Visual GISwaps - An Interactive Visualization Framework for Geospatial Decision Making

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Keywords: GIS, Decision Making, Interactive Visualization, GISwaps.

Abstract: Different visualization techniques are frequently used in geospatial information systems (GIS) to support geospatial decision making. However, visualization in GIS context is usually limited to the initial phase of the decision-making process, i.e. situation analysis and problem recognition. This is partly due to the choice of methods used in GIS multi-criteria decision-making (GIS-MCDM) that usually deploy some non-interactive approach, leaving the decision maker little or no control over the calculation of overall values for the considered alternatives; the role of visualization is thus reduced to presenting the final results of the computations. The contributions of this paper are twofold. First, we introduce GISwaps, a novel, intuitive interactive method for decision making in geospatial context. The second and main contribution is an interactive visualization of the choice phase of the decision making process. The visualization allows the decision maker to explore the consequences of trade-offs and costs accepted during the iterative decision process, both in terms of the abstract relation between different decision variables and in spatial context.

1 INTRODUCTION

The main purpose of geographic information systems (GIS) is providing support for decision making. The term GIS in itself is not easily defined, as it has come to symbolize a technology, an industry, a way of doing work etc. (Chrisman, 1999). In this paper we use the term as defined in Cowen (1988), where GIS is recognized as "a decision support system involving the integration of spatially referenced data in a problem solving environment".

In the last couple of decades, a whole new interdisciplinary field of study, commonly referred to as GIS-based multi-criteria decision analysis (GIS-MCDA), has emerged. In Malczewski and Rinner (2015), GIS-MCDA is defined as "a collection of methods and tools for transforming and combining geographic data and preferences (value judgments) to obtain information for decision making." The most commonly accepted generalization of the decision-making process is suggested by Simon (1960), with *intelligence*, *design* and *choice* as three major phases. During the intelligence phase, a problem or a situation that calls for a decision is identified and formulated. This phase involves data collection, exploration and preprocessing. The alternatives, or the set of possi-

ble solutions, are defined in the design phase. Finally, in the choice phase, the alternatives are evaluated and the most appropriate alternative or set of alternatives is selected.

There is a vibrant community within the GIS-MCDA performing research in the field of visualization. However, Andrienko and Andrienko (2003) point out that, while visualization plays an important role in the initial phase of the decision-making process, it is rarely used in the design phase and the choice phase. GIS decision-making requires more extensive use of visualization and interaction even during the decision process itself, i.e. during making the actual choices. In order to decide whether or not a trade-off to be made is feasible or admissible, the decision maker needs to see how an option is positioned in both the geographical and the attribute space, as well as how it compares to other options (Andrienko and Andrienko, 2003). Limited use of visualization in the choice-phase in geospatial decision making is in part related to the choice of multi-criteria decision-making methods used in GIS-MCDM. This phase of the decision-making process in GIS context is most commonly performed using some weighting method to derive the weights of the criteria, i.e. the weights associated with attribute map layers, and

some compensatory aggregation method to obtain the score for each alternative in the set (Milutinovic et al., 2017). This non-interactive approach leaves the decision maker with little or no control over the score calculation once the criteria weights have been set, and the role of visualization is thus reduced to presenting the final results of the computations.

The contributions of this paper in the context of GIS-MCDA are twofold. We first introduce GISwaps, a novel approach to multi-criteria decision making in geospatial applications. GISwaps is an adaptation of the Even Swaps method (see Hammond et al. (1998, 1999)) to geospatial problems. It presents an intuitive strategy to simplify the decision process through iterative reduction of decision criteria. Following a review of related work in Section 2, the mechanism of GISwaps and its relation to EvenSwaps are described in Section 3. The second and main contribution of this paper, which is presented in Section 4, is an interactive visualization allowing the decision maker to explore the consequences of trade-offs and costs accepted during the iterative decision process, both in terms of the abstract relation between different decision variables and in spatial context. In Section 5 we discuss the benefits of using this interactive visualization in GISwaps for identifying situations and dependencies between alternatives that would otherwise remain unrevealed.

