Open Loop based Time Optimal PID Control Synthesis

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Abstract: This paper deals with PID control tuning. The main objective being to minimize stabilizing/settling time of linear control feedback. The presented method is based on an open loop time optimal control framework reformulation into a closed loop system. Numerical tools such as orthogonal functions and optimization algorithms are used to determine PID parameters that match a certain equivalent bang-bang control. Numerical simulation of the obtained results shows the effectiveness of the proposed approach.

1 INTRODUCTION

Minimum time control has received a great attention from researchers, this is particularly relevant since the introduction of the maximum principe by (Pontryagin et al., 1962). The methods used to solve such problems are numerous. Some of the approaches are analytical (shen et al., 2013) and are based on some affine mapping and graphical resolution. Others are numerical (Lasserre et al., 2005) and uses computational algorithms (Piccagli and Visioli, 2007) etc. Linear systems minimum time control, due to the saturation on both the input and output, feature a generalized bang-bang solution. Specifically, the solution input is a combination of bang-bang functions and linear combination of modes associated to the zero dynamics (Consolini and Piazzi, 2006).

On the other hand, researchers have used the orthogonal polynomials in many fields like tracking control (Warrad et al., 2015) and model order reduction (Qi et al., 2014). Some have used them to solve the minimum time control problem like the work presented in (Piccagli and Visioli, 2007). Where Chebyshev polynomials had been used to find out the solution.

Furthermore, the orthogonal functions are a widely used tool in the control field. They have been used for state estimation (Chou and Horng, 1986) and used for systems that feature delays (Mohan and Kar, 2010). To find a general solution for LTI system with complex poles, orthogonal function were used to solve the open loop problem (Bichiou et al., 2016). In fact orthogonal function or polynomials like block pulse functions (BPFs) are a powerful tool to solve these problems numerically. It offers the advantage of reducing the differential equations to a linear system of algebraic equations through the use of the operational matrix of integration and vector forms. It is also noted that using piecewise orthogonal functions such as block pulse instead of polynomial orthogonal functions results in more relevant control. It captures better discontinuities in the inputs (the control saught is of type Bang-Bang).

In practice, one of the most popular control structures is the PID controller. This is due to its simplicity and ability to achieve the desired performance with various possible technologies (Åström and Hägglund, 1995). In this paper a PID controller of a particular structure (Pradhan and Ghosh, 2015) is adopted in order to find out the feedback signals that allows the studied system to reach either its equilibrium state or a desired steady state both in a minimum time.

Some issues in literature dealing with PID time optimal control exists (Piccagli and Visioli, 2009). In (Piccagli and Visioli, 2009), authors are interested to derive a feedforward control for a closed loop system with PID controller already designed. Then the objective is to improve the set-point following performance of the controller. The system being constrained in both input and output, so the open loop control is not bang-bang.

In the proposed approach, we are basically searching for PID parameters k_p, k_i and k_d that permit to the closed loop system to behave as it was controlled with a minimum time bang-bang control.

The formulation of this problem, is mathematically simplified when using orthogonal function. Then all system variables are expanded over that ba-

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sis. Since then, in all the development, we will replace states, output and control input with their coefficients when the development is truncated to a finite number of functions.

Moreover, introducing, operational matrix of integration and some tensor properties could help us to pose properly the basis of the framework.

The paper is organised as follows The second section is reserved for the description of the used orthogonal functions and their algebraic properties. In the third section, a formulation of the time optimal open loop problem is given. The formulation of the optimization problem and simulation results are presented in the fourth section. In the fifth section, a PID controller structure is introduced. Discussion of the simulation results and discussion are shown in the sixth section. Finally, concluding remarks and future works are presented.

2 ORTHOGONAL FUNCTIONS AND ALGEBRAIC PROPERTIES

Using orthogonal functions (OF) to construct operational matrices was approached in the study of dynamic systems for modeling (Bouafoura et al., 2009), identification (Rao and Sivakumar, 1981); (Pacheco and Steffen, 2002) and control purposes (Mohan and Kar, 2010).

