Robust $PI^{\lambda}D^{\mu}$, H^{∞} and Smith Predictor Controller Design for Time Delay Systems

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Keywords: Robust Control, Fractional Order Controller, Smith Predictor Controller, H^{∞} Controller, Identification, Industrial System, System with Time Delay.

Abstract: This paper present optimal robust control with different controllers design used in the industrial (didactic or process) system. We designed a controller base on Smith's predicator controller and Fractional order PID ($PI^{\lambda}D^{\mu}$) controller and H^{∞} controller. These control techniques has been used with different controller's types to ensure an optimum control in term of dynamic and static performances of a complex didactic industrial process in accordance with the required specifications. We have described in more details the process, the mathematical model, the structure of FOPID controller and the approximation method (singularity function method of Charef) used to approximate fractional order. The principle of control is decried as well with the different types of controllers used in this study. Finally several simulation and real results are presented which have proved the efficiency of this new control design in term of: stability, robustness and precision.

1 INTRODUCTION

To be robust, an industrial process must be well controlled. Indeed, competitiveness requires keeping process values as close as possible to its required optimum performance and process conditions: such as the products quality, production flexibility, energy saving and safety of personnel, facilities and the environment. The main role of industrial controller is to keep the process under control with the guarantee of a good dynamic and static behaviour performance. Which can be achieved by adjusting and adapting the transfer function parameters in order to as close as possible to the real process? In general, an industrial process is modelled by a non-linear, linear (after linearization) or linear mathematical model with a time delay (Boyd, 1991). Regardless if these models are stable or not are required a controller (control action) to ensure the desired performance. The objective of automatic regulation or servo-control of a process is to keep the process values as close as possible to its optimum of operating points, predefined by the process specification (imposed conditions or performance). Safety aspects of staff and facilities should be taken into accounts, such as those relating to energy and respect for the environment. The specifications define qualitative criteria to be imposed, which are usually translated by quantitative criteria, such as stability, precision, speed or evolution laws. Before going ahead and develop the controller architecture and structure and in case of unknown process parameters, an identification phase is mandatory. Different methods of identification exist in the literature (Broida, Strejc, etc.) (Boyd, 1991; Ljung, 1999; Barraud 2006), in our study we have used Ident a Matlab identifications toolbox function and we did a study of a flow control system (Figure 4) by computing its mathematical model (Abraham, 2015) via applying a different identification methods (Broida, Strejc, etc.) and synthesis of its control laws using several types: FOPID, Smith predictor and H^{∞} controllers (Barraud, 2006), and then at the end we checked the simulation results with the process experiments.

2 CONTROLLERS DESIGN

In the literature, it exist a large number of linear or discrete linear controllers adequate to control an industrial process which have a linear system

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behavior (Kumar, 2014). Among the most common and most used controllers are PI, PD and PID different structures (Shamsuzzoha 2008). Also, there is another type of controller which is more robust than the Standart PID such as the Fractional order PID controller (FOPID) (Bettou, 2008; Bouras, 2013; Djari, 2014). Other types of controllers are developed specifically to control the systems with time delay such as Smith's predictor. This controller was proposed for the first time by OJ Smith in 1957 (Esmaeilzade, 2014).The main idea behind Smith's predictor is that, since it is well known to correct systems without time delay with a corrector (PID for example) (Aidan, 1996; Resceanu, 2009). It does not correct the system without delay but the output will then be estimated by delaying it by the value of the time delay of the system. This very simple approach leads to the following structure:

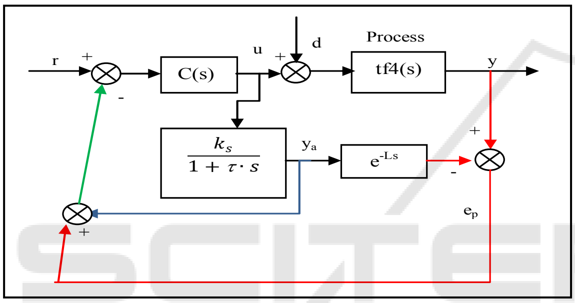

Figure 1: Structure of Smith Predictor (L=Td; Ks=Kp; τ=Td).