2 RELATED WORK

In a survey of ways of visualizing alternatives in the context of multiple criteria decision making, Miettinen (2014) gives an overview of commonly known techniques, such as bar charts, scatterplots and value paths, as well as a number of techniques using circles, polygons or icons, techniques based on hierarchical clustering and projection-based techniques. A number of different approaches to interactive visualization in decision making have been suggested. Carenini and Loyd (2004) propose ValueCharts, a set of interactive visualization methods aimed to help decision maker in inspecting linear models of preferences and evaluation. The concept is further developed in Bautista and Carenini (2006), with ValueCharts redesigned in order to support all phases of preferential choice. A Decision Ball model, aimed to assist decision maker by visualizing decision process based on even swaps, is presented in Li and Ma (2008). Their study is of limited use for our method, though, as it i) is limited to decision making in discrete choice models (assumes small number of alternatives), and ii) operates by cancellation of alternatives rather than

criteria, abandoning the very principle of the Even Swaps method. Kollat and Reed (2007) present a framework for Visually Interactive Decision-making and Design using Evolutionary Multi-objective Optimization (VIDEO). The framework is capable of visually representing up to four objectives (three on X, Y and Z, and a fourth as a colour) and allows visual navigation through large sets of alternatives, exploring and visualizing trade-offs.

The need for interactive visualization in spatial decision making in continuous choice models is significant, due to a large number of alternatives. Andrienko and Andrienko (2003) underline the decision maker's need to see how an option is positioned in both the geographical and the attribute space, as well as how it compares to other options. Based on that principle, the authors have developed CommonGIS, an own software system for exploratory analysis of spatial data including spatio-temporal data (Andrienko et al., 2003; Andrienko and Andrienko, 2003, 2004) Also Malczewski and Rinner (2015) state the importance of interactive visualization in GIS-MCDM. The authors make a distinction between *geovisualization of MCDA input* (visualizing criteria, visualizing alternatives and visualizing scaled values and criterion weights) and *geovisualization of MCDA results* (visualizing combination rules and parameters and visualizing model sensitivity). Each of the aforementioned visualization objectives should assume and support interactivity (Malczewski and Rinner, 2015). Jankowski et al. (2001) raise the question of effective means of using maps as a support to spatial problems exploration and structuring. The main role (objective) of using maps in spatial MCDM is the consideration of geographical locations in the process of deciding trade-offs among the decision criteria. Simultaneous representation of both criterion and decision space helps the decision maker define his/her preferences not only on the base of the attribute data, but also geography. Jankowski et al. (2001) find that highly interactive depiction of both criteria and decision spaces would be more productive for understanding the structure of the decision situation than static display. Thus, decision procedures should be facilitated by highly interactive visualization.

The interactive visualization framework presented in this paper is inspired and guided by the above principles. The framework is designed as an integral part of GISwaps, a novel method for decision making in continuous choice models based on Even Swaps.

3 GISwaps METHOD

In this section we give a brief overview of the GISwaps method, as well as an overview of Even Swaps method on which GISwaps is based. This overview should facilitate the reader in understanding the main features of the proposed visualization framework.

3.1 Even Swaps

Even Swaps is a simple, yet powerful multi-criteria decision-making method developed by Hammond et al. (1998). The main principle of the method is adjusting the consequences of considered alternatives in order to render them equivalent in terms of the chosen criterion. In that way, that criterion may be cancelled out as irrelevant for further analysis. Hammond et al. (1999) define the method by the following five steps:

1. Determine the change necessary to cancel out chosen reference criterion (R).

2. Assess what adjustments need to be done in chosen response criterion (M), in order to compensate for the needed change.

3. Make the even swap. An even swap is a process of increasing the value of an alternative in terms of one criterion and decreasing the value by an equivalent amount in terms of another. After the swaps are performed over the whole range of alternatives, all alternatives will have the same value on R and it can be cancelled out as irrelevant in the process of ranking the alternatives.

4. Cancel out the now-irrelevant criterion R.

5. Eliminate dominated alternative(s). Alternative *a* is said to be dominated by alternative *b* if it is inferior to *b* on at least one criterion and not superior to *b* on any other criterion.

These steps are repeated until a single alternative remains.

Let us consider a simple multi-criteria decision problem. Assume that X wants to buy a house, and is considering four alternatives: A, B, C and D. The goal is to find the best alternative considering price (P), size (S) and distance from the city center (D_{ist}). Values for the alternatives in terms of each of the criteria are presented in Table 1.

Table 1: Values for alternatives A, B, C and D.

