Mathematical Programs and Computations for a Class of Anti-aircraft Mission Planning Problems

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Abstract: The theater defense distribution is an important problem in the military that determines strategies against a sequence of offensive attacks in order to protect his targets. This study focuses on developing mathematical models for three important defense problems that generate anti-aircraft mission plans for a group of missile battalions. The simple Anti-aircraft Launching Assignment problem specifies number of missiles should be launched from each battalion to each fleet of attacking aircraft to maximize the defensive effectiveness, provided that the locations of missile battalions are given. On the other hand, the Anti-aircraft Mission Planning problem maximizes the defender's effectiveness using all his available battalions, while the Inverse Anti-aircraft Mission Planning problem computes necessary weapon resources (battalions and their missiles) to obtain a given defensive effectiveness value. The proposed formulations are Integer Programs and proved as NP-hard. A comprehensive set of experiments is then evaluated to show that these proposed programs can be applied to solve fast real-life instances to optimality.

1 INTRODUCTION

The Theater Distribution Model (TDM) is specifically designed to support the combatant commander to ensure his effective plans within area of operations (Shalikashvili, 1996). Most of researches in the literature focus on the military theater distribution problem associated with determining positions of defender's missile battalions within a potentially geographical region (Shalikashvili, 1993). In most cases, the defender faces with so many risks from terrorist attacks of all kinds. The work we research here is directly motivated by such one risk assessment: surface-to-air missile battalions defense against many fleets of penetrating aircraft. Beside the role of specializing anti-aircraft, missile battalions in an air defense system also protect a target such as capital, political area, economy or military center. The Department of Air Defense and Air Force has conducted some risk assessment exercises and estimated risks of many attack possibilities. One area that is still undeveloped is an algorithm for determining an intelligent package of missile battalion's actions to aircraft attack. A missile battalion package consisting of only one missile type might be ineffective but when several types of missiles are combined properly, they confidently destroy the attacking aircraft and protect their target. More precisely, we develop mathematical formulations for anti-aircraft mission planning effectively for a group of missile battalions. A missile battalion is considered as a fundamental tactical building block which recruits or conscripts in one geographical area assigned by a feudal lord. Given some threat, the defender must decide where to locate defensive missile battalions among potential locations and how they should engage fleets of attacking aircraft. We express the defender's courses of action as following mathematical optimization problems. The first and simple problem is the Anti-aircraft Launching Assignment (ALA) that computes number of missiles should be launched from each battalion to each fleet of attacking aircraft to maximize the defensive effectiveness, provided that the locations of missile battalions is given. On the other hand, the Anti-aircraft Mission Planning problem (AMP) calculates not only the launching assignment for the battalions but also locations of the battalions while Inverse Anti-aircraft Mission Planning problem (IAMP) computes necessary weapon resources, battalions' locations, and a launching assignment to obtain a given effectiveness threshold. The most valuable contribution of this paper comes from the statements and practical mathematical formulations for these three crucial problems in the TDM class. The ALA arises

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in the crowded cities where there are not many choices for locating missile battalions while the AMP is necessarily tackled in sparsely populated area. On the contrary, the IAMP is necessarily considered when the protected target should not be fallen in any cases. Although these problems are proven as NP-hard, the computational results show the efficient of our formulations since they can be applied directly to solve fast real-life instances to optimality. The message is that, with the validation of experienced soldiers, we have gained confidence that the formulations have significant implementation in any defender's combat field. The rest of paper is organized as follows. In Section 2, we state the problems, formulate them as mixed integer programs as well as prove their hardness. The experimental results are reported and analyzed in Section 3. Finally, we conclude the paper and draw some future directions in Section 4.