Different structures of Smith predictor has been proposed in literature with different controllers. Note that, the implementation of a Smith predictor controller needs a very good model of the process. In our study we have used only Fractional order PID (FOPID) controller and with Smith predictor. The structure type of the FOPID controllers is Fractional order controller: $PI^{\lambda}D^{\mu}$. In control theory, the conclusion about fractional control system is that it can increase the stability region and robustness (Esmaeilzade, 2014) moreover it gives performances at least as good as its integer counterpart (Grimble, 2006). The transfer function of a FOPID controller, which was initially proposed by Podlubny in 1999 (Esmaeilzade, 2014), is given by:

$$G_c(s) = \frac{U(s)}{E(s)} = K_p + K_I \frac{1}{s^{\lambda}} + K_D s^{\mu}, (\lambda, \mu > 0) \quad (1)$$

Where Kp, KI, KD∈ R and λ,μ∈ R+: are the controller tuning parameters and the controller design problem is to determine the suitable values of these unknown parameters in such way it responds to all control objectives (Grimble, 2006). Many methods in literature have been proposed for FOPID approximation (Bouras, 2013). In this work we have used singularity function approximation method of Charef (Bettou, 2011), applied in FOPID controller. The fractional-order integrator $s^{-\lambda}$, λ∈ R+ is approximated as:

$$H(s) = \frac{1}{s^{\lambda}} \cong \frac{K_I}{\left(1+\frac{s}{\omega_c}\right)^m}, \quad with\ 0 < m < 1, \lambda \in R^+ \quad (2)$$

To have a good tuning parameters of the $PI^{\lambda}D$ (Kc, Ti, λ) we have used the following algorithm (Bouras, 2013) described in the steps below:

Step1: calculate the parameters θ_i for 0≪i≪2

$$\theta_0 = \frac{1}{2}, \theta_1 = \frac{-m}{4\cdot\omega_u}, \theta_2 = \frac{m}{4\cdot{\omega_u}^2} \quad (3)$$

ω_u: the unit magnitude frequency of reference model;
m: the derivation fractional order of the reference model;
θ_i: calculated with the reference model parameters.

Step 2: calculate the parameters y_i for 0≪i≪2
Using the following formulas:

$$y_0 = \sum_{k=0}^{N} G_P(kT) \cdot e^{-KT\omega_u} \quad (4)$$

$$y_1 = -\sum_{k=0}^{N} (kT) \cdot G_P(kT) \cdot e^{-KT\omega_u} \quad (5)$$

$$y_2 = \sum_{k=0}^{N} G_P(kT) \cdot (kT)^2 \cdot e^{-KT\omega_u} \quad (6)$$

With y_i: calculated from the transfer function Gp(s) compared to the variable s at the point ωu; N : samples number.

Step 3: calculate the parameters X_i for 0 ≪ i ≪ 2
As per the following formulas:

$$X_0 = \frac{\theta_0}{y_0(1-\theta_0)} X_1 = \frac{\theta_1}{y_0(1-\theta_0)^2} - \frac{x_0.y_1}{y_0} \quad (7)$$

$$X_2 = \frac{\theta_2}{y_0(1-\theta_0)^2} + \frac{2{\theta_1}^2}{y_0(1-\theta_0)^3} - \frac{2.x_1.y_1 + x_0.y_2}{y_0} \quad (8)$$

With X_i: derived from the controller transfer function C(s).

Step 4: calculate the parameters K_c, T_i, λ with the following formulas:

$$\lambda = -\frac{\omega_u . x_2}{x_1} - 1, T_i = -\frac{\omega_u^{(1+\lambda)} . x_1}{\lambda} \quad (9)$$

$$K_c = x_0 - T_i . \omega_u^{-\lambda} \quad (10)$$

3 OPTIMAL CONTROL WITH H$^{\infty}$

Several representations are possible to solve the control problems of the closed loop system, such as H_∞ and H_2 optimization. Therefore it is practical to have a general formula, in order to have a "standard problem" for this type of control. The configuration of the closed loop system with the various specifications (weighting functions) is shown in Figure (2).