	A	B	C	D
P (1000 \$)	110	120	115	160
S (m^2)	115	130	125	145
D_{ist} (km)	4	4,5	3	2

There are no dominated alternatives to be removed, so X proceeds and chooses P as the reference criterion (R) and S as the response criterion (M). After adjusting P-value for all the alternatives to the minimum price of 110000\$, X modifies S-value for each alternative in such way that, depending on X's preferences and judgement, this modification compensates for the performed adjustment on P. The result is shown in table 2.

Table 2: The values in terms of S are adjusted in order to compensate for the proposed change on P.

	A	B	C	D
P (1000 \$)	110	110 ~~120~~	110 ~~115~~	110 ~~160~~
S (m^2)	115	120 ~~130~~	118 ~~125~~	115 ~~145~~
D_{ist} (km)	4	4,5	3	2

Now that all four alternatives have the same value on P, P is cancelled out as irrelevant. Furthermore, A is dominated by D and is eliminated. X proceeds by choosing S as the reference criterion and D_{ist} as the response criterion. The result after making adjustments is given in table 3.

Table 3: The values in terms of D_{ist} are adjusted in order to compensate for the proposed change on S.

	B	C	D
S (m^2)	120	120 ~~118~~	120 ~~115~~
D_{ist} (km)	4,5	3,5 ~~3~~	2,8 ~~2~~

As S is now cancelled out as irrelevant, alternative D is chosen as the best alternative with the value of 2,8 in terms of D_{ist} (the only remaining criterion).

3.2 GISwaps

The Even Swaps method, as presented in Section 3.1, is only suitable for decision-making in discrete choice models, when the number of alternatives is relatively small. Many GIS-related decision-making situations concern a continuous choice model, where the number of alternatives is only constrained by the limits of the used data representation model. For example, finding the optimal location for a certain purpose in an area represented by a 1000 x 1000 grid, the number of alternatives could be as high as 1 000 000, depending on the grid resolution and specific constraints of the problem. If the mechanism of the Even Swaps method is to be applied to geospatial problems, the process of making trade-offs needs to be automatized. For this purpose we introduce GISwaps, an approach that makes decision making based on even swaps in decision problems with large number of alternatives manageable. The method, which is described in detail in Milutinovic et al. (2017), uses input from the

decision maker on a number of virtual alternatives in order to calculate, for each alternative in the set, the coefficient of the value change (swap coefficient) on the response criterion. As the name suggests, the virtual alternatives do not need to actually exist in the set, but are hypothetical alternatives used to fine-tune the value update function. The number of virtual alternatives, as well as assigned descriptive values in terms of each criterion, are set in each step for the current reference and response criteria, respectively.

Unlike Even Swaps, GISwaps does not incorporate elimination of dominated alternatives. In geospatial decision making we usually want to obtain an ordered set of all feasible alternatives, as opposed to a single best alternative. In its basic form, our method can be expressed by the following algorithm:

```
Repeat
    Decide the reference criterion R
    Decide the response criterion M
    Set virtual alternatives
    Obtain trade-off values for virtual alternatives
    Calculate update coefficients for virtual alternatives
    For each alternative in the set
        Calculate the update coefficient for the alternative
        Update the value of the alternative with respect to M
    Discard R
Until a single criterion remains
Rank the alternatives
```

In a typical geospatial site location problem, a number of possibly conflicting criteria is usually considered. Finding the optimal site location for building a dam, for example, requires considering factors such as water discharge, undulation, hydraulic head, distance to urban areas, distance to forests and distance to agricultural areas. Applying GISwaps on such case would take five swap turns, one of the criteria being cancelled out in each turn. In the following example, we explain our method on one possible turn, where *hydraulic head* is used as the reference, and *undulation* as the response criterion. We use 16 virtual alternatives in order to fine-tune the value update function. Each of the alternatives is assigned a pair of values: a value from the array V_R (four reference values in terms of the reference criterion R) and a value from the array V_M (four reference values in terms of the response criterion M). We use following reference values for the virtual alternatives in terms of R and M, respectively:

$$V_R = [R_{min}, R_{min} + R_q, R_{min} + 2R_q, R_{min} + 3R_q]$$
$$V_M = [M_{min} + M_q, M_{min} + 2M_q, M_{min} + 3M_q, M_{max}] \quad (1)$$

where

R_{min} : minimum value with respect to R
R_{max} : maximum value with respect to R
$R_q = (R_{max} - R_{min})/4$

M_{min} : minimum value with respect to M
M_{max} : maximum value with respect to M
$M_q = (M_{max} - M_{min})/4$