2.1 Function Development over OF Principle

Any analytical function absolutely integrable on the time interval $[0,\ T]$ can be approximated as follows:

$$f(t) = \sum_{i=0}^{\infty} f_i \phi_i(t) \tag{1}$$

where $\phi_i(t)$ are the elementary orthogonal functions and the coefficients f_i are evaluated by the following scalar product:

$$f_i = \int_0^T f(t)\phi_i(t)dt \tag{2}$$

For numerical purposes, a truncation of equation (1) until a convenient number of elementary functions must be done:

$$f(t) \cong \sum_{i=0}^{N-1} f_i \phi_i(t) = F_N^T \Phi_N(t) \tag{3}$$

where $\Phi_N^T = [\phi_0(t), \phi_1(t), \cdots, \phi_N(t)]$ constitutes the orthogonal basis and $F_N^T = [f_0, f_1, \cdots, f_N]$ is the coefficient vector.

Integrating (3) transforms it as follows:

$$\int_0^t f(t) \cong F_N^T P_N \phi_N(t) \tag{4}$$

where $P_N \in \mathbb{R}^{n\times n}$ is the operational matrix of integration depending on the considered orthogonal basis.

This approximation can fit both orthogonal piecewise functions and orthogonal polynomials, in fact their operational matrices in the integer case were largely found out and applied in systems science.

As a result, the differential equations describing dynamic processes can be reduced into algebraic relations allowing important simplifications in the synthesis problems.

2.2 Properties of the Block Pulse Functions

Block pulse functions (BPFs) constitute a complete set of orthogonal functions and are defined over the time interval $[0,T]$ as follows (Wu et al., 2000):

$$\varphi_i(t) = \begin{cases} 1 \ \ if\ t \in [iT\ ,\ (i+1)T] \\ \quad i = 0,...,N-1 \\ 0 \ \ otherwise \end{cases} \tag{5}$$

A function $f(t)$ can be approximately represented by:

$$f(t) \simeq \sum_{i=0}^{N-1} f_i \varphi_i(t) = F^T \phi(t) \tag{6}$$

with
$F = [f_0, f_1, ..., f_{N-1}]^T$ is the coefficient vector.
$\phi = [\varphi_0(t), \varphi_1(t), ..., \varphi_{N-1}(t)]^T$ is the block pulse basis vector.
f_i is given by:

$$f_i = \frac{T}{N}\int_{iT}^{(i+1)T} f(t)\phi_i(t)dt \tag{7}$$

where N is the dimension of the operational matrix. The operational matrix for the block pulse functions is given as follows:

$$P_N = \frac{T}{N}\begin{bmatrix} \frac{1}{2} & 1 & 1 & \dots & 1 \\ 0 & \frac{1}{2} & 1 & \dots & 1 \\ \vdots & \ddots & \frac{1}{2} & \dots & 1 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & 0 & \frac{1}{2} \end{bmatrix} \tag{8}$$

3 TIME OPTIMAL OPEN LOOP PROBLEM

In this part, a general minimum time problem is formulated for SISO linear time invariant systems.

We are particularly interested with the class of systems described by:

$$\begin{cases} \dot{x}(t) &= Ax(t)+Bu(t) \\ y(t) &= Cx(t) \end{cases} \tag{9}$$

where $x \in \mathbb{R}^n$ is the vector of states, $u \in \mathbb{R}^m$ is the control input and $y \in \mathbb{R}^p$ is the output.

The objective here is to determine the controller sequence that allows the system to move from a known point **A** to another point **B** in the least possible time. For this, time optimization is required. That leads to minimizing this cost function (Kirk, 1970):

$$J = \int_{t_0}^{t_f} dt \tag{10}$$

Applying The Pontryagin Maximum Principle (PMP) (Pontryagin et al., 1962) and using the Hamiltonian (Kirk, 1970), the the following control is derived. This can be written as follows:

$$u(t) = \begin{cases} u_{min}, & if \quad \lambda^T B < 0 \\ u_{max}, & if \quad \lambda^T B > 0 \end{cases} \tag{11}$$

Thus, the obtained control is bang-bang.

To determine the control we need to determine the sign of λ which is the co-state vector.

We remind here that this solution of the minimum time control problem is solved in open loop.

4 BPFs BASED OPTIMAL CONTROL SOLUTION

4.1 Formatting of the Control Problem

Finding the minimum time solution for system (9) means solving multiple differential equations which is a very difficult task. In order to make this problem easier to solve, BPFs based approach is used.