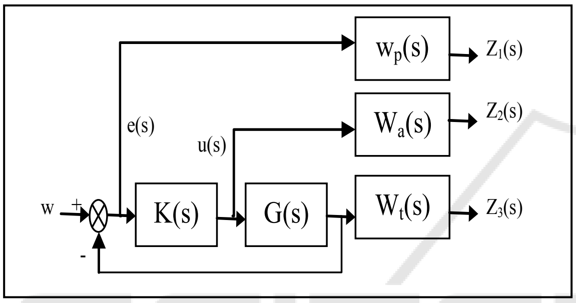

Figure 2: Problem formulation Standard.

Where: $\boldsymbol{W_t(s)}$: transfer matrix of the stability specification; $\boldsymbol{W_a(s)}$: transfer matrix relating to the additive error; $\boldsymbol{W_p(s)}$: matrix for transferring the performance specification.

The general configuration of the standard problem (Tsai, 2014) is presented in Fig.7 (LFT, Linear Fractional Transformations representation).

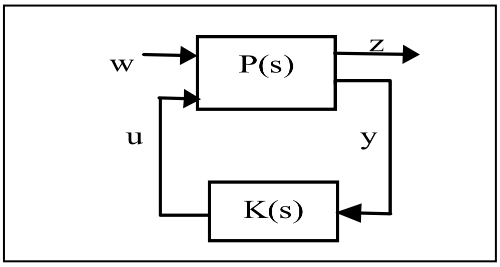

Figure 3: Standard problem (LFT representation).

Where: u: system commands (dimension "m");
w: disturbance inputs (dimension "l");
y: measurements on the system (outputs) (dimension "q");
z: controlled outputs (dimension "p"); x: state vector (dimension "n")

The solution of the standard problem (generalized mixed sensitivity problem) is found by computing a control law u - delivered by a controller K(s) - such that: $u = K(s).y$ minimizing the influence of the perturbation signal w on the output signal z, namely:

$$\left\| \begin{bmatrix} W_p S \\ W_a R \\ W_t T \end{bmatrix} \right\|_\infty < 1 \quad (11)$$

With:

T(s): Complementary Sensitivity defined by

$$T(s) = L(s)(I + L(s))^{-1} \quad (12)$$

L(s): is the Open loop L(s) = G(s) K(s)
R(s): Transfer to Control defined by

$$R(s) = K(s)(I + L(s))^{-1} \quad (13)$$

S(s): Sensitivity defined by:

$$S(s) = (I + L(s))^{-1} \quad (14)$$

We have associated with the standard problem the following cost function $\mathbf{T_{zw}}$:

$$T_{zw}(s) = P_{11}(s) + P_{12}(s)K(s) + [I - P_{22}(s)K(s)]^{-1}P_{21}(s) \quad (15)$$

With:

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \quad (16)$$

$$z(s) = T_{zw}(s)w(s) \quad (17)$$

And we have illustrated the steps for obtaining the K(s) controller parameters by solving the problem H_∞. The problem of optimization by H_∞ is to find a controller K(s) which stabilize the process, so as to minimize the transfer between the inputs w and the outputs z.

$$\|T_{zw}(j\omega)\|_\infty = \max_{\omega} \bar{\sigma}(T_{zw}(j\omega)) \quad (18)$$

The structure of the central controller K(s) is given by the following function:

$$K(s) = -Z_\infty L_\infty (sI - A_\infty)^{-1} F_\infty \quad (19)$$

The association of the sensitivity function S(s) will improve our controller performance in term of closed-loop stability and attenuates the resonance peaks on the maximum singular value of the sensitivity S(s) (Tsai, 2014). The solution to the problem of optimization by H∞ mentioned earlier will be realized by the iteration on the parameter γ then the optimal robust controller K(s) will have to satisfy the condition: $\|Tzw(j\omega)\|_\infty \leq \gamma$. The parameter γ has to satisfy the compromise "Stability/Performance". The different steps for the robust controller's determination are described as follow. All these calculations steps can be considered long before obtaining controller structure, because they must be carried out for each value of the parameter γ. Therefore it is preferable to use a calculation algorithm, which computes the robust controller parameters quicker with very good accuracy. The robust controller parameters algorithm is exposed as below:

