# A Procedure to Generate Discrete MIMO Closed-loop Benchmark Via LFT with Application to State Space Identification

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Abstract: In this paper we use the conformal transformation known as linear fractional transformation (LFT), with the purpose of generating a discrete multivariable closed-loop benchmark from continuous multivariable closed-loop control system, having in mind state space identification. To reach this objective we propose a procedure based on the general framework representation (GFR) and on the multi input multi output (MIMO) LFT bilinear discretization process. We first use the LFT tool to obtain the continuous joint control-output (augmented) system form for representing the canonical closed-loop continuous system. Afterwards, we discretize the augmented continuous closed-loop system in order to obtain an augmented discrete model, then, we calculate the discrete plant and controller in the state space form. An application to the multivariable control of a continuous chemical reactor is presented and also we use the discrete benchmark generated to identify a state space model an example of the potential of the our proposal.



## 1 INTRODUCTION

The use of multivariable benchmarks allows the comparison of new methods with classical methods at low cost. In several areas such as robotics (Aly et al., 2017), systems control (Wu et al., 2017), systems identification (Ase and Katayama, 2015), among others, testing algorithms and comparing results are essential to evaluate the new methods under development and then their comparisons with the already existing ones.

In order to generate a discrete benchmark for the canonical form presented in Figure 1 and in the augmented form (joint control-output), we present in this work a procedure to obtain discrete benchmarks having in mind the identification problem. The proposed procedure shows how to obtain the LFT augmented representation of the continuous closed-loop system widely used in identification methods.

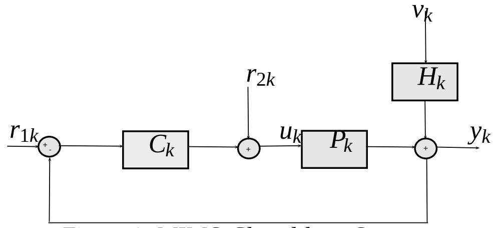


Figure 1: MIMO Closed-loop System.

Our goal in this paper is to propose a simple but powerful methodology to generate discrete MIMO closed-loop benchmarks. It is based on the discretization of MIMO continuous closed-loop control systems in the LFT augmented form representation

The method proposed here guarantees the features preservation of the continuous system by the use of a conformal transformation known as Linear Fractional Transformation (LFT), widely used in control theory, usually for robust control analysis and synthesis. Indeed this multivariable conformal mapping is a Möbius transformation, a classical and fundamental concept in theory of complex analysis and its multiple applications (Nehari, 1952; Cohn, 1967; Ungar, 1997; Richter et al., 1999a; Richter et al., 1999b; Lui et al., 2007). For our proposal we used the LFT as a general framework representation connecting the state space and the input-output representations for control systems (Doyle, 1984), with the following purposes: i) to represent augmented continuous/discrete MIMO LTI systems in closed-loop, and ii) to discretize continuous systems to generate multivariable benchmarks.

This procedure can supply discrete MIMO LTI benchmarks exploring the discretization of continuous MIMO control systems in the augmented rep-



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resentation of Figure 1 and contribute very effectively for discrete state space identification of MIMO closed-loop systems.

This work is organized as follows: first, a brief introduction of the concepts of augmented systems and linear fractional transformation are presented; then, the methodology for the representation of an augmented system via LFT is shown. Immediately afterwards the discretization procedure via LFT of the augmented continuous system is presented, and then the calculation of the discrete plant model and discrete control model from the discrete augmented system are presented. Finally applications of the procedure to obtain multivariable benchmarks for a multivariable chemical-reactor control system and the subspace identification of the augmented system are presented .

## 2 LINEAR FRACTIONAL TRANSFORMATION

The linear fractional transformation (Nehari, 1952; Zhou et al., 1996; Doyle et al., 1991) for a complex variable $s \in \mathbb{C}^1$ is a function $F : \mathbb{C} \mapsto \mathbb{C}$ that can be generalized for the matrix case with the complex matrix of coefficients:

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \in \mathbb{C}^{(p1+p2)\times(q1+q2)}, \quad (1)$$

and the matrix $\triangle_\bullet \in \mathbb{C}^{(q2\times p2)}$.