Considering the system (9) written in state space form, in order to derive the final time, a variable change is considered as follows:

$$t = \tau t_f \tag{12}$$

This transformation allows us to go from the time domain $t \in [0, t_f]$ to $\tau \in [0, 1]$, the system states become:

$$x(t) = \tilde{x}(\tau) \tag{13}$$

Notice that, the latter variable change lead to a constant time interval [0,1], for the used series. Since, the final time t_f is unknown, we deduce,

$$\dot{x}(t) = \frac{d\tilde{x}(\tau)}{d\tau}.\frac{d\tau}{dt} = \frac{1}{t_f}\dot{\tilde{x}}(\tau) \tag{14}$$

The original system (9) is now equivalent to:

$$\frac{1}{t_f}\dot{\tilde{x}}(\tau) = A\tilde{x}(\tau) + B\tilde{u}(\tau) \tag{15}$$

The use of orthogonal functions consists on developing both, system states and input over that base:

$$\begin{aligned} \tilde{x}(\tau) &= \tilde{X}_N^T \cdot \phi_N(\tau) \\ \tilde{u}(\tau) &= \tilde{u}_N^T \cdot \phi_N(\tau) \end{aligned} \tag{16}$$

Furthermore, integrating equation (15) leads to

$$\frac{1}{t_f}(\tilde{X}(\tau) - \tilde{X}(0)) = A\int_0^\tau (\tilde{X}(\sigma))d\sigma + B\int_0^\tau \tilde{u}(\sigma)d\sigma \tag{17}$$

Introducing, coefficients of $\tilde{x}(\tau)$, $\tilde{u}(\tau)$ and operational matrix of integration we obtain:

$$\int_0^\tau \tilde{X}(\sigma)d\sigma = \tilde{X}_N^T \int_0^\tau \phi_N(\sigma)d\sigma = \tilde{X}_N^T P_N \phi_N(\tau) \tag{18}$$

then we can write:

$$(\tilde{X}_N^T - \tilde{X}_{N_0}^T)\phi_N = t_f(A\tilde{X}_N^T P_N + B\tilde{u}_N^T P_N)\phi_N \tag{19}$$

After the integration of equation (15) and the introduction of coefficients $\tilde{x}(\tau)$,$\tilde{u}(\tau)$ (Bichiou et al., 2016), we can write:

$$\tilde{X}_N^T - \tilde{X}_{N0}^T = t_f(A\tilde{X}_N^T P_N + B\tilde{u}_N^T P_N) \tag{20}$$

where $\tilde{X}_{N0}$ depends on the chosen set of functions.

For the block pulse functions, the vector $\tilde{X}_{N0}$ is of the following form:

$$\tilde{X}_{N0} = \begin{bmatrix} \tilde{X}(0) & \tilde{X}(0) & \cdots & \tilde{X}(0) \end{bmatrix}$$

4.2 Optimization Algorithm

Consider the system model defined in (9).

To find the transition time from the initial position to the target one need to solve the following nonlinear problem (NLP):

$$\min_{t_f, u_N, X_N} (t_f) \tag{21}$$

subject to

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bu(t) \quad 0 \le t \le t_f \\ &u \in [u_{min}, u_{max}] \\ &x(0) = x_0, x(t_f) = x_f \end{aligned} \tag{22}$$

then this problem is reported to the domain $[0, 1]$. The optimization algorithm in the orthogonal base is of the following form (OFNLP):

$$min\ (tf) \tag{23}$$

subject to linear constraints:

$$\begin{aligned} \tilde{u}_{Nmin} \le \tilde{u}_N\ \phi_{N,bp} \le \tilde{u}_{Nmax} \\ \tilde{X}_{Nf,bp} = \begin{bmatrix} 0 & 0 & \cdots & x_f \end{bmatrix} \end{aligned}$$

and nonlinear constraints:

$$\tilde{X}_N^T - \tilde{X}_{N0}^T = t_f(A\tilde{X}_N^T P_N + B\tilde{u}_N^T P_N) \quad (24)$$

The resolution of such optimization problem can be done through interior point routines like the function ”fmincon” implemented in Matlab. As a result the final time t_f and the control sequence may be obtained.

5 SYSTEM WITH PID CONTROL

5.1 Closed Loop with PID

In this section, a PID controller is introduced to improve the system performances. the considred feedback control is described by figure(1).

