Using Flexible Time Scale to Explore the Validity of Agent-based Models of Ecosystem Dynamics: Application to Simulation of a Wild Rodent Population in a Changing Agricultural Landscape

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Abstract: Identifying parameters value is a major issue in model engineering. In discrete time agent-based models, time step is an important one as it determines the frequency at which agents realize their activity step. This parameter is commonly defined as a fixed constant during the model design stage. In particular cases, this may lead to biases as it may be sometimes difficult to determine if agents efficiently realize their activity step once each 1, 2 seconds, hour or the like. A simulation model of a rodent population has been used to study the effect of using a flexible time scale on its outcomes. Three types of processes have been considered as time dependent in the model, environment sensing, movement and life cycle (maturity, gestation…). A time step sensitivity analysis constitutes the principal result of this study. For the widest range of time step values, model's behaviour is unrealistic and bound to algorithms artefacts. A very small range of time steps leads to simulation of a perennial rodents' population. Biases bound to variable time step implementation are discussed. Using flexible time scale approach proved efficient to get insight into the model's behaviour and fruitful clues to assess agents' processes frequency in the actual ecosystem.

1 INTRODUCTION

Agent-based models are recognized as powerful approaches to formalize ecological processes (White, 2016; Fu and Hao, 2018). This formalism is wide-spread in social systems modelling (Squazzoni, 2010), whether animal or specifically human systems, as it can make emerge organization patterns out of agents' interaction (Whitley, 2016). As for other models, one important focus must be put in agent-based models on calibration of parameters used to describe the simulated populations (Stanilov, 2011). Indeed, following Watts (2016), an agent-based model whose parameters are not conveniently fitted may be useless, even with a good representation of its agents' logic.

Several directions are proposed in the literature to simulate agent-based models with a particular distinction between discrete time and discrete events simulation (Buss *et al.*, 2010). Among these alternatives, discrete time simulations are widely used (Railsback *et al.,* 2017) as they constitute a practical and easier to implement approach (Floudas *et al.*, 2004) to formalize concrete systems, be them natural (Singh et al, 2018), social (Sauser *et al.*, 2018) or economical (Ponomarenko *et al.*, 2018). In discrete time simulations, agents are sequentially allowed to perform one cycle of activity each given time step. As a general rule, parameters calibrations are realized for a fixed time step uniformly incremented (Al Rowaei *et al.*, 2011). Recent work on this question put forward the significant impact that using a fixed time step could have on the outcomes of such type of models (Buss and Rowaei, 2010, Kuo, 2015). Indeed, one cycle usually implies agents' decision processes about their environment such as perception-deliberation-execution in a PDE scheme (Ferber and Müller, 1996) or Belief-Desire-Intention in a BDI scheme (Caillou *et al.*, 2017). Whatever the scheme however, it is often difficult, if possible, to determine if one agent has to process the selected scheme once each second, two seconds, minute, hour, day or the like.

In this study we are interested in configuring a classical agent-based model of a rodent population in the wild. The aim is to evaluate the optimal time step duration to fulfil the need of the model's objective, that is to say, make evolve a perennial population in a changing landscape. Beyond the

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model design with its environment, agents' behaviour, etc., we designed the model so as it could be run at various time scales in order to determine the convenient time step necessary for this purpose and thereafter use the model accordingly.

The article is first devoted to the presentation of the model and the approach used to implement a flexible time scale. The use case is then described along with the simulation protocol and its associated time-scale sensitivity analysis. The results section presents the outcomes of the model for a range of time steps used. Results and the method used to formalize time scale changes are then discussed before concluding on perspectives and possible improvements.

2 MODEL AND USE CASE DESCRIPTION

2.1 General Model Overview

The general model used is described in Le Fur *et al.* (2017). It is coded in Java using the Repast Simphony Platform (North *et al.*, 2005). It is a combination of three connected class hierarchies; one for substrates at different spatial aggregation levels, one for genes and genomes that define agents' life traits (age at maturity, gestation length, max age, …) and one to describe agents' behaviours; the latter being a compound of moving, reproduction and social behaviours mechanisms.