The LFT has two forms, the lower one given by:

$$\mathcal{F}_l(M, \triangle_l) \triangleq M_{11} + M_{12} \triangle_l (I - M_{22} \triangle_l)^{-1} M_{21} \quad (2)$$

and the upper:

$$\mathcal{F}_u(M, \triangle_u) \triangleq M_{22} + M_{21} \triangle_u (I - M_{11} \triangle_u)^{-1} M_{12} \quad (3)$$

supposing that $(I - M_{22} \triangle_l)^{-1}$ and $(I - M_{11} \triangle_u)^{-1}$, exist.

### 2.1 Continuous Augmented Systems

Closed-loop continuous systems presented in Figure 2, can be represented as augmented systems (Verhaegen, 1993; van der Veen et al., 2013; Ljung, 1999); they have taken this name because the size of the state vector is increased as:

$$x(t) = \begin{bmatrix} x_p(t) \\ x_c(t) \end{bmatrix},$$

where $x_p(t) \in \mathcal{R}^n$ is the state vector associated to the plant, and $x_c(t) \in \mathcal{R}^m$ is the state vector associated to the controller.

can be formulated as in the Figure 3,

The set plant/controller is given by:

$$\begin{aligned} \bar{x}_p(t) &= A^c x_p(t) + B^c u(t) \\ y(t) &= C^c x_p(t) + D^c u(t) \end{aligned} \quad (4)$$

and

$$\begin{aligned} \dot{x}_c(t) &= A_c^c x_c(t) + B_c^c [r_1(t) - y(t)] \\ u(t) &= r_2(t) + C_c^c(t) x_c(t) + D_c^c [r_1(t) - y(t)] \end{aligned} \quad (5)$$

where $A^c$, $B^c$, $C^c$, $D^c$, $A_c^c$, $B_c^c$, $C_c^c$, $D_c^c$, are the continuous matrices of the plant and the controller, respectively. The signals $u(t) \in \mathcal{R}^{nu}$, $y(t) \in \mathcal{R}^{my}$, $r_1(t) \in \mathcal{R}^{nr_1}$ and $r_2(t) \in \mathcal{R}^{nr_2}$, are the inputs, outputs and the exogenous inputs.

The augmented system can be expressed by:

$$\begin{aligned} \dot{x}(t) &= \bar{A}_{TC} x(t) + \bar{B}_{TC} \tilde{u}(t) \\ \tilde{y}(t) &= \bar{C}_{TC} x(t) + \bar{D}_{TC} \tilde{u}(t) \end{aligned} \quad (6)$$

the continuous matrices $\bar{A}_{TC}$, $\bar{B}_{TC}$, $\bar{C}_{TC}$, $\bar{D}_{TC}$ describe the continuous augmented system (the calculation of these matrices are presented in Section 3); this set of matrices has adequate sizes. The signals

$$\tilde{u}(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix},$$

$$\tilde{y}(t) = \begin{bmatrix} y(t) \\ u(t) \end{bmatrix}$$

in the augmented system in (6) represent the joint inputs and joint outputs respectively.

## 3 DISCRETE AUGMENTED SYSTEMS VIA LFT REPRESENTATION

The system in Figure 1[2] with plant and controller is given by:

$$\begin{aligned} x_{pk+1} &= A x_{pk} + B u_k \\ y_k &= C x_{pk} + D u_k \end{aligned} \quad (7)$$

and

$$\begin{aligned} x_{ck+1} &= A_c x_{ck} + B_c [r_{1k} - y_k] \\ u_k &= r_{2k} + C_c x_{ck} + D_c [r_{1k} - y_k] \end{aligned} \quad (8)$$

and the problem of representing the control-output set can be given as an output/input relationship.

[1]the set of complex variables is denoted by: $\mathbb{C}$

[2]In Equations (7) and (8), $A$, $B$, $C$, $D$, $A_c$, $B_c$, $C_c$, $D_c$, are the discrete matrices of the plant and the controller, respectively. The signals $u_k \in \mathcal{R}^{nu}$, $y_k \in \mathcal{R}^{my}$, $r_{1k} \in \mathcal{R}^{nr_1}$ and $r_{2k} \in \mathcal{R}^{nr_2}$, are the discrete inputs, outputs and the exogenous inputs.