For example, for hydraulic head values in range 1-46 and undulation values in range 1-136, it gives the following values for V_R and V_M, respectively:

$$V_R = [1, 12.3, 23.5, 34.8]$$
$$V_M = [35, 68, 103, 136] \quad (2)$$

Based on his/her judgement and preferences, the decision maker now needs to perform even swaps. He/she chooses a compensation value $M_{u(i,j)}$ of criterion M for each virtual alternative, i.e. for each pair of reference values $(V_{R(i)}, V_{M(j)})$; $i, j \in [1..4]$. Each value $M_{u(i,j)}$ is chosen so that the decision maker is indifferent between the differences $(R_{max} - V_{R(i)})$ and $(V_{M(j)} - M_{u(i,j)})$. The compensation values are stored in a 4x4 matrix M_u (Equation 3). With the input from the decision maker as shown in Figure 1, the M_u matrix contains following values:

$$M_u = \begin{bmatrix} 18 & 40 & 57 & 87 \\ 20 & 44 & 66 & 91 \\ 22 & 50 & 70 & 95 \\ 28 & 59 & 84 & 106 \end{bmatrix} \quad (3)$$

Update coefficients for each element in M_u are stored in corresponding matrix M_c. Each value $M_{c(i,j)}$ is calculated as

$$M_{c(i,j)} = \frac{V_{M(j)} - M_{u(i,j)}}{R_{max} - V_{R(i)}} \quad (4)$$

For the input used in the example, the M_c matrix contains following values:

$$M_c = \begin{bmatrix} 0.37 & 0.63 & 1.01 & 1.09 \\ 0.44 & 0.73 & 1.08 & 1.33 \\ 0.57 & 0.82 & 1.44 & 1.82 \\ 0.60 & 0.85 & 1.66 & 2.69 \end{bmatrix} \quad (5)$$

In order to calculate the update coefficient for an alternative a, we need to determine index i using a_R (the value of a in terms of the reference criterion R) and index j using a_M (the value of a in terms of the response criterion M). The indexes are determined as follows:

$$i = k \text{ for } V_{R[k]} \leq a_R < V_{R[k+1]}, \quad k \in [1..3]$$
$$i = 4 \text{ for } a_R \geq V_{R[4]}$$
$$j = 1 \text{ for } a_M \leq V_{M[1]}$$
$$j = k \text{ for } V_{M[k-1]} < a_M \leq V_{M[k]}, \quad k \in [2..4] \quad (6)$$

The value update coefficient for any alternative a can now be calculated as

$$a_c = M_{c(i,j-1)}$$
$$+$$
$$(M_{c(i,j)} - M_{c(i,j-1)}) \cdot \frac{a_M - V_{M(j-1)}}{V_{M(j)} - V_{M(j-1)}} \quad (7)$$

The remaining four turns are performed in the same manner. After cancelling out five of six criteria, the grid containing the values for the only remaining criterion would be the *de facto* result of the decision process that can be shown in an appropriate GIS software.

While GISwaps enables interactive definition of compensation values and a fully automated update of response criteria in a quasi-continuous multi-criteria decision model, it requires suitable visualizations of the outcomes in the iterative process of updating coefficients and elimination of reference criteria. In the following section we present our approach to visualizing the complex relationships between the choice of reference and response criteria, the design of the update coefficients, and their effect on the huge amount of alternatives both in attribute space and geographical space.

4 VISUAL GISwaps

The GISwaps method is an interactive method for spatial decision making. Through a semi-automatized process explained in Section 3.2, the decision maker sets the actual trade-off values between alternatives with respect to a pair of criteria. Throughout the process, the decision maker is active and in full control. The interactive visualization presented in this section allows the decision maker to explore the consequences of trade-offs and costs accepted during each step of the iterative decision process.

The main window of the application that implements the GISwaps method as well as the visualization framework is given in Figure 1. The image shows the setup of the example used in Section 3.2, after the decision maker has selected reference (in the application referred to as *measuring stick*) and response criteria, and set compensation values in terms of the latter. The example is used in this section with the same trade-off values as in Section 3.2.