1. *Choice of specifications W_t, W_p and W_a.*
2. *Realization of the augmented plant P(s).*
3. *Take $\gamma = 1$, synthesize controller H_∞.*
4. *Calculation of the cost function Tzw.*
5. *If $\|T_{zw}(j\omega)\|_\infty \leq \gamma$ go to 7.*
6. *Otherwise adjust γ and go to 2.*
7. *Frequency Evaluation's and temporal results.*
8. *If the results are satisfactory go to 10.*
9. *Otherwise adjust γ and go to 1.*
10. *End.*

The implementation of the controller will be obtained by MATLAB software's via Robust Control Toolbox.

4 DIDACTIC INDUSTRIAL PROCESS

The process illustrated in FIG. 4 consists of numerous components and accessories (Abraham, 2015). The accessory components are pre-installed on plates.

Figure 4: Experiment setup of a flow control (Abraham, 2015).

The basic module contains one storage tank: 75L (1), Centrifugal pump (2), Compressed air controller with pressure gauge (0-2,5bar) with quick coupling for supplying experiments (3), orifice with Differential Pressure Sensor (Electro-pneumatic control valve) (4), flow Rate Sensor (Electromagnetic) (5), rotameter (6), valve (7) and Switch cabinet (8). The Controlled System Flow is operated with water as the working medium and consists of a variable area flow meter. The flow resistance can be configured using a valve (7), which changes the flow properties in the controlled systems.

One particular benefit of these controlled systems is that, thanks to the float, all changes in the flow rate caused by interference or behaviour of a controller can be observed directly. The training system has an electronic sensor with display for measuring flow rate. It is suitable for measuring flow rates of liquids in closed tubes. The measurement variable is the flow rate. The ideal flow velocity is 1- 3m/s.

The measurement principle is electromagnetic induction according to Faraday's law. Electromagnets or coils generate a magnetic field, in which a conductor moves. This induces a voltage. Here, the medium flowing in the flow rate sensor corresponds to the moving conductor. The magnetic field is generated by pulsed direct current of alternating polarity. The identification methods used to identify our process are described in the following section.

5 PROCESS IDENTIFICATION

The search of an industrial process model is necessary and must result in a model correctly representing the behaviour of the process. However, the model must not be too sophisticated, at the risk of being incompatible with the available corrector, or be too simplistic not to mask certain aspects that are detrimental to proper functioning.

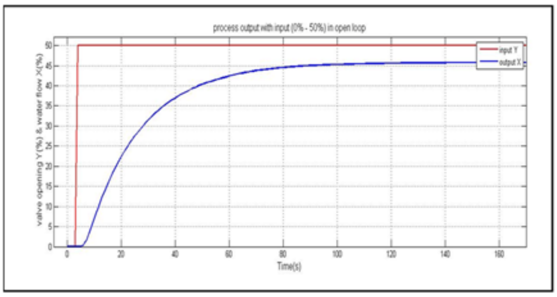

Figure 5: Process step response with input 0 %→ 50% in open loop.

The choice of a model, like its determination, must be judicious (Liung, 1999). The system can then be excited by a step signal with different values. In principle, the output and input must be of the same type with linear system (figure 5). If not, the system is nonlinear (Barraud, 2006).