The model is implemented using the so-called 'mechanistically rich' approach (De Angelis and Mooij, 2003, Topping *et al.*, 2010) combining abiotic, trophic, physiological, behavioural, social, demographic and environmental mechanisms, all being formalized in the most parsimonious way. The expected outcome of this approach is to formalize the dependency of each underlying causal chains to gain an insight into the overall complex patterns observed in the natural environments within which agents evolve. The 'mechanistically rich' approach leads to simulation models producing complex patterns that cannot be systematically interpreted but that can be studied by modifying the model's logic or parameters.

Environment is simulated using a discrete grid where substrate within each cell can be characterized and modified (road, crop, house, hedge …). It is superimposed with a continuous space where agents' moves and sensing can be computed precisely. Within the use case presented, cells formalize a heterogeneous agricultural landscape with fields of different kinds such as corn, rape, meadow, alfalfa… (Figure 2). Each field characteristic is modified through time by simulated agricultural practices (sowing, mowing, growing, ploughing…) which leads to modify the interest or danger of each cell for the simulated rodent agents. Moreover, each year, the nature of each field may be modified so as to simulate crop rotations that are usual in this type of environment. Agents hence are submitted to a perpetually changing environment which influences their distribution or population size.

Agents are individual rodents bearing different statuses (mature, immature, male, female, pregnant, weaning, etc.); they evolve in the domain fulfilling several desires such as foraging, reproduction, fleeing, suckling... Foraging agents react to their environment by selecting and moving to the area for which they perceive themselves to have the highest affinity. They select a destination (or choose to remain where they are) on the basis of their physiological state, location, and the perception of their surroundings. This is taken into account in the model using a 'perception-deliberation-decision-action' scheme *(e.g.,* Ferber, 1999).

In this study, the decision process of the rodent is limited to aiming to a selected destination and interacting with its target once arrived. Agent's speed, sensing radius and deliberation processes affect its response to its environment (Figure 1). A controller schedules the agents' steps and manages the seasonal fluctuation of the landscape.

```
Update physiological status
If current place is dangerous or overloaded
    Flee (remove target, select an aim and move at high speed)
Else
    If already gets target (other relative, burrow system, crop)
        If arrived
            Process target (eat, suckle, mate, enter burrow…)
            Update 'cognitive' status (target, desire)
        Else
            If moving target re-compute target's position
                Move towards target
    Else
        Perceive objects within sensing area
        Select desire (forage, reproduction, none, spawn, suckle)
        Elaborate set of alternatives
                (deliberate out of perceived objects given desire)
        Select target (out of possible alternatives
(closest+random)
        If target found
            Compute target position
            Move towards target
        Else wander (choose random aim and move)
Grow older (increment age)
Check death (age dependent death probability)
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Figure 1: simplified pseudocode for the processes performed by each rodent agent during one time step. Bold: sub-models not detailed here; italics: comments on the corresponding sub-model; underlined: processes involving time-scale dependency.

2.2 Use Case Description

A theoretical domain is used as a support for simulation. The simulated space mimics one real situation encountered in the French Poitou-Charentes region (*e.g.*, 46°16'9.91"N 0°24'26.07"W), an area colonized by the rodent species simulated. It is a square of 53x53 cells of 7.48 m side representing one 15.72 ha area. Various types of crops are arbitrarily disposed in the domain as well as human habitation, road and a motorway.

Rodents reproduce from April to October, during this period, reproduction prevails on foraging. When male mature agents perceive mature females they mate; females then produce offspring's after a gesta-tion length (mating latency and weaning are also formalized). Burrow systems are the third spatial entity considered. They are dug by female rodent agents and disappear within a week when they are empty. Burrow systems thus exist for limited periods of time; they are located in both the discrete and continuous space in which the agents move.