Our interactive visualization framework integrates fundamental visualization techniques, and consists of three conceptual units: 1) visualization of alternatives in attribute space, 2) visualization of alternatives in geographical space, and 3) visualization of value functions. In each step of the iterative process, reference and response criteria are selected by the decision maker. The scatterplot representing the attribute space for the set of alternatives is constructed as a 2D-plot, with position on the X-axis determined by the value in terms of the response criterion, and position on the Y-axis determined by the value in terms of the reference criterion. This representation gives

the decision maker an insight into the distribution of data in terms of the currently chosen response criterion. An extra dimension may be added to the plot by color-coding samples in order to show values of the alternatives in terms of a third criterion, that we will refer to as comparison criterion. Even though the values of the alternatives in terms of the comparison criterion are not changed, and the comparison criterion should not be directly considered when determining trade-off values for the response criterion in the current iteration, this extra information gives the decision maker a deeper insight and better understanding of the consequences of the performed trade-offs. The comparison criterion is chosen by the decision maker from the list of non-cancelled criteria. An example of a non-color-coded and a color-coded plot is given in Figure 2. The positions of the alternatives are updated and the plot redrawn each time the decision maker changes a value in terms of the response criterion of any of the virtual alternatives. In order for the decision maker to be able to explore and make judgements regarding the consequences of a change, the original positions of alternatives are shown in the plot in light blue color. This gives the decision maker a better understanding of not only the magnitude of change resulting from the proposed trade-off values, but even the distribution of change, i.e. how many alternatives are affected by the proposed trade-off. By clicking on and highlighting a point in the plot, the decision maker gains access to further information about the selected alternative. A vector representing the magnitude of change in terms of the response criterion after the suggested trade-off is shown in the plot as the distance between the original and current value (position), as a complement to the numeric information. Furthermore, the selected alternative is highlighted in the map panel, giving the decision maker information about the position of the alternative in the geographical space. An example is shown in Figure 3. The part of the visualization that concerns geographical space consists of two maps of the geographic area where the decision problem is situated. The two maps simultaneously show color-coding for both response criterion and comparison criterion (if any). The geographic distribution of the alternatives in the left map is always color-coded by values of alternatives in terms of the response criterion. The color-coding in the right map is related to the color-coding in the scatterplot, i.e. it shows the values of the alternatives in terms of the comparison criterion if there is one selected, and is not color-coded otherwise. In Figure 3, Discharge was chosen as comparison criterion.

Visualization of value functions is given in form of a multi-line chart, with the number of lines de-
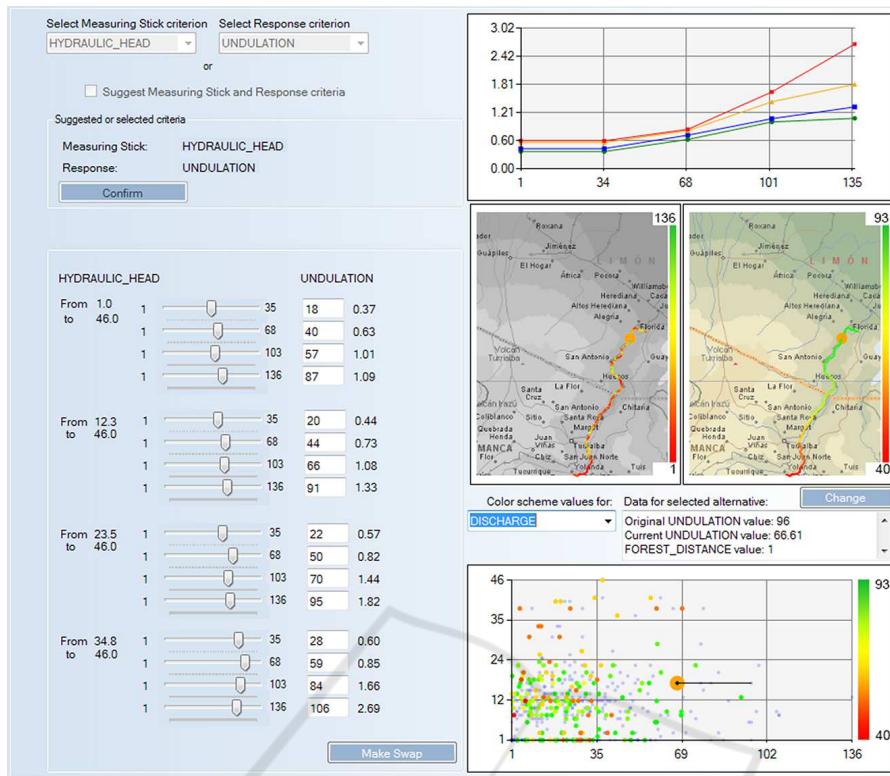
Figure 1: Main window of the application implementing GISwaps and the visualization framework. Adjusting the values in terms of the response criterion (*Undulation*, in the example), in order to compensate for the changes made in the reference criterion (*Hydraulic head*), is performed in the left panel. The right panel contains attribute space visualization scatterplot (bottom), geographical space visualization maps (middle), and value functions visualization (top).
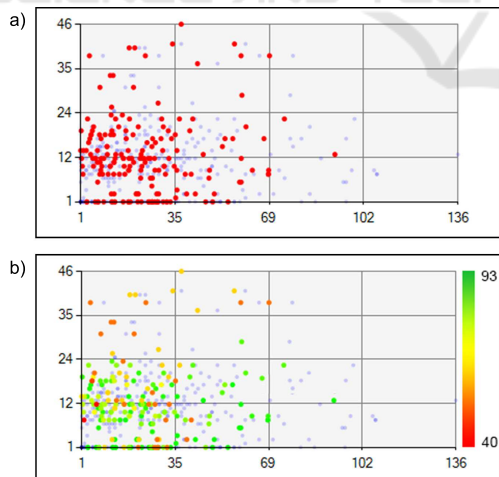