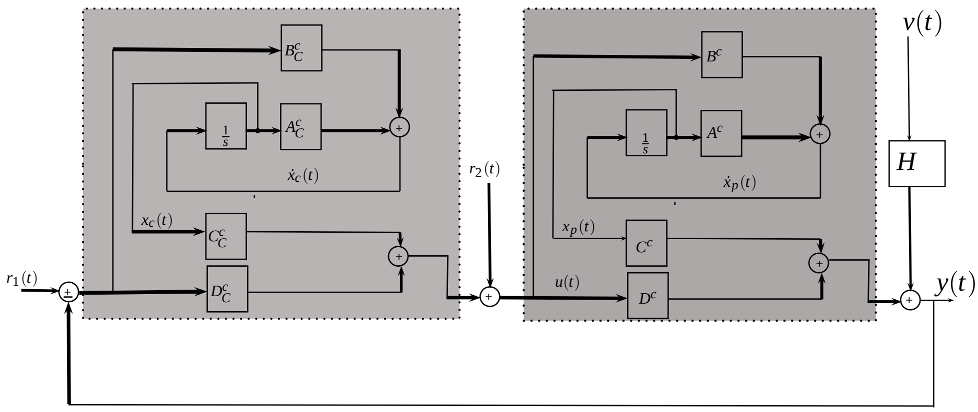


Figure 2: Continuous Closed-loop MIMO System.

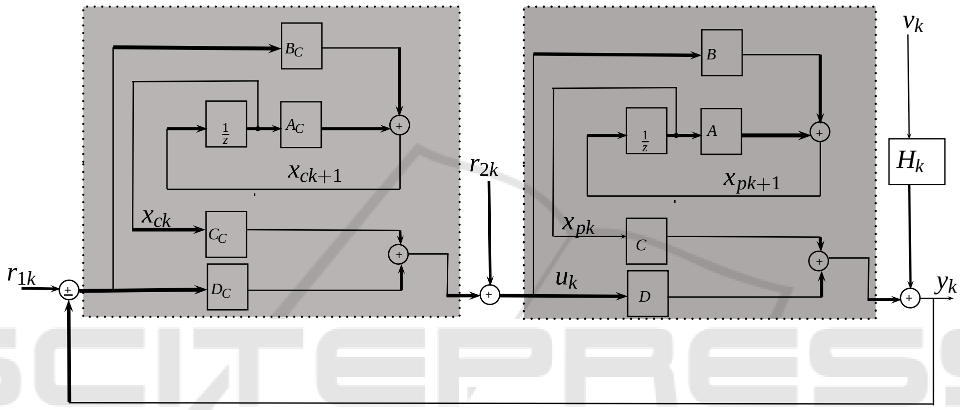


Figure 3: Discrete Closed-loop MIMO System.



With the assumption that $v_k = 0$ in Figure 2, we have that the control-output[3] set is given by:

$$\begin{bmatrix} x_{pk+1} \\ x_{ck+1} \\ u_{sk} \\ y_k \\ u_k \end{bmatrix} = M \begin{bmatrix} x_{pk} \\ x_{ck} \\ r_{sk} \\ r_{2k} \\ r_{1k} \end{bmatrix} \tag{9}$$
$$r_{sk} = Du_k$$

where $M$ is the matrix calculated from the topology on Figure 2 as a general framework representation via LFT given by:

$$M = \left[\begin{array}{cc|c|cc} A-BD_cC & BC_c & -BD_c & BD_c & B \\ -B_cC & A_c & B_c & B_c & 0 \\ \hdashline -D_cC & C_c & -D_c & D_c & I \\ \hdashline C & 0 & I & 0 & 0 \\ -D_cC & C_c & -D_c & D_c & I \end{array}\right] \tag{10}$$

[3] $u_k$ in the Figure 3 is splitted in two parts, the signal $u_k$ before the grey box is called $u_k$, and the signal $u_k$ after the grey box is called $u_{sk}$

or

$$M = \left[\begin{array}{c|c|c} A_0 & B_0 & B_2 \\ \hdashline C_0 & D_{00} & D_{01} \\ \hdashline C_2 & D_{10} & D_{11} \end{array}\right] \tag{11}$$