A common simulation output is presented on Figure 2. Rodents distribute themselves through time depending on the reproduction season and the evolution of the field statuses. They usually preferentially occupy perennial fields of meadow or alfalfa as well as roadside verges or field borders as described in the literature (*e.g.*, Briner et al., 2005, Topping et al., 2010). Population size (Figure 2 middle) shows a seasonal fluctuation with births occurring during the reproduction season. Mortality peaks occur when ploughing happens in a crop occupied by a colony of rodents. At a yearly scale, population may undergo acute decline (*e.g.*, year 7) leading to either population collapse or restoring. Mean dispersal (Figure 2 bottom) remains steady and fits with the observed vital domain of this species (Quéré and Le Louarn, 2011), maximum dispersal fluctuates at a value near the simulated domain side with less dispersal for females which remain more sedentary because of their childcare activity.

2.3 Time Scale Mechanisms Involved

Three major categories of processes are bound to the time scale used and vary accordingly to the time step chosen for simulation. The first involves the duration of each phase of the rodent life cycle (weaning, maturity, gestation length, etc.); the second concerns agents' sensing:

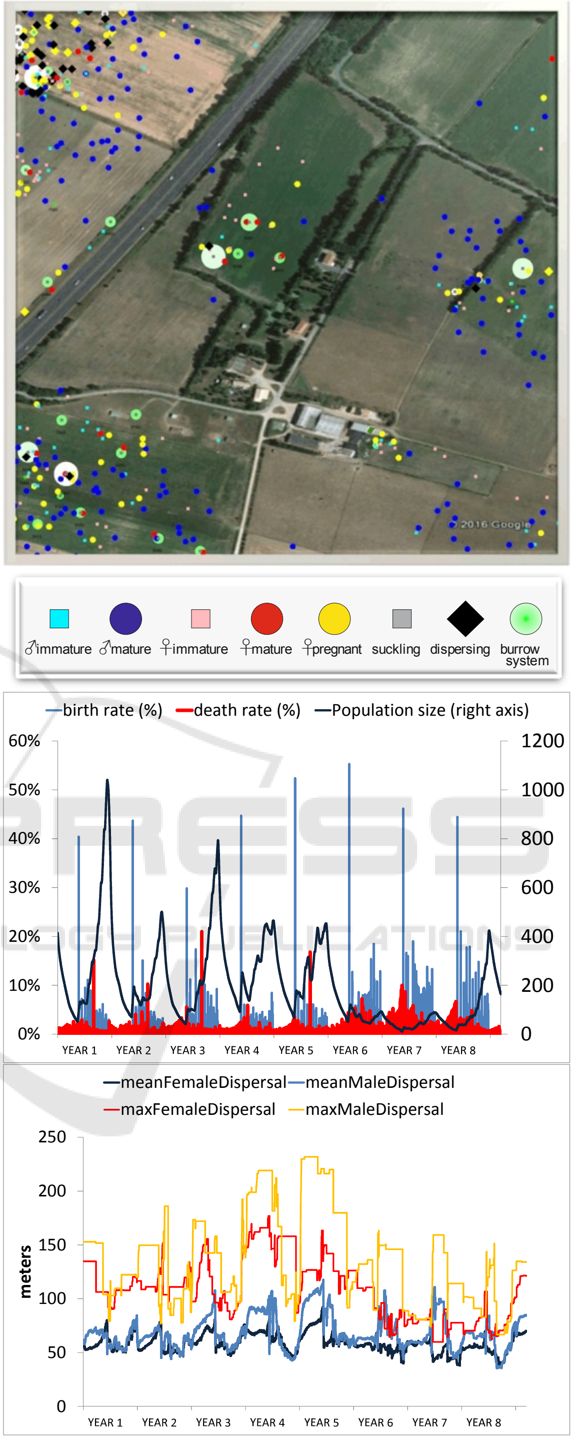

Figure 2: standard simulation outputs of the studied use case - common vole rodents in a fragmented agricultural landscape. Top: snapshot of the population distribution within the simulated domain; middle, population size and birth/death rates; bottom: evolution of mean and maximum dispersal within the period (simulation time step: 3hours).