We have used Matlab function: ident from the identification toolbox. The structure of parametric estimation method is a simple transfer function in continuous time that describes a linear dynamic system. This model is characterized by a static gain, time constants and time delay. If some parameters are known, we need just enter their values and tick the box "Known". The estimation algorithm will use these values for the model. The behaviour of the system is close to the first-order systems with a small time delay, so we start from this principle and we have made the identification with the four datasets. The general form of the transfer function is given by the following formula:

$$\mathrm{tf(s)} = \frac{\mathrm{X(s)}}{\mathrm{Y(s)}} = \frac{\mathrm{K_p}}{\mathrm{T_p} \cdot \mathrm{s} + 1} \cdot e^{-\mathrm{T_d} \cdot \mathrm{s}} \qquad (20)$$

The obtained model with this method is illustrated in the following formula:

$$\mathrm{tf_4(s)} = \frac{0.89686}{20.539 \cdot \mathrm{s} + 1} \cdot e^{-0.5 \cdot \mathrm{s}} \qquad (21)$$

The tf4 is the model that represents better the real system. The index response of the open-loop model (tf4) is illustrated in the following figure:

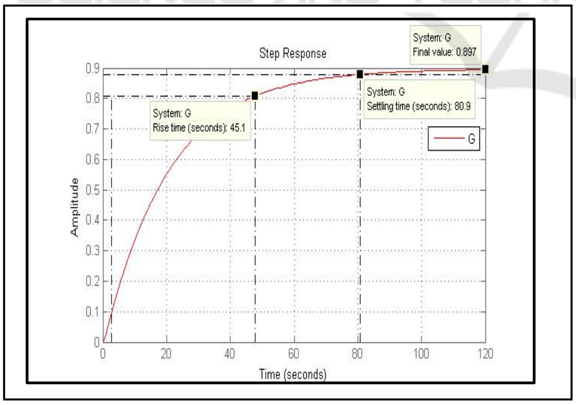

Figure 6: Step response of model tf4 in open loop.

The open loop characteristics are not satisfactory (the system is very slow, final value different of 1) (figure 6). Hence the need to used a controller to ensure the optimal characteristics and improved the stability of process. In the following section different controllers used in this study has been described and on particularly the Smith's predictor controller with new structure.

6 SIMULATION

The simulation is done on a closed loop with an step input. The simulation Parameters are as follows:

FOPID controller: m=0.9; KI =12.3231

H∝ controller:

τ = 20.539;   System time constant in open loop

a = 10;   Acceleration parameter

$w_o = 1/(a^*\tau)$;

$w_1 = \frac{w_0^2 \cdot (s+20)}{(s+w_0)^2}$: Performances specification

$W_2 = []$; $w_3 = \frac{(s+1)}{(0.17 \cdot s+1)}$: stability specification

PID controller: kp1=10.5; ki1=0.808; kd1=1.84;

Smith Predictor: Delay1=0.5s and delay2=2.5s; Disturbance equal 1 at t=40s; Simulation Time =100s; the simulation is organized as follows:

- First study: controlling the system with FOPID and PID controllers without Smith predictor.
- Second study: controlling the system with S FOPID and PID controllers with Smith predictor.
- Finally controlling the system with H∝ robust controller

The block diagram of the control is as follows:

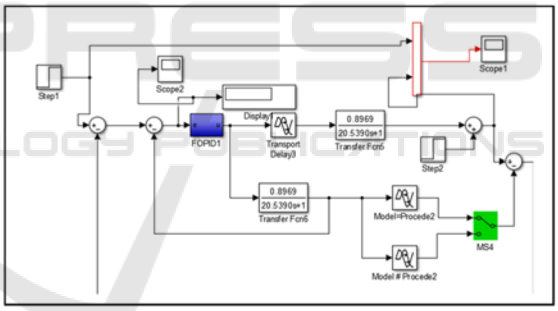

Figure 7: Block diagram of Smith predictor and FOPID controller.

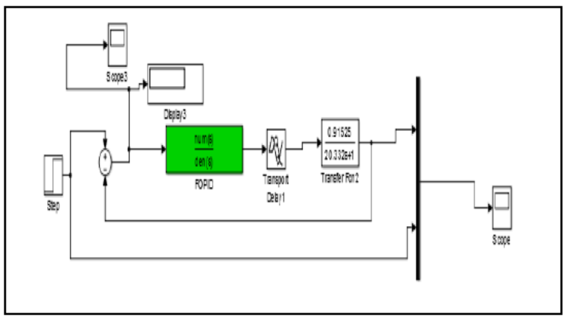

Figure 8: Block diagram with FOPID controller.