Then the system can be represented by the LFT as:

$$G(z) = \mathcal{F}_u\left\{\mathcal{F}_l(M,D), z^{-1}\right\} \tag{12}$$

with the direct transfer matrix $D \neq 0$ in (12), the system can be represented by Figure 4

The LFT in (12), can be simplified if $D = 0$, in this case the system can be represented by:

$$M = \left[\begin{array}{cc|cc} \overbrace{A-BD_cC \quad BC_c}^{\bar{A}} & & \overbrace{BD_c \quad B}^{\bar{B}} & \\ -B_cC & A_c & B_c & 0 \\ \hdashline C & 0 & 0 & 0 \\ \underbrace{-D_cC \quad C_c}_{\bar{C}} & & \underbrace{D_c \quad I}_{\bar{D}} & \end{array}\right] \tag{13}$$

The system in (9), with $D = 0$ is expressed by:

$$\begin{bmatrix} x_{pk+1} \\ x_{ck+1} \\ y_k \\ u_k \end{bmatrix} = \left[\begin{array}{c|c} \bar{A} & \bar{B} \\ \hdashline \bar{C} & \bar{D} \end{array}\right] \begin{bmatrix} x_p \\ x_c \\ r_{2k} \\ r_{1k} \end{bmatrix} \tag{14}$$

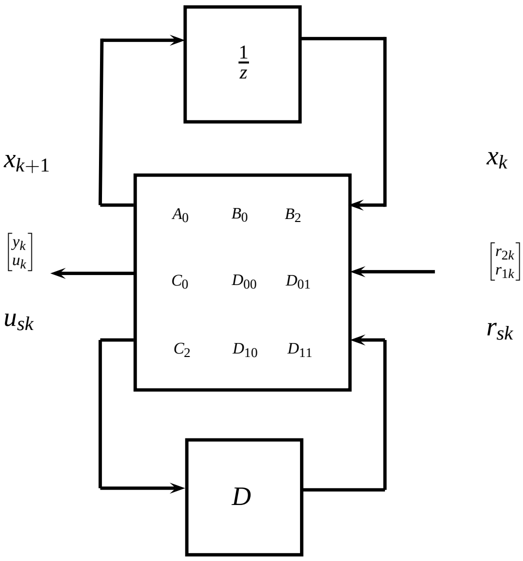


Figure 4: LFT Closed-loop System Diagram.

Then the discrete augmented system is given by:

$$\begin{aligned} x_{k+1} &= \bar{A}x_k + \bar{B}\tilde{u}_k \\ \tilde{y}_k &= \bar{C}x_k + \bar{D}\tilde{u}_k \end{aligned} \tag{15}$$

where $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ are the discrete state matrices with adequate sizes and the discrete signals

$$\tilde{u}_k = \begin{bmatrix} r_{2k} \\ r_{1k} \end{bmatrix} \in \mathcal{R}^{nr_1+nr_2},$$

$$\tilde{y}_k = \begin{bmatrix} y_k \\ u_k \end{bmatrix} \in \mathcal{R}^{my+nu}$$

and

$$x_k = \begin{bmatrix} x_{pk} \\ x_{ck} \end{bmatrix} \in \mathcal{R}^{n+m}$$

represents the joint input , and the joint output, respectively.

Finally, the control and the plant, calculated from the discrete augmented system are given by:

$$P_k = \left[ \begin{array}{c|c} A_0 - B_2 D_{11}^{-1} C_2 & B_2 D_{11}^{-1} \\ \hline C_1 & 0 \end{array} \right] \tag{16}$$

and

$$C_k = \left[ \begin{array}{c|c} A_0 - B_2 D_{11}^{-1} C_2 & B_0 - B_2 D_{11}^{-1} D_{10} \\ \hline D_{11}^{-1} C_2 & D_{11}^{-1} D_{10} \end{array} \right] \tag{17}$$

## 4 AUGMENTED CONTINUOUS SYSTEM DISCRETIZATION

In this section using the properties of the LFT representation and the bilinear approximation (18), we obtain the discrete model given in the equation (15). If the relationship between the $s$ and $z$ complex frequencies, is given by:

$$s \approx \frac{2}{T_d}\left(\frac{z+1}{z-1}\right) \tag{18}$$

then $s$ can be expressed as an upper LFT given by:

$$\frac{1}{s} \approx \mathcal{F}_u(N, z^{-1}I) \tag{19}$$

with matrices:

$$N = \begin{bmatrix} -I & -\frac{\sqrt{2}Td}{2}I \\ \sqrt{2}I & \frac{Td}{2}I \end{bmatrix},$$

and

$$\triangle = z^{-1}I$$

where $T_d$ represents the sampling period.

