊖</sup>



(a) Original image



(b) 1-D fuzzy entropy result



(c) 2-D non-fuzzy entropy result



(d) 2-D fuzzy entropy result

Figure 6: Example 3 of image segmentation.

As can be seen from Table 1, the 2-D non-fuzzy method outperforms the 1-D fuzzy and 2-D fuzzy method in terms of computing time, and the advantage is obvious.

## 5 CONCLUSIONS

In this paper, the parameters for the pdf s are estimated first, and then the maximum likelihood ratio test is applied to segmentation. The iterative algorithm is employed to improve the accuracy of segmentation and it is also extended to the segmentation of images with arbitrary pdf s. The experimental results indicate that the spatial information of the pixels should be taken into consideration when selecting the threshold, and the 2-D fuzzy approach is superior to the 2-D direct

maximum amount of entropy approach and 1-D fuzzy entropy approach.

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