Figure 2: Scatterplot for the example decision problem with no comparison criterion selected (a) and with Discharge selected as the comparison criterion (b).

termined by the number of elements in V_R and the number of points defining each line determined by the number of elements in V_M (see Equation 1 in Section 3.2). An example is given in Figure 4. This visual representation makes potential deviations in a value function evident, allowing the decision maker to adjust suggested values if the deviation is due to an error, rather than personal preference and judgement.

5 DISCUSSION AND CONCLUSIONS

In Keeney and Raiffa (1976), the essence of the issue of trade-offs under certainty is described as "How much achievement on objective 1 is the decision maker willing to give up in order to improve achievement on objective 2 by some fixed amount?". As the authors point out, the trade-off issue usually requires subjective judgement of the decision maker. As a trade-off method based on even swaps, GISwaps is an intuitive and efficient tool capable to address one of the big challenges in the context of decision support systems, namely, how to handle the preferences of the decision maker. Instead of being concerned with the abstract concept of the relative importance of decision criteria, using GISwaps the decision maker
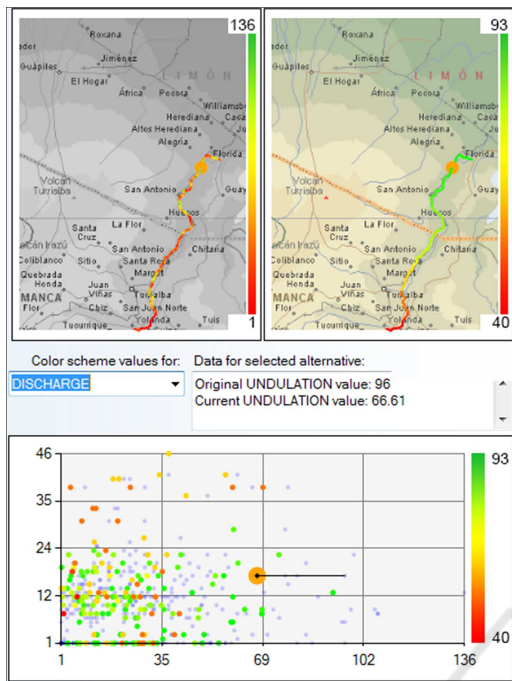
Figure 3: When an alternative is selected, its magnitude of change is shown, as well as its position in the geographical space (highlighted in the map panel).
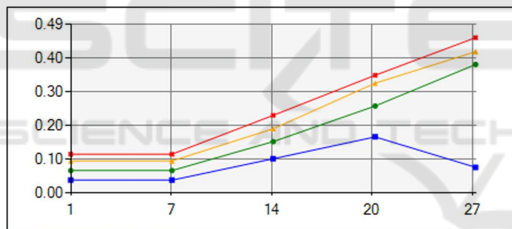

Figure 4: Visualization of value functions. The number of functions (lines) is determined by the number of elements in V_R.

works with concrete values and differences, which we believe makes our method more flexible than the commonly used approach of combining weighting and aggregation methods.