Agents have a sensing area encompassing any object or agent (substrate nature, relatives, burrow systems) perceptible within one time step. It is defined as a fixed circle with a parameterized radius (*e.g.*, Jia *et al.*, 2018) corresponding to the vital range of this type of animal (Quéré and Le Louarn, 2011). The sensing area moves with the agent and is computed precisely from the continuous space coordinates. The radius value is declared in m/day and is adjusted to the time step (or tick) scale used by converting it into m/tick or cell/tick depending on the behaviour mechanism involved in rodent's activity.

The third category of process depending on time scale is the common speed of the agent which is also expressed in m/day and converted into m/tick. For a given time step, the rodent speed is fixed except in cases where either its current place has exceeded the cell or burrow system carrying capacity or if it arrived in a dangerous area (*e.g.*, road, motorway…). In such cases the rodent flees from its current place at a speed four times its normal speed until it reaches a place that is not overloaded.

2.4 Flexible Time-scale Implementation

To ensure the integrity of the multiple scales units and conversions dealt with and secure model's verification, we have first suffixed most methods or properties names with the units that characterize them (*e.g.*, meter, day, cell, tick, gramPerDay).

Time and space conversion is realized using an extension of the standard java Gregorian calendar which constitutes the time reference within the model. This class manages both a time amount and a time unit (*e.g.,* 3+hours). We also plugged a converter class providing all the necessary utilities to operate the needed conversions between time step units and universal units managed by the calendar. This permits conversion of speeds and sensing spheres depending of the time or space units in the continuous space and within the grid (*e.g.*, meter per day into meter per tick or into grid cells per tick).

2.5 Simulation Conditions and Sensitivity Analysis Performed

The rodent population is initialized with 400 individuals and 50 burrow systems representing a pioneer population density of 25 ind./ha. Simulations are run using time steps ranging *(i)* from 5 min to 90 min each 5 min, *(ii)* from 90 min to 48 hours each 10 min and *(iii)* from 48 hours to 9 days each 30 min. Two constraints are imposed to stop simulations. The first correspond to a maximum of three years simulation duration, giving a one-year cycle to allow the model to escape from initial conditions and two supplementary yearly cycles with similar cyclic patterns. Simulations are stopped at the beginning of the reproduction season where rodents' population is at its lowest. The second stop condition is triggered when either a maximum population of 6.000 individuals evolving within the domain is reached, that is a signature of a pullulating population, or when no female remains, hence signing a collapsing population. Two indicators are selected to study the effect of changing time step, the first one is the duration of the simulation; either max allowed time or population life before collapse. The second is the size of the population at the end of the simulation

3 RESULTS

Depending on the initial parameter values the simulated population may persist a few days to several centuries before collapsing. In the current model the latter case is rare and the population often collapses in the complex environment within which it evolves. This is expressed in Figure 3 where the time step values tested almost always result in the early extinction of the population, except for small tick values.

The range of values used in this sensitivity analysis is intentionally larger than the supposed realistic range of time step values; this makes it possible to highlight the artefactual behaviours related to the model function and the simplification that it brings. Thus, the right of the graph shows an increase in the lifetime of the population as the time step increases with a phase transition at a time step of 190 hours leading to a plateau. In those extreme situations from 20 to 190 hour per time step, the increase in the population life span is related to the increase in rodent speed and perception that allows them to reach their target more and more quickly during a single cycle (as of a time step equals to 63 hours, rodents acquire a complete perception of the domain at each tick). Detailed simulations observed there indicate a boundary conditions effect that becomes preponderant with a significant rodent density observed abutting the limits of the domain.