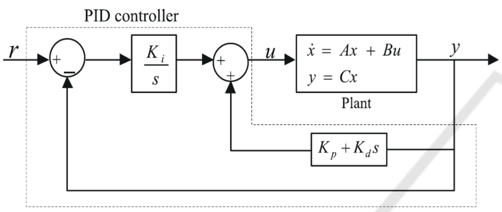

Figure 1: Feedback control structure.

the control effort is then given by:

$$u = k_p Cx + k_i \xi + k_d C\dot{x} \quad (25)$$

where

$$\xi(t) = \int_0^t (r - y)d\tau \quad (26)$$

In this work, we aim to find the triplet $\{k_p, k_d, k_i\}$ that steer the system from an arbitrary initial state to a final one. Hence, both stabilization and tracking may be investigated.
Equation (25) could be written:

$$u = k_p Cx + k_i \xi + k_d C(Ax + Bu) \quad (27)$$

Then the control u becomes:

$$u = (1 - k_d CB)^{-1}[k_p Cx + k_i \xi + k_d C(Ax + Bu)] \quad (28)$$

Let us note $\overline{K} = [\overline{k}_p \overline{k}_i \overline{k}_d] = (1 - k_d CB)^{-1}[k_p k_i k_d]$.
Hence, in the following section, we are interested to investigate $\overline{k}$, then pid controller gains could be recovered as follows:

$$k_d = \overline{k}_d(1 + CB\overline{k}_d)^{-1} \quad (29)$$

$$k_p = \overline{k}_p(1 - CB\overline{k}_d) \quad (30)$$

$$k_i = \overline{k}_i(1 - CB\overline{k}_d \quad (31)$$

From the above, it is clear that finding PID controller is possible iff both $I - k_d CB$ and $I + CB\overline{k}_d$ are invertible. Moreover, in (Zheng et al., 2002), it is proven that the existence of invertibility $I - k_d CB$ is necessary and sufficient for well-posedness of the MIMO PID control problem.

5.2 PID Synthesis Formulation

In order to compute time optimal PID controller gains, we propose firstly to consider the open loop system and find the optimal coefficients of control $\tilde{u}_N^T$ and system trajectory $\tilde{X}_N^T$. Let us denote $\tilde{u}_N^{T*}$ and $\tilde{X}_N^{T*}$ solutions of the optimization problem (OFNLP) presented in the last section.

The obtained control should obviously verify equation (28):

$$\tilde{u}_N^{T*} = \overline{k}_p C\tilde{X}_N^{T*} + \overline{k}_i(r_N^T - C\tilde{X}_N^{T*})P_N + \overline{k}_d CA\tilde{X}_N^{T*} \quad (32)$$

For numerical convenience, the latter expression of the control should be introduced into the state equation with respect to the time domain of resolution. Then it comes:

$$\tilde{X}_N^{T*} - \tilde{X}_{N0}^T = t_f(A\tilde{X}_N^{T*}P_N + B[\overline{k}_p C\tilde{X}_N^{T*} + \overline{k}_i(r_N^T - C\tilde{X}_N^{T*})P_N + \overline{k}_d CA\tilde{X}_N^{T*}]P_N) \quad (33)$$

Rearranging equation (33), one may obtain:

$$\frac{1}{t_f}(\tilde{X}_N^{T*} - \tilde{X}_{N0}^T) - A\tilde{X}_N^{T*}P_N = B\overline{K}\begin{bmatrix} C\tilde{X}_N^{T*}P_N \\ (r_N^T - C\tilde{X}_N^{T*})P_N^2 \\ C\tilde{X}_N^{T*}P_N \end{bmatrix} \quad (34)$$

Then $\overline{K}$ could be found with the following formula:

$$vec(\overline{K}) = (F^T \otimes B)^+ vec(G) \quad (35)$$

with respect to the result (Brewer, 1978):

$$vec(XYZ) = (Z^T \otimes X)vec(Y)$$

and where F is the last term of equation(34), while G stands for the left hand side of the same equation, $\otimes$ denotes the Kronecker product (Brewer, 1978), *vec* is the vectorization operator and $(.)^+$ is the Moore-Penrose pseudoinverse of a matrix.

6 SIMULATION RESULTS AND DISCUSSION

6.1 Example 1: Application to a Bridge Crane

In this part, we intend to use the validated results on a geometric model of a bridge crane (Ermidoro et al., 2014).

The system is composed of the bridge, moving along the Y axis and the trolley, moving along the X axis. The load is connected to the trolley by a rope and can oscillate along any direction as described in figure (2).