The TDM has been motivated and validated for both defensive side and offensive side in general ((Robert, 2006), (Crino and Moore, 2004), (Jackson, 1989), (Studies and Agency, 1992), (Brian, 1994), (Seichter, 2005), (Moore, 2002), (Brown et al., 2008)). Some of these studies have been developed into mathematical models which are related to the AMP can be classified as the Weapon-Target Assignment (WTA)and the Defender- Attacker Model (DAM). The WTA is the assignment of weapon to the hostile target in order to protect assets, which can be formulated as a nonlinear integer programming problem and is known to be NP-hard complete. Although several heuristic methods have been propose for solving the WTA, such as genetic algorithm, tabu search, simulated annealing, and variable neighborhood search ((Murphey,), (Tokgoz and Bulkan, 2013)), an integer programming and network flow-based lower bounding method has been introduced in (Ahuja Ravindra and James, 2007). An instance of DAM in a fast theater model is introduced in (Seichter, 2005), which is built upon the existing air model. The air strike attacker's main objective is maximizing target value destroyed by killing as many targets with high values as possible, while the ground combat wants to minimize its own losses. The resulting model is a Mixed Integer Program (MIP) finding an optimal, actively defensive actions by the ground force that can significantly reduce the air attacker's effectiveness. The defender-attacker problem is continued studying in (Moore, 2002), in which a new methodology for strategy optimization under uncertainty has been proposed. The authors describe the implementation of a genetic programming algorithm to determine an optimized evasion strategy for the extended two-dimensional pursuer-evader problem under conditions of uncertainty about the type of pursuer. The DAM model is also applied to defender's risk assessment and mitigation,(Brown et al., 2008).

2 PROBLEM FORMULATIONS

We now state and formulate the anti-aircraft mission planning problems. The complexity of these problems is also discussed at the end of this section.

2.1 Problem Descriptions

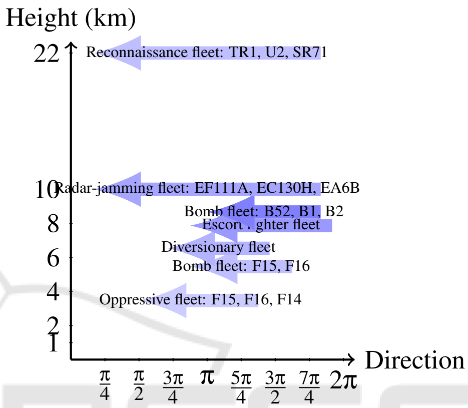

Figure 1: An example of an attacking plan.

Suppose that the attacker's plan can be observed by an intelligence system of the defender and is described as follows. In the offensive side, the attacker strikes the target by a group of fleets of attacking aircraft. For a sake simplicity, from now on, the term "fleet" is used stead of "fleet of attacking aircraft". Each fleet is organized by a group of aircraft which have same missions such as carrying bombs or making radar noise; enter the theater at same height, direction; and fly with same velocity. Each fleet is associated with a weight of importance depending mainly on its mission. Figure 1 illustrates an attacking plan, in which the horizontal axis represents the directions of the fleets, considered as angles between attacking directions and a predefined axis; while the vertical axis represents the height of the fleets. There are seven fleets at different heights, directions and velocities, drawn by seven arrows. The color of the arrows reflects the fleets' weights. For instance, the darkest arrow describes the fleet with the highest weight, carrying bombs such as B52, B1, B2. Further, flying positions of aircraft in a fleet must be captured in detail, for example, a flying position of a four-aircraft fleet is illustrated in Figure 2.

In the defensive side, the defender's responsibility is engaging fleets to protect its point target. In or-

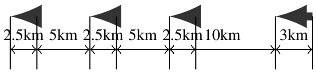

Figure 2: An example of an attacking fleet.