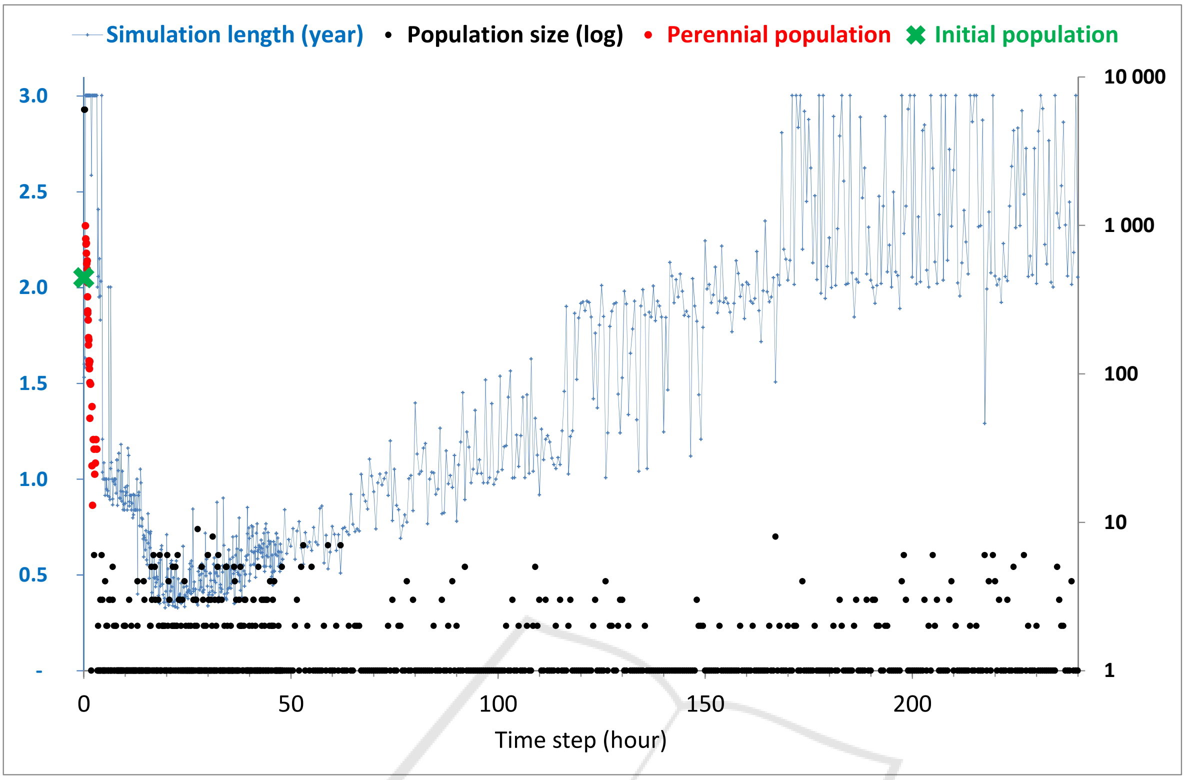

Figure 3: Selected output indicators of the time step sensitivity analysis. Dots: population size at the end of the simulation; dotted line: duration of the simulation. Simulations are stopped when the rodent population collapses, when it exceeds 6.000 individuals (proliferation), or when the duration reaches 3 years.

The last plateau to the right of the figure starts at the time step 190 hours when rodents acquire a speed per tick allowing them to traverse the whole domain modelled during a single time step. At this stage, any target is instantly reached. However, this functionality does not allow the population to persist and in this value range no population is viable. The observed outputs indicate high mortality peaks during the winter season. These peaks are attributed to the non-optimal positioning of rodents related to their excessive displacements.