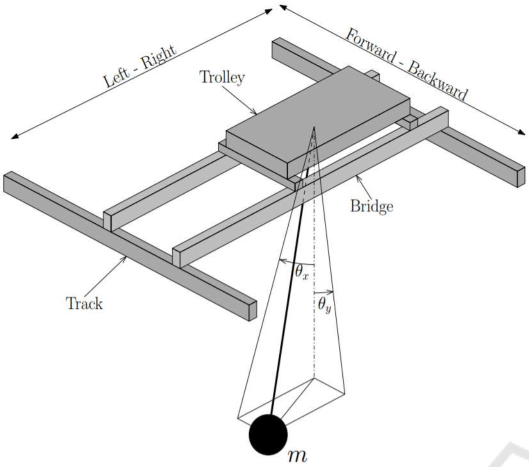

Figure 2: The typical structure of a bi-dimensional bridge crane.

The model of the bridge crane is described by a classical state space model:

$$\begin{cases} \dot{x}(t) & = & Ax(t)+Bu(t) \\ y(t) & = & Cx(t) \end{cases} \tag{36}$$

where

$$A=\begin{bmatrix} 0 & 1 \\ -\frac{g}{r} & -\frac{b}{mr^2} \end{bmatrix} \quad B=\begin{bmatrix} -\frac{1}{r} \\ -\frac{b}{mr^3} \end{bmatrix} \quad C=\begin{bmatrix} 1 & 0 \end{bmatrix} \tag{37}$$

Note that the control input is saturated and $u \in [u_{min}, u_{max}]$.

The state vector $x=[x_1,x_2]^T=[\theta,\dot{\theta}]^T$. We need to move the system from $x_0=[\theta_0,0]^T$ to $x_r=[0,0]^T$. Hence, in this example, we assume that $r=0$, The system is shifted to its equilibrium point.

The physical parameters of the bridge crane are as follows:

$$m=1000kg, r=5m, g=9.81m/s^2, b=12000$$

The bounds of the controller is $[-10,10]$. As found in (Ermidoro et al., 2014) for time optimal control $t_f = 4.2717s$.

The control sequence of the system in open loop when minimizing (OFNLP) is illustrated in figure (3).

The state trajectories of the system are given in figure(4).

In this example, we deal with a stabilization problem, then we propose to use a previous result (Bichiou et al., 2016) to obtain a state feedback gain. Then a method transforming that result to PID with the same structure (figure 1) is applied (Pradhan and Ghosh, 2015).

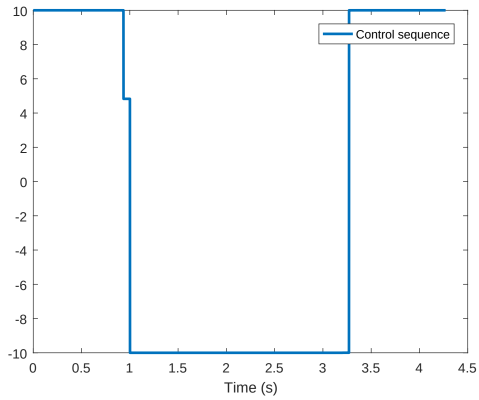

Figure 3: Bridge crane control sequence in open loop.

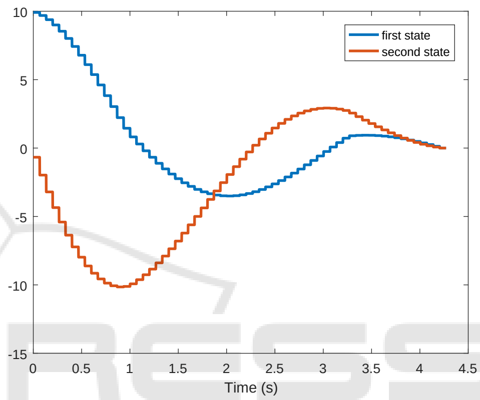

Figure 4: Bridge crane state trajectories in open loop.

The PID parameters are: $k_p = 488.7322$, $k_i = 37.5069$ and $k_d = 112.1876$.

The system state trajectories are given in figure (5).

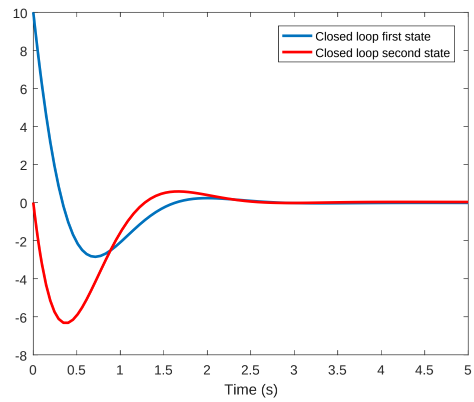

Figure 5: Bridge crane state trajectories using PID controller.