der to formulate a class of anti-aircraft mission planning problems, we take into account following factors. The first one is *critical radius* corresponding to each fleet that defines a critical circle centered at the target. The defender has to make a defensive plan such that no attacking aircraft in that fleet is able to get inside that circle. This critical radius is computed depending on the height, velocity of that fleet and type of bombs carried by that fleet. For instance, in Figure 3, the point target is described by the green rectangle with critical radius OX corresponding to a fleet coming from the AO direction. The second factor is *action range* of a battalion corresponding to a fleet that can be understood as a fleet's flying path where the fleet will be intercepted by that battalion. In Figure 3, the action range of battalion B_1 is CD segment, where D is the intersection of the attacking direction AO and the critical circle, and C is the intersection of the direction AO and the circle centered at B_1 of radius B_1C which is defined as the long range of missiles belonging to that battalion. Similarly, the action range of battalion B_2 is BD segment. The third factor *maximum launches* representing firepower capabilities of a battalion can be seen as maximum number of missiles that can be launched from the battalion to the fleet in its action range. This number is calculated basing on the type of missile, number of missiles in that battalion, as well as the shortest time between two successive launches. The fourth factor is *the expected number of killed aircraft* of a fleet caused by a battalion in a number of launches. This value can be estimated by number of missiles, probability of kill, defensive mode related to each fleet, and set of coefficients corresponding to each missile battalion such as technical coefficient, control coefficient, and complex coefficient of combat. Lastly, a minimum distance between each pair of battalions is required to avoid radar jamming between missiles in the battalions. Note that if two battalions locate at a same position, this constraint can be ignored. As a defender, we would like to measure the result of our defense. An effective criterion is then introduced as *defensive effectiveness*, calculated by fraction between value of killed aircraft and value of all aircraft in attacking fleets. The effectiveness is strongly influenced by battalions' locations, that motivates us to study an adaptable, efficient, and cost-effective process to analyse missile battalion defense against aircraft threats. Three problems proposed in this paper, the ALA, AMP and IAMP, belong to the defensive side, where the defender preserves a point target. The attacker's plan including all information about the offensive side stated above is supposed to be observed clearly by the intelligence system of the defender. While in the AMP, the defender not only finds optimal locations for missile battalions but also indicates numbers of missiles launched from battalions to fleets; in the ALA, the defenders is not required to find locations for battalions. We consider now somewhat inverse version of the AMP, the IAMP. Assume, in particular, that the defender has a set of battalions, in stead of using all battalions to maximize his defensive effectiveness, one generates a defensive mission plan such that the corresponding effectiveness is greater than or equal to a given value. The detailed mathematical programs of these models are proposed in Section 2.2.

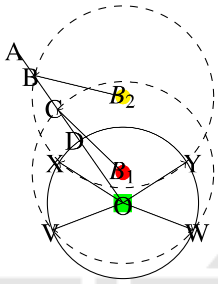

Figure 3: An example of influence of battalion's locations on their firepower capabilities.

2.2 Mathematical Programs

For a simplicity of presentation, we first introduce some indices and notations.

Inputs

- B: set of missile battalions;
- F: set of fleets of attacking aircraft;
- L: set of potential locations for battalions;
- For a battalion $b \in B$, denote $m(b)$ and $c(b)$ by number of missiles distributed to b and cost of a missile of b, respectively;
- For a fleet $f \in F$, denote $n(f)$ and $w(f)$ by number of aircraft in fleet f and weight of that fleet, respectively;
- For a pair (b, f) $(b \in B, f \in F)$, denote $t(b, f)$ by the maximum number of missiles that battalion b is able to launch to fleet f in its action range;
- Last, for a triple (b, f, t) $(b \in B, f \in F$ and $t \in \mathbb{Z}^+)$, denote $e(b, f, t)$ by the expected number of killed aircraft in fleet f caused by t missiles launched from battalion b to fleet f.

2.2.1 ALA Mathematical Program

Given the locations of battalions, the objective of the ALA is to compute number of missiles should be launched from each battalion toward each fleet such that the defensive effectiveness is maximized. This program arises naturally in real-life instance when the target and battalions are firmly positioned in a given area. To formulate the ALA program, we introduces binary variables $x_{b,f,t}$ where $b \in B, f \in F$ and $t \in \mathbb{Z}^+$ indicating that whether t missiles are launched from battalion b to fleet f or not. The mathematical program of the ALA problem can be stated as a mixed integer program as follows.

$$Max \sum_{f\in F} w(f) \frac{\sum_{b\in B} \sum_{t=1}^{t(b,f)} e(b,f,t)x_{b,f,t}}{n(f)} \tag{1a}$$

$$s.t. \quad \sum_{f\in F} \sum_{t=1}^{t(b,f)} t x_{b,f,t} \leq m(b), \;\; \forall b \in B \tag{1b}$$

$$\sum_{b\in B} \sum_{t=1}^{t(b,f)} e(b,f,t)x_{b,f,t} \leq n(f), \;\; \forall f \in F \tag{1c}$$

$$x_{b,f,t} \in \{0,1\}, \forall b \in B, f \in F, t \in \{1,\dots,t(b,f)\}. \tag{1d}$$

In this program, the objective function (1a) is the defensive effectiveness which can be calculated as the fraction between the value of killed aircraft and the value of aircraft in these fleets. Constraints (1b) simply stipulate that the number of launched missiles of each battalion should not be greater than its given load-out. Constraints (1c) ensure the number of aircraft destroyed in a fleet do not exceed the number of aircraft in that fleet; while the launching times constraints are hidden in (1b) and (1c) since t takes values in $[1,t(b,f)]$. The variables are restricted to be binary in constraints (1d).