From $N$ and $z^{-1}$ in (19) we obtain the discrete closed-loop system LFT represented in Figure 5, where the star product between the state matrices and

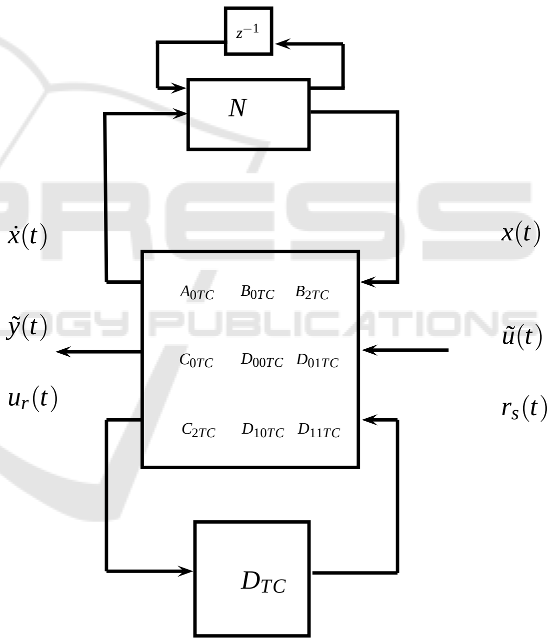


Figure 5: LFT Closed-Loop System Discretization Diagram.

the $N$ matrix, gives

$$\mathcal{F}_u\left\{\mathcal{F}_l\left\{N \star \begin{bmatrix} A_{0TC} & B_{0TC} & B_{2TC} \\ C_{0TC} & D_{00TC} & D_{01TC} \\ C_{2TC} & D_{10TC} & D_{11TC} \end{bmatrix}, D_{TC}\right\}, z^{-1}\right\}$$

where

$$N \star \begin{bmatrix} A_{0TC} & B_{0TC} & B_{2TC} \\ C_{0TC} & D_{00TC} & D_{01TC} \\ C_{2TC} & D_{10TC} & D_{11TC} \end{bmatrix} = \tilde{M}$$

is given by (20), and $\mathcal{F}_u\left\{\mathcal{F}_l\left\{\tilde{M}, D\right\}, z^{-1}\right\}$ contains the discretized matrices of the continuous system.

$$\tilde{M} = \left[\begin{array}{c|c|c} (I+\frac{T_d}{2}A_{0TC})(I-\frac{T_d}{2}A_{0TC})^{-1} & \sqrt{2}\frac{T_d}{2}(I-\frac{T_d}{2}A_{0TC})^{-1}B_{0TC} & \sqrt{2}\frac{T_d}{2}(I-\frac{T_d}{2}A_{0TC})^{-1}B_{2TC} \\ \hline \sqrt{2}C_{0TC}(I-\frac{T_d}{2}A_{0TC})^{-1} & C_{0TC}\frac{T_d}{2}(I-\frac{T_d}{2}A_{0TC})^{-1}B_{0TC}+D_{00TD} & C_{0TC}\frac{T_d}{2}(I-\frac{T_d}{2}A_{0TC})^{-1}B_{0TC}+D_{01TD} \\ \hline \sqrt{2}C_{2TC}(I-\frac{T_d}{2}A_{0TC})^{-1} & C_{2TC}\frac{T_d}{2}(I-\frac{T_d}{2}A_{0TC})^{-1}B_{2TC}+D_{10TD} & C_{2TC}\frac{T_d}{2}(I-\frac{T_d}{2}A_{0TC})^{-1}B_{2TC}+D_{11TD} \end{array}\right] \tag{20}$$

## 5 BENCHMARK GENERATION

The proposed procedure presented here can be summarized by the following steps: i. Represent the control system in closed-loop as an augmented model in the joint control-output form. ii. Discretize the continuous augmented model via LFT, and iii. Calculate the discrete controller and plant from the discrete augmented model .