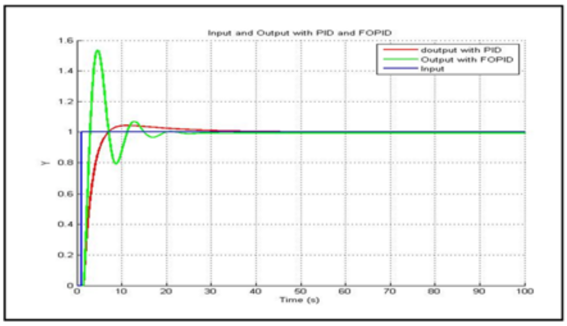

Figure 9: Input and Output curve, with FOPID and PID controller.

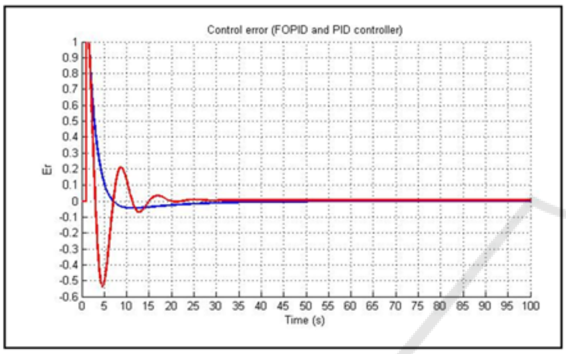

Figure 10: Control error with FOPID (red curve) and PID controller (bleu curve).

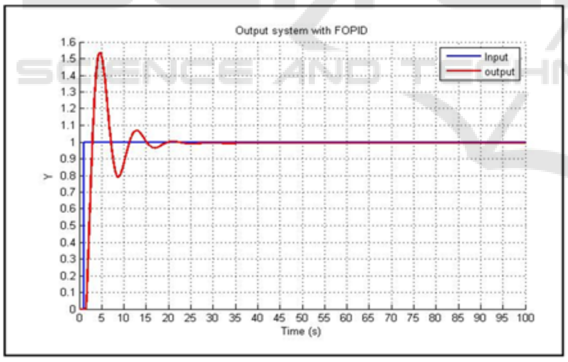

Figure 11: Input and Output curve with FOPID controller.

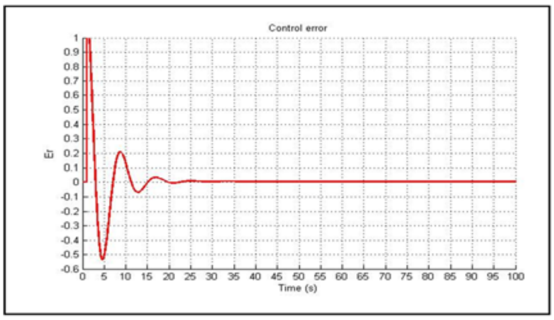

Figure 12: Control error curve with FOPID controller.

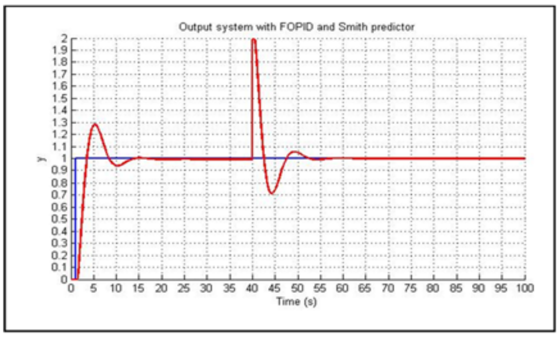

Figure 13: Input and Output curve (Process= model), with smith Predictor and FOPID controller.

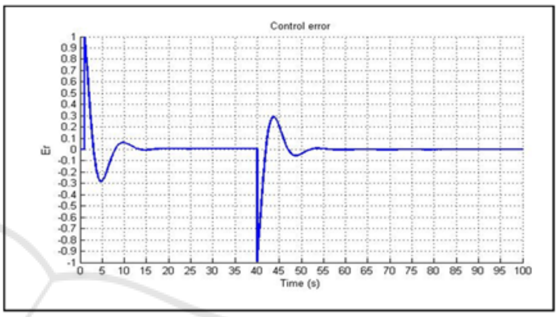

Figure 14: Error control (Process = model), with FOPID controller.