2.2.2 AMP Mathematical Program

We consider now a situation that the defender need to decide where to locate missile battalions among a set of limited number of available locations and missiles to distribute a defensive mission plan that maximizes his defensive effectiveness. The AMP can be viewed as an extension of the ALA model. The AMP formulation introduces more variables representing locations of battalions and missile-distance constraints defining lower bounds for geographic distances between battalions at different locations. In addition to notations, variables and constraints introduced in the ALA formulation, the AMP considers more categories of factors as follows.

Inputs:

- For a pair of locations (l_i,l_j) $(l_i,l_j \in L)$, denote $d(l_i,l_j)$ by the geometric distance between l_i and l_j;
- For a pair of battalions (b_i,b_j) $(b_i,b_j \in B)$, denote $\overline{d}(b_i,b_j)$ by the minimum distance between b_i and b_j if they are located at two different positions;
- For each triple (b,l,f) $(b \in B, l \in L, f \in F)$, denote $t(b,l,f)$ by the maximum number of missiles that battalion b located at l is able to launch to fleet f in its action range;
- For a quadruple (b,l,f,t) $(b \in B, l \in L, f \in F$ and $t \in \mathbb{Z}^+)$, $e(b,l,f,t)$ indicates the expected number of killed aircraft in fleet f caused by t missiles launched from battalion b located at l to fleet f.

Further, we introduce binary variables $y_{b,l}$ where $b \in B, l \in L$, indicating whether battalion b locates at location l or not. Second, binary variables $z_{b,l,f,t}$ where $b \in B, l \in L, f \in F$ and $t \in \mathbb{Z}^+$, state that if t missiles are launched from battalion b located at location l to fleet f. In addition to constraints given in the ALA formulation, the AMP requires minimum distance between any two battalions such that if battalions b_i and b_j locate at location l_i and l_j respectively, then

$$d(l_i,l_j) \geq \overline{d}(b_i,b_j), \;\; \forall b_i \neq b_j, l_i \neq l_j \tag{2}$$

The MIP model for the AMP problem can be represented as:

$$Max \sum_{f\in F} w(f) \frac{\sum_{l\in L}\sum_{b\in B} \sum_{t=1}^{t(b,l,f)} e(b,l,f,t)z_{b,l,f,t}}{n(f)} \tag{3a}$$

$$s.t. \quad \sum_{l\in L} y_{b,l} = 1, \;\; \forall b \in B \tag{3b}$$

$$\sum_{l\in L}\sum_{f\in F} \sum_{t=1}^{t(b,l,f)} t z_{b,l,f,t} \leq m(b), \forall b \in B \tag{3c}$$

$$(2 - y_{b_i,l_i} - y_{b_j,l_j})\infty + y_{b_i,l_i} d(l_i,l_j) \geq \overline{d}(b_i,b_j) \quad \forall b_i,b_j \in B, b_i \neq b_j, \forall l_i,l_j \in L, l_i \neq l_j \tag{3d}$$

$$\sum_{l\in L}\sum_{b\in B} \sum_{t=1}^{t(b,l,f)} e(b,l,f,t)z_{b,l,f,t} \leq n(f), \forall f \in F \tag{3e}$$

$$\sum_{f\in F} \sum_{t=1}^{t(b,l,f)} z_{b,l,f,t} \leq y_{b,l}, \forall b \in B, l \in L \tag{3f}$$

$$y_{b,l}; z_{b,l,f,t} \in \{0,1\}, \forall b \in B, l \in L, f \in F, \quad t \in \{1,\dots,t(b,l,f)\}. \tag{3g}$$

In the AMP formulation, the defender's objective is maximizing its effectiveness (3a) which is the sum of fractions of expected number of killed aircraft and number of aircraft $n(f)$ for all fleets while considering additionally the importance weights of these fleets. Constraints (3b) simply limit each missile battalion to one location. Constraints (3c) stipulate that the number of launches from each battalion should not be greater than its given number of missiles. Constraints (3d) describes the missile-distance constraints (2), showing that if two missile battalions b_i and b_j are at locations l_i and l_j where $l_i \neq l_j$, respectively, the distance between these battalions must be greater than a required minimum distance $\bar{d}(b_i, b_j)$. Constraints (3e) define upper bounds for number of aircraft in these fleets. Constraints (3f) tell us that there are some missiles launched from a location to a fleet if and only if there exists at least one missile battalion located at that location. Lastly, constraints (3g) indicate that $x_{b,l}, y_{b,l,f,t}$ where $b \in B, l \in L, f \in F, t \in \{1, \ldots, t(b,l,f)\}$ are binary variables.