In (MacFarlane and Kouvaritakis, 1977) is presented the design of a controller for a continuous chemical reactor; this model has been widely used in the literature. First we obtain the augmented continuous system representation according to the procedure described above:

$$M_{TC} = \left[\begin{array}{cc|cc} \multicolumn{2}{c|}{\overbrace{\begin{matrix} A_{TC}-B_{TC}D_{cTC}C_{TC} & B_{TC}C_{cTC} \\ -B_{cTC}C_{TC} & A_{cTC}\end{matrix}}^{\bar{A}_{TC}}} & \multicolumn{2}{c}{\overbrace{\begin{matrix} B_{TC}D_{cTC} & B_{TC} \\ B_{cTC} & 0 \end{matrix}}^{\bar{B}_{TC}}} \\ \hline \multicolumn{2}{c|}{\underbrace{\begin{matrix} C_{TC} & 0 \\ -D_{cTC}C_{TC} & C_{cTC}\end{matrix}}_{\bar{C}_{TC}}} & \multicolumn{2}{c}{\underbrace{\begin{matrix} 0 & 0 \\ D_{cTC} & I \end{matrix}}_{\bar{D}_{TC}}} \end{array}\right] \tag{21}$$

The coefficient matrices of the augmented continuous system (21), are given in (22).

Then the discretization of the continuous system is performed. The matrix $\tilde{M}$ is calculated by (20), and given by:

$$\tilde{M} = \left[\begin{array}{c|c} \overbrace{\begin{matrix} A-BD_cC & BC_c \\ -B_cC & A_c\end{matrix}}^{\bar{A}_d} & \overbrace{\begin{matrix} BD_c & B \\ B_c & 0 \end{matrix}}^{\bar{B}_d} \\ \hline \underbrace{\begin{matrix} C & 0 \\ -D_cC & C_c\end{matrix}}_{\bar{C}_d} & \underbrace{\begin{matrix} 0 & 0 \\ D_c & I \end{matrix}}_{\bar{D}_d} \end{array}\right]$$

The coefficients matrices of the augmented discrete system, are given in (23). Finally the discrete plant and controller, are calculated by (16) and (17). The plant matrices are given in (24), an the controller matrices by (25)

### 5.1 Closed-loop State Space Identification of the Augmented System

In this section, we show how to use the benchmark in (23). First we use the joint input $\tilde{u}_k$ to excite the discrete augmented model in (23) in order to obtain the joint output $\tilde{y}_k$. The second step is the use of a subspace method to identify the augmented system; in this work we use a Canonical Correlation Analysis identification method presented in (Katayama and Picci, 1999; Forero et al., 2015), to obtain the state space matrices. The discrete augmented matrices identified are presented in (26).

## 6 CONCLUSION

In this work a simple and efficient procedure is proposed to obtain discrete multivariable benchmarks for closed-loop control systems from continuous MIMO control systems, widely used to design, to evaluate and to test its performance. The procedure allows to find benchmarks for data generation, in the joint control-output form, which are very useful for closed-loop systems identification. It also allows the use of the canonical feedback form with MIMO plant and controller models supposedly known for discrete MIMO state space identification. The features of the continuous system, due to the augmented LFT representation of the discrete system are conserved.

Finally, *i)* a discrete MIMO benchmark of a chemical reactor system is provided by our proposal for tests and comparisons of multivariable discrete identification techniques in closed-loop and *ii)* a state space augmented closed-loop identification is provided using the discrete benchmark and the Canonical Correlation method for LTI systems identification.