6.2 Example 2: Fighter Aircraft

In this part, we intend to use a linearized longitudinal dynamics of the McDonnell Douglas Tailless Advanced Fighter Aircraft (TAFA) (Kefferpütz and Adamy, 2011) for tracking purposes.

The general representation of the system is of the following form:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{cases} \quad (38)$$

where

$$A = \begin{bmatrix} -1 & 1 \\ 6 & -2 \end{bmatrix}; \; B = \begin{bmatrix} 0 \\ 8 \end{bmatrix} \quad (39)$$
$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

with x_1 is the deviation of the angle of attack (rad) and x_2 is the body axis pitch rate (rad/s).

The control variable is limited to $|u| \leq 20/180\pi$ rad/s.

The system control is given in figure (6).

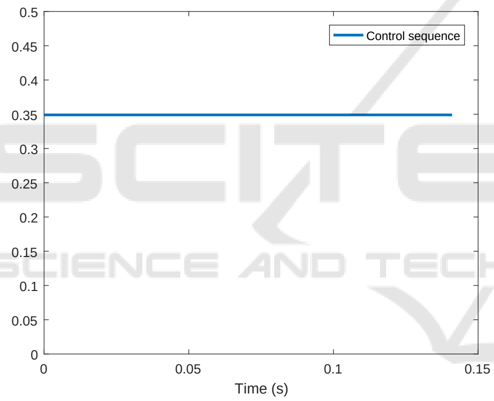

Figure 6: Fighter aircraft control in open loop.

The PID parameters are: $k_p = -0.6447$, $k_i = 6.8510$ and $k_d = 0.0064$.

The state trajectories of the system are given in figure (7).

The system control is given in figure (8).

The system output is given in figure (9).

7 CONCLUSIONS

In this paper, a based minimum time control problem for linear systems is proposed. In fact, the open loop control determined with orthogonal function optimization is transformed to a PID controller. Hence, dynamics of the closed loop system may be enhanced and final time could be reached in tracking as in a classical bang-bang control. The whole development is carried with block pulse functions and related operational matrices. Simulations illustrate the validity of the approach.

In future work, we intend to generalize the PID time optimal control problem to some classes of nonlinear systems.

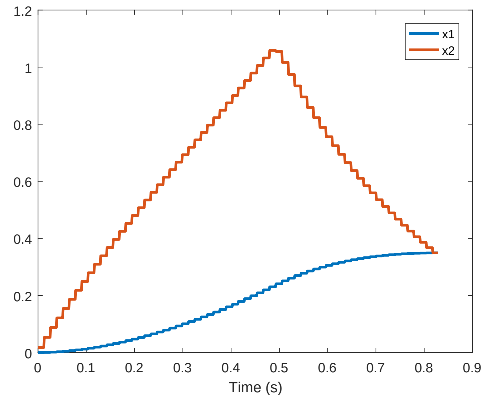

Figure 7: Fighter aircraft state trajectories in open loop.

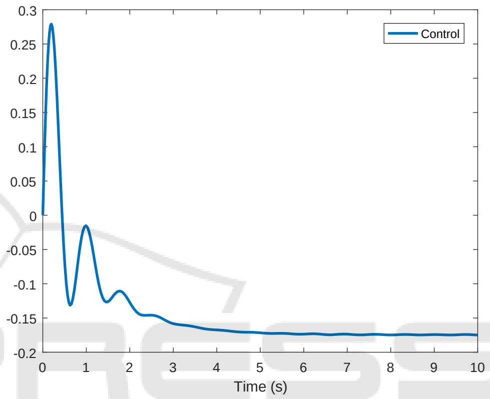

Figure 8: Fighter aircraft PID control.

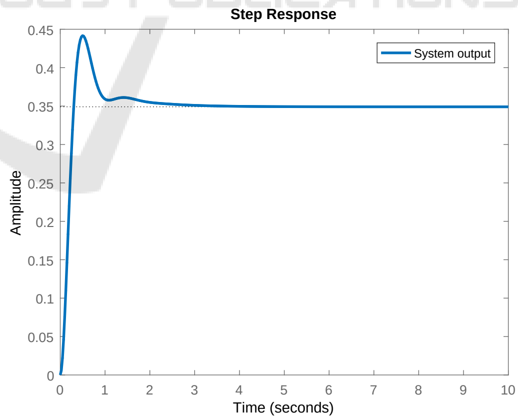

Figure 9: Fighter aircraft output with PID controller.

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