2.2.3 IAMP Mathematical Program

An other significant situation that usually appears in the real combat field is the inverse AMP problem, the IAMP. Assume, in particular, that the defender has a set of battalions, in stead of using all battalions to maximize his defensive effectiveness, one generates a defensive mission plan such that the corresponding effectiveness is greater than or equal to a given value with the lowest cost. Intuitively, one would like to locate defensive battalions as well as compute number of missiles with the lowest cost such that its effectiveness is at least γ (such as $\gamma = 0.5, 0.6, \ldots$). In addition to the inputs introduced in the ALP and the AMP, the IAMP program need some more notations.
Inputs:

- For a battalion $b \in B$, denote $c(b)$ by the cost of a missile of b;
- For each pair (b,l) of a battalion $b \in B$ and a location $l \in L$, $c(b,l)$ refers to the cost of allocating battalion b at location l;

Further, the IAMP mathematical formulation introduces more integer variables $u_b \in \mathbb{Z}^+$, indicating number of missiles launched from battalion $b \in B$. The mathematical program for the IAMP is given as:

The objective of this formulation is to minimize the total cost (4a) that is calculated by sum of establishing battalion cost and launched missile cost on these battalions. Constraints (4b) are different from (3b) since not every missile battalion is required to locate at some location. Constraint (4d) requires the obtained effectiveness be at least a given value, γ. The number of missiles on each battalion is set at constraints (4e) while its upper bound has been given in constraints (4f). The other constraints are similar to ones in the AMP formulation given in constraints (4c), except that number of launched missiles are integer variables as in constraints (4g).

$$Min \sum_{b\in B}\sum_{l\in L} c(b,l) y_{b,l} + \sum_{b\in B} c(b) u_b \tag{4a}$$

$$s.t. \ \sum_{l\in L} y_{b,l} \leq 1, \ \ \forall b \in B \tag{4b}$$

$$Constraints (3c), (3d), (3e), (3f), (3g) \tag{4c}$$

$$\sum_{f\in F} w(f) \frac{\sum_{l\in L}\sum_{b\in B}\sum_{t=1}^{t(b,l,f)} e(b,l,f,t) z_{b,l,f,t}}{n(f)} \geq \gamma, \tag{4d}$$

$$u_b - \sum_{l\in L}\sum_{f\in F}\sum_{t=1}^{t(b,l,f)} t z_{b,l,f,t} = 0, \ \ \forall b \in B \tag{4e}$$

$$u_b \leq m(b) \sum_{l\in L} y_{b,l}, \ \ \ \forall b \in B \tag{4f}$$

$$u_b \in Z^+, \ \ \forall b \in B \tag{4g}$$

where $\gamma \in [0,1]$ is a given effectiveness value.

Note here that, the ALA and the AMP always result in an optimal solution with the effectiveness belonging to $[0,1]$, while the IAMP sometimes returns to no solution in case the weapon resources are not sufficiently available to reach the expected effectiveness. Further, one of the difficulties to deal with these programs is to define the values of parameters that supposed to be known. This study clarifies how to compute the expected number of killed aircraft, the maximum number of launches and the minimum distance between two missiles for special situation of a defensive missile battalion plan against aircraft attack, that can be found in the Appendix.

2.3 NP-Hardness

The NP-completeness is indicated as the complexity of these mathematical programs in this section.

Theorem 1. *The ALA is NP-complete.*

Proof. To prove this, we reduce the 3-Partition Problem (3PP) to the decision problem of ALA since the 3PP is known as NP-complete (Garey and Johnson, 1990).

Recall that in the 3PP, we are given a multi-set S of $3b$ positive integers $i_1, i_2, \ldots, i_{3b}$, where the value of every element in the set belongs to interval $(\frac{C}{4}, \frac{C}{2})$, for a positive integer C; and we are asked to decide if there are b disjoint subsets of S such that sum of all elements in each subset equals to C.