For much shorter time steps (Figure 4), simulations indicate a range of tick values (in red) that enables a sustainable population over the medium term (*i.e.,* beyond the period presented here). Within this interval, the population remains at a sufficient level to resist the hazards of its environment. This range of values also reflects the adequate frequency of agents' deliberation/execution process. It lies in this case between 25 minutes and 3 hours with optimal value at about 45 min corresponding to an almost steady population (see illustration on Figure 2).

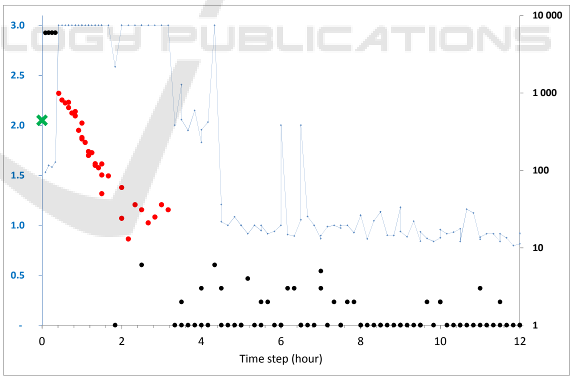

Figure 4: Sensitivity analysis outputs for small values of the time step: focus on the extreme left part of Figure 3; same caption used.

It can be also noticed that within this interval, the more the time step increases, the more the dynamics of the population deteriorates with a smaller and smaller size at the end of the simulation. This phenomenon can be attributed to a less efficient adaptation of the virtual population to its simulated environment. It can be also interpreted as a bias related to the method used for computing the rodents' sensing area according to the time step, which will be discussed in the next section.

For very small tick values (5 to 20 min), the observed phenomenon is a rodent outbreak. Detailed observation of each of these simulations (not figured) suggests that, using these small tick values, rodents' moves remain very limited from one time step to another. The burrow systems then constitute foci where rodents maintain themselves in dense groups that reproduce intensely. In addition, when burrows are established in stable areas, resident populations may be less subject to the hazards of the environment than when they move further.

4 DISCUSSION

Performing a sensitivity analysis of the model on a wide range of time scales provided two types of insights. On the one hand it permitted to get better understanding of the model function and limitations. On the other hand it provided a mean to infer a reasonable range of validity from the logic of the modelled processes, such as here the frequency of decision/action processes performed by rodents over a period of time and leading to a perennial population in a given environment. The valid range of frequency here suggests that rodents in the wild would perform a deliberation process from each 3 min to each 3 hours. To our knowledge, this value is not accessible to experimentation or sampling. However, it could constitute a clue to estimate the order of magnitude of the cognitive activity that these small animals realize in their environment. Nevertheless, these results have to be considered with caution and as only indicative since they come from a single parameter sensitivity analysis, that is, all other things being equal otherwise. It is almost certain that the model is also sensitive to numerous other aspects such as the spatial resolution or the initial conditions imposed. Changing values for these parameters would be susceptible to modify the resulting optimal time scale that rose out of the analysis. Multi-criteria sensitivity analysis (*e.g.*, Saltelli *et al.*, 2004) would therefore be necessary to get more confident insight into the model's potential.

Simulations indicated large variation of the selected indicator outputs; the population life time and size. In an ideal scheme, the expected outcome of such analysis would be that the simulated population dynamics and indicator values would remain unchanged whatever the time scale chosen.

Some contexts permits to reach such objective. These occur when relationships between time dependent parameters and time scale are linear. This was here the case for life traits parameters such as gestation, weaning duration, ageing.... Changing time scale did not change the rodent agents' life cycle whatever the time step chosen. Kuo *et al.* (2012) developed an epidemiological stochastic agent-based model where probabilities could be adjusted relative to time scale. In this case also, their study led to almost reproducible results whatever the time scale chosen. When however relationships between time step and time-dependent parameters are not linear, discrepancies appear and increasing biases occur with increasing changes in time steps.