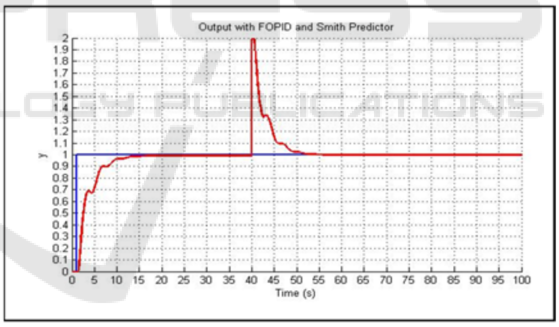

Figure 15: Input and Output curve (Process ≠ model, time delay=2.5s), with Smith predictor and FOPID controller.

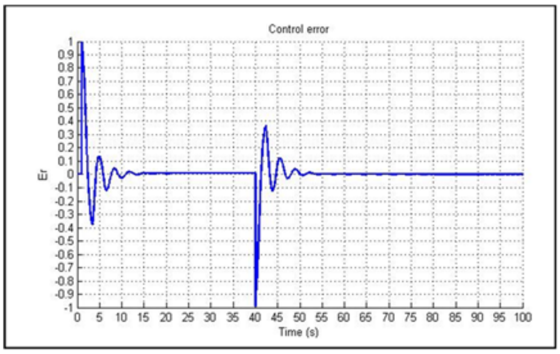

Figure 16: Error control (Process ≠ model, time delay=2.5s), with Smith predictor and FOPID controller.

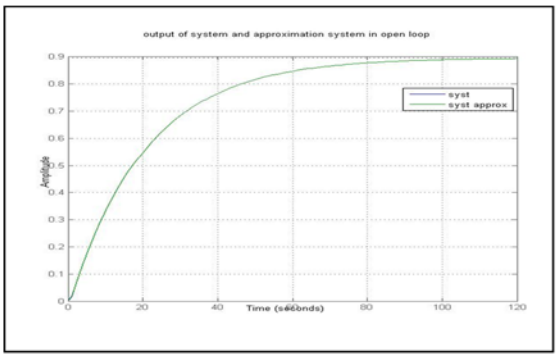

Figure 17: Output of system and approximation system in open loop.

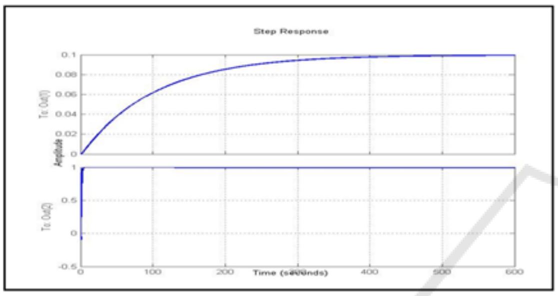

Figure 18: Output of Z1 and Z3 with (W2=0).

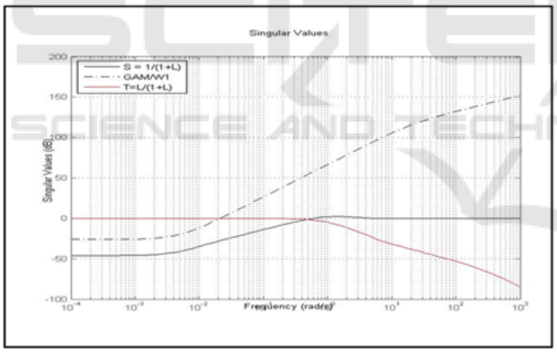

Figure 19: Singular Values.