For any instance of the 3PP, an instance of the decision problem of ALA is simultaneously created as follows: The set of battalion B is initialized by b missile battalions; each battalion is allocated 3 missiles; the set of fleets includes $3b$ fleets, $f_1, f_2, \ldots, f_{3b}$ (each positive integer in the 3PP instance corresponds to a fleet in the instance of the decision problem of ALA); each fleet is formed by just one aircraft; each battalion can launch at most one missile to each fleet; the expected number of killed aircraft is $\frac{i_k}{C}$ ($\frac{i_k}{C} \in (\frac{1}{4}, \frac{1}{2})$) when one missile is launched from any battalion to a fleet f_k; for all $k \in [1, 2, \ldots, 3b]$, and the lower bound on the defensive effectiveness is set to 1.

As a result, the 3PP instance has a partition into b disjoint subsets, sum of elements in each subset is equal to C, if and only if all battalions located at the unique location, each battalion launches exactly 3 missiles to 3 fleets and the defender's effectiveness reaches to its ideal value of 1 or all fleets are destroyed completely. Thus 3PP reduces to the question whether the decision problem of the ALA has a solution or not. Thus, a pseudo polynomial reduction has been given, showing that the ALA is NP-complete. □

The NP-completeness of the AMP and the IAMP are proved in the same way as above.

3 EXPERIMENTS

This section dedicates to report and analyze experimental results on variety of instances for the ALA, AMP and IAMP. For each problem, we generated randomly 15 instances with the help of experienced soldiers to make them real and reliable. The coefficients used to generate instances were carefully extracted from the historical data and technical guides of missiles S-75 and S-125. The mathematical formulations for the ALA, AMP and IAMP were implemented in C++ programming language, using IBM Ilog Cplex Concert Technology, version 12.5. The standard cuts of Cplex were automatically added. Since the number of constraints (3d) in the AMP and IAMP programs is large and they belong to Miller-Tucker-Zemlin class (Miller et al., 1960), these constraints are treated as lazy constraints to reduce the computation time as follows. At the beginning of resolution, these constraints are all relaxed. Each time, an integer feasible solution is counted at a node in the search tree, this solution is checked for the satisfiability of all these constraints, all violated constraints are injected more in the model. This cut-addition strategy always

Table 1: Experimental results for the ALA.

Instances			Branch-and-Cut		
No.	b	f	Gap	Eff.	Time
1	5	5	0	0.246	0.004
2	7	8	0	0.462	0.005
3	6	5	0	0.742	0.012
4	6	8	0	0.546	0.025
5	7	5	0	0.673	0.004
6	10	10	0.02	0.767	7202.291
7	10	10	0	0.706	21.608
8	10	10	0	0.648	0.083
9	10	10	0.04	0.815	7201.130
10	10	10	0	0.787	2.292
11	10	10	0	0.780	142.796
12	10	10	0.02	0.699	7200.970
13	10	10	0	0.521	1.129
14	10	10	0	0.776	0.252
15	10	10	0	0.999	0.024

ensures the returning solution is correct since all constraints (3d) are strictly respected. All computations were performed in multi-thread mode on a computer with an Intel Core i7-479 CPU, 3.6 GHz, and 4 GB RAM running Ubuntu version 16.4, 64 bits. A time limit of 2 hours of CPU time was set for each instance resolution. All computational results for the ALA, AMP and IAMP are respectively indicated in Table 1, 2 and Table 3. The columns in the tables have the following meanings: *No.*: Instance number, b: Number of battalions, m: Number of missiles in each battalion, l: Number of potential locations, f: Number of attacking aircraft fleets, *Gap*: Gap (%) between the optimal solution of the integral relaxation of an integer program and integer feasible solution of that program found so far, *Eff.*: Defensive effectiveness value, and *Time*: Overall CPU time in seconds.

Considering the first five instances in Table 1 where the number of battalions and the number of fleets are small as the size of a regiment, the Cplex solver gives optimal solutions very fast. In the instances $(6, 9, 12)$, it takes the Cplex solver about 2 hours to get its best solution, but not the optimal solution. On the other aspect, it can be seen from instance 8 that the optimal solution is easily achieved by the Cplex solver in lest than 1 second. It may be concluded from the numerical results that the Cplex solver runs fast and achieves optimal solutions in small size instances but sometimes it takes several hours to obtain its best solution.

In reality, the practical AMP usually allocates a missile regiment of 4 to 6 missile battalions to protect an important target which is attacked by 5 to 8 attacking aircraft fleets. Instances of those sizes can be solved so fast by our formulation as report in the

Table 2: Experimental results for the AMP.