This is particularly the case here for time scaling of agents' perception area. Little literature was found on formalization of agents' perception area. Jia et al. (2018) used sensing circle radius as the parameter defining the perception area of an agent. This parameter was also used in this study to define the agents' sensing area and perform the conversion from one time scale to the other. In a fixed time step context, this approach is indeed the more logical and straightforward. However, in a multi-time-scale context, where sensing area must be scaled as a function of time, it is not clear if this approach comes out as a satisfactory solution. Geometry calculations made before this study indicate that, in the case of a straight line movement, the cumulated area perceived by a rodent during several small time steps is greater than that of a circle corresponding to the area perceived on a larger time step equivalent to the sum of the previous ones. At the same time, if one considers that the rodent does not usually move in a straight line but in an erratic or semi-erratic trajectory such as in Lévy flight's (Chechkin *et al.*, 2008), as it is the case during foraging, this travelled area then decreases and converges toward the same order of magnitude than the integrated circle.

In any case therefore, the area actually perceived depends on the detail of the agent's trajectory. It is indeed logical that the perception area computed at any timescale depends on the simulated trajectories of rodents. Since these trajectories are moreover themselves dependent on time and objects, changing time scale produces biases in the model outcome that may be difficult to reduce.

5 CONCLUSION

Exploitation of the model output at different time scales proved valuable to better understand the model potential, limitation and functioning. This approach also provided a better insight on the plausible range of activity of rodents in the wild such as the frequency at which they should react to their environment by mean of the perception/ deliberation scheme, within the limitation of such simplified model.

This work also raises question on the best way to formalize sensing. In this domain, comparative study of different means to formalize time-dependent perception, for example by using a surface, a radius, or making agents' sensing area a time-independent parameter, would help improving modelling of ecosystem-dependent agents.

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REFERENCES

Al Rowaei, A. A., Buss, A. H., and Lieberman, S., 2011. The effects of time advance mechanism on simple agent behaviors in combat simulations. *Proc. Winter Simulation Conference*: 2431-2442.

Balci, O., 1998. Verification, Validation, and Accreditation. *Proc. Winter Simulation Conference*: 41-48.

Briner, T., Nentwig, W. and Airoldi, J.P., 2005. Habitat quality of wildflower strips for common voles (*Microtus arvalis*) and its relevance for agriculture. Agriculture, *Ecosystems and Environment* 105:173–179

Buss, A., Al Rowaei, A., 2010. A comparison of the accuracy of discrete event and discrete time. *Proc. Winter Simulation Conference*: 1468-1477.

Caillou, P., Gaudou, B., Grignard, A., Truong, C. Q. and Taillandier, P., 2017. A Simple-to-use BDI architecture for Agent-based Modeling and Simulation. *Advances in Social Simulation 2015*:15-28.

Chechkin, A., Metzler, R., Klafter, J. Gonchar, V., 2008. Introduction to the Theory of Lévy Flights. *In R. Klages, G. Radons, I.M. Sokolov (Eds), Anomalous Transport: Foundations and Applications*, Wiley-VCH, Weinheim.

DeAngelis, D. L., Mooij, W. M., 2003. In praise of mechanistically rich models. In: Canham, C. D., Cole, J. J., Lauenroth, W. K. (Eds.), Models in Ecosystem Science. *Princeton University Press, Princeton, New Jersey*, pp. 63–82.

Ferber J., 1999. Multi-agent systems: an introduction to distributed artificial intelligence*, Addison-Wesley Reading*.

Ferber, J. and Müller, J. P., 1996. Influences and reaction: a model of situated multi agent systems. *Proc. International Conference on Multi-Agent Systems* (ICMAS-96): 72-79.

Floudas, C. A., and Lin, X., 2004. Continuous-time versus discrete-time approaches for scheduling of chemical processes: a review. *Computers & Chemical Engineering*, 28(11): 2109-2129.