$$\bar{A}_{TC}=\begin{bmatrix}1.3800 & -0.2077 & 6.7150 & -5.6760 & 0 & 0\\ -0.5814 & -61.0800 & 0 & 0.6750 & 0 & 12.7778\\ -30.3930 & -7.0870 & -38.1140 & 37.3530 & 16.5165 & 8.8480\\ 0.0480 & -7.0870 & 1.3430 & -2.1040 & 0 & 2.5560\\ -4.0000 & 0 & -4.0000 & 4.0000 & 0 & 0\\ 0 & -4.0000 & 0 & 0 & 0 & 0\end{bmatrix}\quad \bar{D}_{TC}=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 10 & 1 & 0\\ -10 & 0 & 0 & 1\end{bmatrix}$$

$$\bar{B}_{TC}=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 56.79 & 5.679 & 0\\ 31.46 & 11.360 & 1.136 & -3.1460\\ 0 & 11.36 & 1.136 & 0\\ 4.00 & 0 & 0 & 0\\ 0 & 4.00 & 0 & 0\end{bmatrix}\bar{C}_{TC}=\begin{bmatrix}1.00 & 0 & 1.00 & -1.00 & 0 & 0\\ 0 & 1.00 & 0 & 0 & 0 & 0\\ 0 & -10.00 & 0 & 0 & 0 & 2.25\\ 10.00 & 0 & 10.00 & -10.00 & -5.25 & -2.00\end{bmatrix} \tag{22}$$

$$\bar{A}_d=\begin{bmatrix}1.0013 & -0.0002 & 0.0066 & -0.0056 & 0.0001 & 0.000\\ -0.0006 & 0.9407 & -0.0000 & 0.0007 & -0.0000 & 0.012\\ -0.0299 & -0.0069 & 0.9625 & 0.0367 & 0.0162 & 0.008\\ 0.0000 & -0.0069 & 0.0013 & 0.9979 & 0.0000 & 0.002\\ -0.0039 & 0.0000 & -0.0039 & 0.0039 & 1.0000 & -0.000\\ 0.0000 & -0.0039 & 0.0000 & -0.0000 & 0.0000 & 1.000\end{bmatrix}$$

$$\bar{B}_d=\begin{bmatrix}0.0001 & 0.000 & 0.0000 & -0.0000\\ -0.0000 & 0.039 & 0.0039 & 0.0000\\ 0.0219 & 0.007 & 0.0008 & -0.0022\\ 0.0000 & 0.007 & 0.0008 & -0.0000\\ 0.0028 & -0.000 & 0.0000 & 0.0000\\ 0.0000 & 0.002 & -0.0000 & -0.0000\end{bmatrix}$$

$$\bar{C}_d=\begin{bmatrix}1.3940 & -0.0002 & 1.391 & -1.390 & 0.011 & 0.004\\ -0.0004 & 1.3723 & -0.000 & 0.000 & -0.000 & 0.008\\ 0.0040 & -13.7290 & 0.000 & -0.004 & 0.000 & 3.094\\ 13.9544 & 0.0040 & 13.928 & -13.921 & -7.309 & -2.784\end{bmatrix}$$

$$\bar{D}_d=\begin{bmatrix}0.0155 & 0.000 & -0.0000 & -0.0015\\ -0.0000 & 0.027 & 0.0028 & 0.0000\\ 0.0000 & 9.728 & 0.9724 & -0.0000\\ -9.8554 & -0.003 & 0.0000 & 0.9845\end{bmatrix} \tag{23}$$

$$\bar{A}_{pd}=\begin{bmatrix}1.0014 & -0.0002 & 0.0067 & -0.0057 & -0.0000 & -0.0000\\ -0.0006 & 0.9957 & -0.0000 & 0.0007 & 0.0000 & 0\\ 0.0011 & 0.0043 & 0.9934 & 0.0059 & 0 & 0\\ 0.0000 & 0.0043 & 0.0013 & 0.9979 & -0.0000 & -0.0000\\ -0.0040 & 0.0000 & -0.0040 & 0.0040 & 1.0000 & 0.0000\\ 0.0000 & -0.0040 & 0.0000 & -0.0000 & -0.0000 & 1.0000\end{bmatrix}\quad \bar{B}_{pd}=\begin{bmatrix}0.0000 & -0.0000\\ 0.0040 & 0.0000\\ 0.0008 & -0.0022\\ 0.0008 & -0.0000\\ -0.0000 & 0.0000\\ -0.0000 & -0.0000\end{bmatrix}$$

$$\bar{D}_{pd}=0_{2\times 2}\quad \bar{C}_{pd}=\begin{bmatrix}1.3940 & -0.0002 & 1.3914 & -1.3907 & 0.0115 & 0.0044\\ -0.0004 & 1.3723 & -0.0000 & 0.0005 & -0.0000 & 0.0088\end{bmatrix} \tag{24}$$