Instances				Branch-and-Cut		
No.	b	l	f	Gap	Eff.	Time
1	7	38	8	0	0.717	0.63
2	6	8	5	0	0.922	508.87
3	5	58	8	0	0.619	1.52
4	7	50	5	0	1.000	0.50
5	5	83	8	0	0.488	0.65
6	10	100	10	0	1.000	7.67
7	10	100	10	0	0.701	212.89
8	10	100	10	0	0.7919	5.11
9	10	100	10	0	0.7819	172.56
10	10	100	10	0	0.8143	9.66
11	10	100	10	0	0.633	49.81
12	10	100	10	0.02	0.913	7201.38
13	10	100	10	0.05	0.936	7211.39
14	10	100	10	0.07	0.91089	7217.53
15	10	100	10	0.01	0.7099	7201.81

Table 3: Experimental results for IAMP.

Instances					Branch-and-Cut		
No.	b	m	l	f	Eff.	Gap	Time
1	21	12	56	11	0.72	0	18.6
2	21	12	41	11	0.81	0	5.6
3	22	12	42	12	0.97	0.83	7209.9
4	20	12	45	10	0.52	0	11.7
5	21	12	51	11	0.57	0	20.5
6	24	12	54	14	0.98	0.05	7213.7
6	23	12	48	13	0.46	0	9.8
8	23	12	58	13	0.91	0.12	7210.6
9	22	12	47	12	0.71	0	22.1
10	21	12	41	11	0.56	0	29.8
11	23	12	48	13	0.94	0.12	7225.7
12	20	12	55	10	0.84	0.09	7217.6
13	22	12	47	12	0.44	0	8.6
14	22	12	47	12	0.65	0	15.2
15	22	12	57	12	0.60	Out of memory	

first five rows of Table 2, specially instances $(1,4,5)$ of these sizes were solved in less than one second. This result motivates us to increase the size of test instances up to 10 battalions, 100 locations and 10 fleets to evaluate the tackling ability of the formulation. In the time limit of 2 hours, the 11 easy instances (obtained optimal solutions by Cplex solver) among 15 instances were completely solved to optimality, most of them take less than 1 minutes; while the 4 hard instances (not obtained optimal solutions by Cplex solver) were solved nearly to optimality since the gaps are so small $(0.01\% - -0.07\%)$. Although 10 last test instances have same sizes in term of the number of battalions, the number of fleets and the number of locations, the computational difficulty is different. This diversification comes from the geographic of the potential locations. In comparison to defensive effectiveness of easy instances, the hard ones have greater effectiveness. Sets of available missile battalions in IAMP instances were generated around 20 battalions that are larger than the ones used in Table 2. Among 15 instances in Table 3, there are 9 easy instances solved to optimality in less than 30 seconds, while 5 hard instances were not completely solved in the time limit, and one instance (No. 15) returned to “Out of memory” error during the resolution. Similar to hard AMP instances, the hard IAMP instances generate very large search trees that may cause no feasible solution. It is essentially seen that the easy IAMP instances correspond to reasonable effectiveness values, while the hard IAMP instances usually are associated with high effectiveness values. These experimental results were validated and recommended by the experienced soldiers that this formulation should be packaged for the purpose of training and integrated into a *C4I* system.

4 CONCLUSION

In this paper, we formulated three problems in the class of defensive missile battalions mission planning against aircraft attack model, that support defensive decision makers not only decide where to locate their missile battalions, but also point out that how many missiles should be launched from each battalion to each attacking aircraft fleet. The mathematical formulations are IPs, proved as NP-hard. These mathematical programs were implemented and experimented on test instances generated basing on the help of experienced veterans, in which parameters on probability of kill, maximum number of launches, as well as minimum distance between two battalions, were pre-processed for a particular set of defensive battalions and aircraft attack. The numerical results provide the incidence that the proposed formulations should be widely applied in real-life combat field. As future works, we intend to formulate and tackle other variance of the defensive distribution models.