Fu, Z., Hao, L., 2018. Agent-based modeling of China's rural-urban migration and social network structure. *Physica A: Statistical Mechanics and its Applications*: 1061-1075.

Jia, J., Chen, J., Chang, G., Wen, Y., and Song, J., 2009. Multi-objective optimization for coverage control in wireless sensor network with adjustable sensing radius. *Computers & Mathematics with Applications*, 57(11-12), 1767-1775.

Kuo, C. T., Wang, D. W., and Hsu, T. S., 2012. A Simple Efficient Technique to Adjust Time Step Size in a Stochastic Discrete Time Agent-based Simulation. *In Proceedings of the 2nd International Conference on Simulation and Modeling Methodologies, Technologies and Applications - Volume 1: SIMULTECH*: 42-48.

Le Fur, J., Mboup, P.A., and Sall, M., 2017. A Simulation Model for Integrating Multidisciplinary Knowledge in Natural Sciences. Heuristic and Application to Wild Rodent Studies. *Proc. 7th Internat. Conf. Simul. And Model.Method., Technol.and Applic*. (Simultech), Madrid, july 2017: 340-347.

North, M. J., Howe, T. R., Collier, N. T., Vos, J. R., 2005. The Repast Simphony Development Environment. In, *Proc. Agent 2005 Conference on Generative Social Processes, Models, and Mechanisms*:13-15.

Ponomarenko, A., and Sinyakov, A., 2018. Impact of Banking Supervision Enhancement on Banking System Structure: Conclusions Delivred by Agent-Based Modelling. *Bank of Russia Working Paper Series wps19,* Bank of Russia.

Quéré, J.P. and Le Louarn, H., 2011. Les rongeurs de France: faunistique et biologie. *Quae ed., Paris*, ISBN: 978-2-7592-1033-6, 312p.

Railsback, S., Ayllón, D., Berger, U., Grimm, V., Lytinen, S., Sheppard, C., and Thiele, J., 2017. Improving execution speed of models implemented in NetLogo. *Journal of Artificial Societies and Social Simulation*, vol. 20, no. 3. doi:10.18564/jasss.3282

Saltelli, A., Tarantola, S., Campolongo, F., Ratto, M., 2004. Sensitivity Analysis in Practice. Wiley, New York.

Sauser, B., Baldwin, C., Pourreza, S., Randall, W., and Nowicki, D., 2018. Resilience of small-and medium-sized enterprises as a correlation to community impact: an agent-based modeling approach. *Natural Hazards*, 90(1): 79-99.

Singh, K., Ahn, C. W., Paik, E., Bae, J. W., and Lee, C. H., 2018. A Micro-Level Data-Calibrated Agent-Based

Model: The Synergy between Microsimulation and Agent-Based Modeling. *Artificial life*: 128-148.
Squazzoni, F., 2010. The impact of agent-based models in the social sciences after 15 years of incursions. *History of economic ideas*: 197-233.
Stanilov, K., 2012. Space in agent-based models. *In Agent-based models of geographical systems*: 253-269). Springer, Dordrecht.
Topping C. J., Høye T. T. and Olesen C. R., 2010. Opening the black box – development, testing and documentation of a mechanistically rich agent-based model. *Ecological Modelling* 221 (2) 245-255.
Watts, J., 2016. Scale Dependency in Agent-Based Modeling: How Many Time Steps? How Many Simulations? How Many Agents?. *Uncertainty and Sensitivity Analysis in Archaeological Computational Modeling,* Springer, Cham.: 91-111.
White, A. A., 2016. The sensitivity of demographic characteristics to the strength of the population stabilizing mechanism in a model hunter-gatherer system. *In Uncertainty and Sensitivity Analysis in Archaeological Computational Modeling,* Springer, Cham.: 113-130.
Whitley, T. G., 2016. Archaeological simulation and the testing paradigm. *In Uncertainty and Sensitivity Analysis in Archaeological Computational Modeling* Springer, Cham.: 131-156.