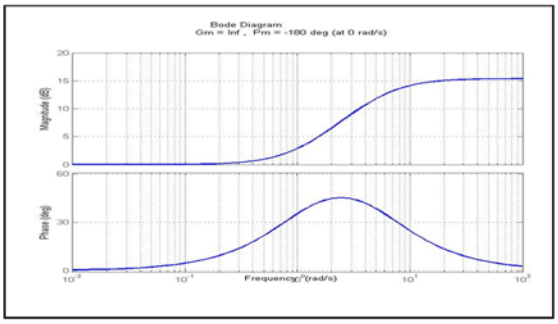

Figure 20: Frequency response of W3.

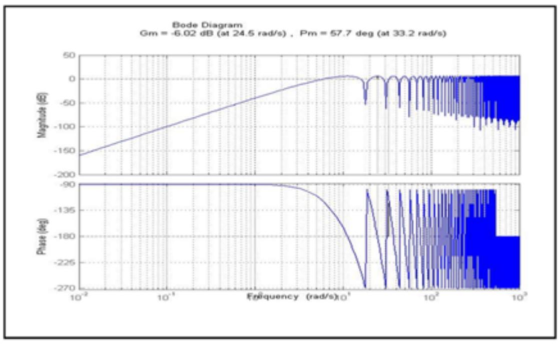

Figure 21: Frequency response of disturbances.

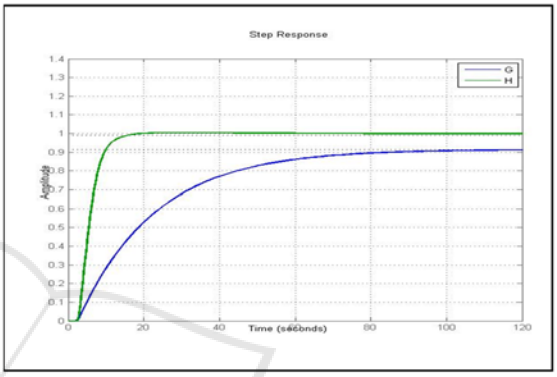

Figure 22: Output system curve in open loop (bleu curve) and in closed loop (green curve).

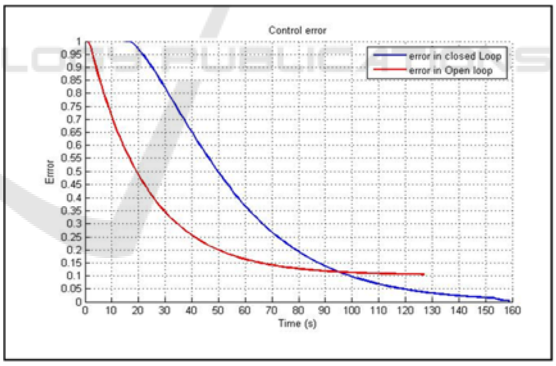

Figure 23: Control error with H∞ controller.

Table 2: Control error.

Control error	FOPID with Smith Predictor	FOPID	PID & Smith Predictor	H_{∞}
Process= model	$1.1.10^{-6}$	6.8. 10^{-3}	$6.5.10^{-4}$	3.5. 10^{-3}
Process ≠ model (delay =2.5s)	$-9.4.10^{-7}$		$8.7.10^{-4}$	

The obtained results illustrated in Fig.9 and Fig.10 show the PID controller is more efficient (short response time).The Fig.11 until Fig.16 illustrated the efficiency of smith predictor with FOPID controller with and without disturbance (very good robustnes, stability and presicion). The Fig.17 until Fig.23 and Table.III show that the H^{∞} controller is more efficient then the FOPID (short response time and good precision). In the table.III we can observed that the designed Smith predictor with FOPID controller gives the best performences and robustnes.

7 CONCLUSION

In this work we have presented a structure of Smith Predictor controller based on PID and Fractional order PID control (FOPID) and robust H^{∞} controller applied to the industrial didactic process, modeled by a linear model with time delay. A detailed description of the system was presented with identification phase. The chosen model has been validated. the obtained results show the new smith predictor sutructure with an Fractional order PID control improves more the performance of the process compared with PID or H_{∞} controller and keep the study open for further optimization of the FOPID parameters in case of a big time delay. Different optimization algorithms can be applied such as PSO or Genetic algorithms.

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