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APPENDIX

4.1 Compute $e(b,f,t)$

Suppose that a battalion $b \in B$ plan to launch t missiles to fleet $f \in F$ that has $n(f)$ aircrafts. We are given coefficient corresponding to each missile battalion $b \in B$, $c^b = c^b_t c^b_c c^b_d$, where c^b_t is technical coefficient, c^b_c is control coefficient and c^b_d is combat complex coefficient. The probability of kill of each missile launched from battalion b to fleet f is known as p ($p \in [0,1]$). Based on defensive mode, we consider following situations:

1. Disperse mode: Suppose that each time a battalion decides to launch 2 missiles to an aircraft of a fleet. Since the probability of kill is $p(b,f) = p$ for all $b \in B, f \in F$, expected number of killed aircraft is $e(b,f,t) = t(1-(1-p)^2)$.
2. Focus mode: Suppose that battalion b launches t times focusing on fleet f, where $t = n(f)t_1 + t_2$, then probability of kill on each aircraft in t_1 launches is $1-(1-p)^{t_1}$. Battalion b has $t_2(t_2 < n(f))$ launches left, inferring probability of kill on one aircraft in each launch is $p(1-p)^{n(f)-1}$. Then, expected number of killed aircraft can be estimated by $eb,f,t = n(f)(1-(1-p)^{t_1}) + t_2 p(1-p)^{n(f)-1}$.
3. Random mode: Let X_i where $i = 1,2,...,n(f)$, be random variables defined as 1 if aircraft i is killed and 0 otherwise. While probability of kill on aircraft i in fleet f is $1-(1-\frac{p}{n(f)})^t$, expected number of killed aircraft can be approximated as $e(b,f,t) = E(\sum_{i=1}^{n(f)} X_i) = n(f)E(X_i) = n(f)(1-(1-\frac{p}{n(f)})^t)$.

4.2 Compute $t(b,l,f)$

Value $t(b,l,f)$ is maximum number of launches that a battalion b located at location l can launch to fleet f. This number depends on following quantities. For a fleet f, we let $v(f)$, $h(f)$ and $l(f)$ be its velocity, height and length, respectively. In an attack, fleet brings different type of bomb that can be verified as $tb(f) = 1$ if fleet f brings nuclear bomb and $tb(f) = 0$ if fleet f brings regular bomb. For a battalion b, we denote d_{max} and r_b by long range of missile on battalion b and distance between that battalion and the target, respectively. We suppose that the shortest time between two consecutive launches, t_{as}, as well as obscured coefficient, δ, are known. Furthermore, angle of battalion location, α_b, and angle of in-coming fleet, α_f, are parameters. Function $t(b,l,f)$ can be computed as follows:

1. Compute critical radius $r_s = 5000tb(f) + v(f)\sqrt{\frac{2h(f)}{g}} - \Delta$ where $5000m$ is active radius of nuclear bomb, $g \approx 9.8m/s^2$ is gravity acceleration, $\Delta = 0.25h(f)$ if $v(f) \leq 300m/s$, $\Delta = 0.4h(f)$ if $v(f) > 300m/s$.
2. Compute shape time of fleet t_{fs}: $t_{fs} = \frac{l(f)}{v(f)}$
3. Compute launching time of battalion t_{bs}:
   (a) Angle between battalion's location and fleet φ: $\varphi = |\alpha_f - \alpha_b|$.
   (b) If ($r_b + r_s > d_{max}$ and $r_b + d_{max} > r_s$ and $r_s + d_{max} > r_b$) then
      i. If $\varphi > \varphi^*$ then $t(b,l,f) = 0$ where $\varphi* = \arccos(\frac{r_b^2 + r_s^2 - d_{max}^2}{2r_b r_s})$.

ii. If $\varphi \leq \varphi^*$ then $t_{bs} = \frac{x - r_s}{v(f)}$ where x is root of equation $x^2 + r_b^2 - d_{max}^2 = 2xr_b \cos\varphi$.

(c) If $(d_{max} \geq r_b + r_s)$then $t_{bs} = \frac{y - r_s}{v_f}$ where y is root of equation $y^2 + r_b^2 - d_{max}^2 = 2yr_b \cos\varphi$.

(d) If $(r_s \geq d_{max} + r_b)$ then $t(b,l,f) = 0$.

(e) If $(r_b \geq d_{max} + r_s)$ then

i. If $\varphi > \varphi^*$ then $t(b,l,f) = 0$ where $\varphi* = \arcsin(\frac{d_{max}}{r_b})$.

ii. If $\varphi \leq \varphi^*$ then $t_{bs} = \frac{2\sqrt{d_{max}^2 - r_b^2 \sin^2\varphi}}{v_f}$.

4. Compute $t(b,l,f) = 1 + \frac{\delta t_{bs} + t_{fs}}{t_{as}}$.