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$$\bar{A}_{cd} = \begin{bmatrix} 1.0014 & -0.0002 & 0.0067 & -0.0057 & -0.0000 & -0.0000 \\ -0.0006 & 0.9957 & -0.0000 & 0.0007 & 0.0000 & 0 \\ 0.0011 & 0.0043 & 0.9934 & 0.0059 & 0 & 0 \\ 0.0000 & 0.0043 & 0.0013 & 0.9979 & -0.0000 & -0.0000 \\ -0.0040 & 0.0000 & -0.0040 & 0.0040 & 1.0000 & 0.0000 \\ 0.0000 & -0.0040 & 0.0000 & -0.0000 & -0.0000 & 1.0000 \end{bmatrix} \quad \bar{B}_{cd} = \begin{bmatrix} -0.0001 & -0.0000 \\ 0.0039 & -0.0390 \\ -0.0211 & -0.0101 \\ 0.0008 & -0.0079 \\ 0.0000 & 0.0000 \\ -0.0000 & 0.0001 \end{bmatrix}$$

$$\bar{D}_{cd} = \begin{bmatrix} -0.0000 & 10.0045 \\ -10.0105 & -0.0040 \end{bmatrix} \quad \bar{C}_{cd} = \begin{bmatrix} 0.0041 & -14.1182 & 0.0000 & -0.0048 & -0.0000 & 3.1820 \\ 14.1740 & 0.0042 & 14.1480 & -14.1406 & -7.4246 & -2.8284 \end{bmatrix} \tag{25}$$

Sampled period for discretization $T_d = 1 \times 10^{-3}$

$$\bar{A}_{id} = \begin{bmatrix} 0.9689 & -0.01256 & -0.007291 & -0.004276 & -0.006158 & 0.01881 \\ -0.01378 & 0.9934 & -0.004539 & -0.0001598 & -0.001776 & 0.005776 \\ -0.00137 & 0.008931 & 0.9626 & -0.003548 & -0.0101 & 0.01289 \\ -0.00589 & -0.004834 & 0.006397 & 0.9987 & 0.0006793 & 0.001603 \\ 0.001286 & -0.0001574 & 0.001982 & -0.0002916 & 0.9988 & -0.001973 \\ 0.03105 & 0.01546 & 0.0002839 & 0.00601 & 0.00698 & 0.9799 \end{bmatrix}$$

$$\bar{B}_{id} = \begin{bmatrix} 0.0005063 & 0.002138 & 0.0002813 & -0.0001329 \\ 0.0006053 & 0.0008545 & 3.315 \times 10^{-5} & 0.0001523 \\ -0.002271 & 0.0008675 & 0.0001492 & 0.000295 \\ 0.0003701 & 0.0002612 & 0.0004898 & -6.727 \times 10^{-5} \\ -0.0002263 & 0.0003804 & -0.0003407 & 0.0001433 \\ -0.0001153 & -0.001831 & -0.0001196 & 9.156 \times 10^{-5} \end{bmatrix}$$

$$\bar{C}_{id} = \begin{bmatrix} 4.72 & 5.83 & -10.86 & -1.127 & -2.734 & 2.343 \\ 12.95 & 5.012 & 4.647 & 2.117 & 3.557 & -8.537 \\ -126.9 & -49.75 & -46.05 & -22.24 & -34.22 & 84.22 \\ 48.55 & 46.58 & -102.8 & -13.17 & -28.54 & 25.91 \end{bmatrix}$$

$$\bar{D}_{id} = \begin{bmatrix} 0.01549 & 6.057 \times 10^{-6} & -1.328 \times 10^{-8} & -0.001548 \\ -1.104 \times 10^{-8} & 0.02757 & 0.002755 & 1.147 \times 10^{-9} \\ 1.104e \times 10^{-7} & 9.729 & 0.9724 & -1.147 \times 10^{-8} \\ -9.855 & -0.003829 & 1.089 \times 10^{-5} & 0.9845 \end{bmatrix} \tag{26}$$

Sampled period$T_d = 1 \times 10^{-3}$


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