Geospatial multi-criteria decision problems tend to be complex, with a large number of conflicting criteria to be taken into consideration, and thousands or even millions of alternatives to be evaluated and compared. In order for the decision maker to have a good understanding of the consequences of his/her choices, numeric data needs to be complemented by a visual representation of relevant information that would be unavailable otherwise. In an interactive decision-making method, this needs to be done at each step of the decision-making process.

The positive impact of the interactive visualization presented in this paper is multifaceted:

- The scatterplot representation of the alternatives in the attribute space gives the decision maker an insight into the distribution of data in terms of the currently chosen response criterion. From Figure 2, for example, it is obvious that only a small number of alternatives has the undulation value in the upper half of the range, i.e. over 68.

- It reveals potential outliers. Outliers, both positive and negative, may have a significant impact on the reliability of the values interpolated from the values assigned to virtual alternatives; for that reason, they should not pass unnoticed.

- It gives the decision maker the possibility to get a closer look at the alternatives of interest. Extra information available on demand includes the original value, the current value and the magnitude of change in terms of the response criteria, the value of the alternative in terms of any of the non-cancelled criteria, and the geographic location of the alternative. This information is of great importance for understanding the impact of a suggested trade-off.

- The map view helps the decision maker make more sensitive choices. By being able to see the geographic location of the selected alternative, the decision maker can, based on his/her knowledge and preferences, re-evaluate suggested trade-offs that would affect the alternative(s) of interest.

- Our visualization saves the decision maker time and effort. The prime assumption for any trade-off based method for decision making to be successful is that the trade-offs are balanced and carefully performed. Deciding trade-off values for virtual alternatives that might not have any impact on the outcome of the decision process is therefore better avoided. By means of visual representation of alternatives in the attribute space, the decision maker can see if there are empty value intervals, i.e. value ranges with no alternatives. In the example in Figure 5, there are seven intervals out of sixteen with no alternatives. Consequently, it is unnecessary to decide trade-off values for the virtual alternatives that define those intervals.

- It helps the decision maker make consistent trade-offs. The significance of the value functions chart increases with the number of virtual alternatives. In many cases, the decision maker wants the trade-off values to be consistent over the set of virtual alternatives. Making consistent trade-offs based on numeric values only is a demanding task, and the complexity of the task increases with the number of virtual alternatives. Visual representation makes potential deviations in a value function

evident (see Figure 4; the function represented by the blue line shows a deviation for x = 27), allowing the decision maker to adjust suggested values if the deviation is due to an error, rather than personal preference and judgement.

The GISwaps method and the interactive visualization presented here are designed and developed in close cooperation with experts in multi-criteria decision making and geographic information systems. At this point, though, they have not been used by practitioners in the field of GIS-MCDM. In a forthcoming study, the GISwaps method will be evaluated with respect to efficiency and usability, in comparison with the combination of AHP and the weighted summation method, which is the most commonly used approach in geospatial decision-making. Based on the results, a second study will be carried out, where the impact of the interactive visualization framework presented in this paper will be evaluated.
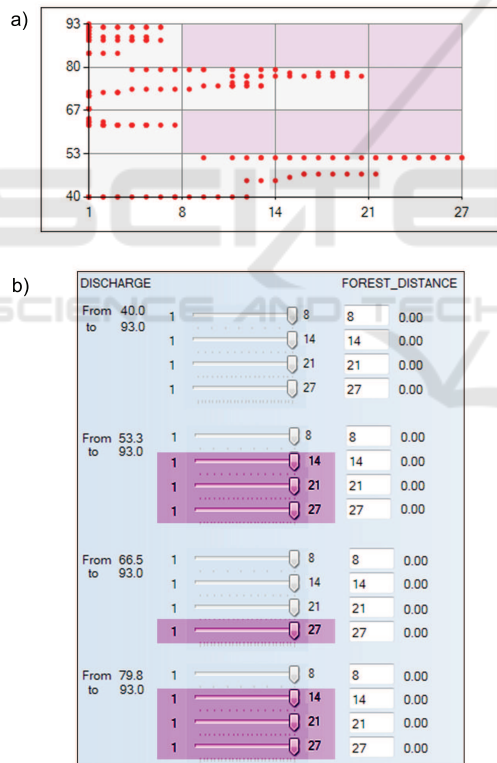

Figure 5: a) Empty intervals, i.e. intervals that contain no alternatives; b) Virtual alternatives for which it is unnecessary to set trade-off values, as they define empty intervals.

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