CSG Ray Tracing Revisited: Interactive Rendering of Massive Models Made of Non-planar Higher Order Primitives

Seyedmorteza Mostajabodaveh[1,2], Andreas Dietrich[1,2], Thomas Gierlinger[1,2], Frank Michel[1,2] and André Strok[1,2]

[1] *Fraunhofer IGD, Darmstadt, Germany*

[2] *Technische Universität Darmstadt, Darmstadt, Germany*

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Abstract: In many scientific and engineering areas, CAD models are constructed by combining simple primitives using Boolean set operations. Rendering such a dataset usually requires a preprocess, where the surface of the CAD model is approximated by an often highly complex triangle mesh.

Real-time ray tracing provides an alternative to triangle rasterization as it allows for the direct visualization of (higher-order) solid and planar primitives without having to triangulate them. Additionally, Boolean compositing operations can be performed implicitly per ray, primitives have low storage requirements, and curved surfaces appear pixel-accurate. In this paper we demonstrate these properties using massive real-world CAD models.

1 INTRODUCTION

Although *constructive solid geometry* (CSG) is one of the oldest techniques for modeling solid objects, it is still found in CAD packages due to its intuitive concept. Complicated solid objects are constructed by combining simple solid primitives using Boolean set operators (typically union, intersection, and difference). While CSG is often applied to model solids, it is rarely found in rendering systems. Rather than directly rendering CSG primitives and composites, such models are usually preprocessed and converted into a surface representation, most often a triangle mesh, as it can be efficiently rendered on current graphics hardware. This is a viable approach for small and mid-size scenes. However, for complex datasets, preprocessing can become prohibitively time consuming, moreover it can result in a huge number of triangles, potentially too many to fit into GPU memory.

An alternative is to directly ray trace complex CSG models (Glassner, 1989). With the public availability of real-time ray tracing frameworks, such as NVIDIA's OptiX (Parker et al., 2010) or Intel's Embree (Wald et al., 2014), it has become feasible to ray trace even huge CAD models at interactive frame rates. Additionally, these frameworks scale to multiple GPUs (NVIDIA Optix) or CPUs (Intel Embree) which is a big advantage compared to rasterization frameworks. Our CSG ray tracing approach has a number of attractive benefits:

On-the-fly Compositing. Boolean set operations are performed on a per-ray basis immediately during rendering. It is therefore not necessary to apply them in a preprocess, which enables the user to change the set operators while inspecting the scene.

Low Storage Requirement. Since no a priori triangulation of primitives is necessary, they can be described by a small set of parameters. For example, for a cylinder only height and radius need to be stored. (Additionally a transformation matrix may be needed, but it is not always required to store a full 4×4 matrix as we will see further below.) This enables holding scenes in memory, which would not fit in a triangulated form (see Figure 1).

Pixel-accurate Higher Order Surfaces. Higher order primitives, such as cylinders or tori, can be directly tested for intersection with a ray by solving the appropriate equations without having to triangulate first. Consequently, no discretization artifacts are visible, and curved surfaces appear perfectly smooth (see Figure 5).

The main contribution of this paper is to present a method to apply CSG operations in real-time during

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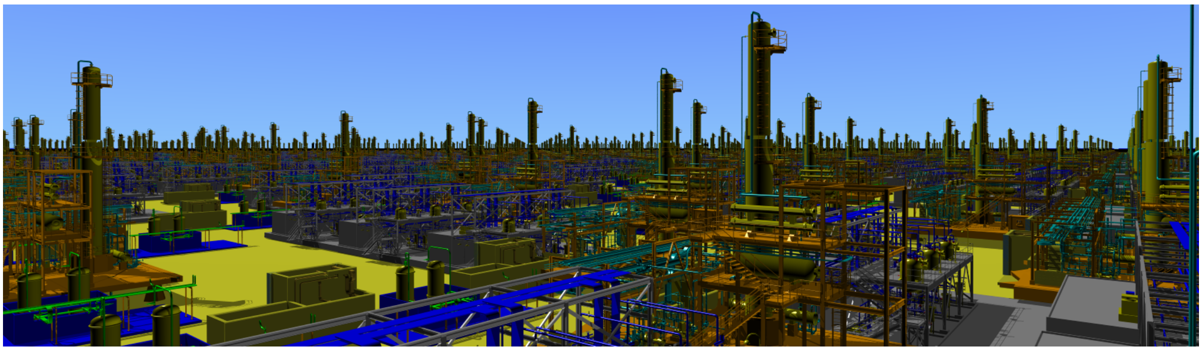


Figure 1: A complex CAD scene built out of 8,000 individual plant models. In total, the scene consists of more than 100,000,000 non-planar second-order (cylinders, cones, etc.) and forth-order (tori) primitives, ray traced at more than 10 frames per second on 16 CPU cores, including pixel-accurate shadows.

ray tracing while keeping the memory overhead low. Additionally, we have evaluated this method in comparison to two well-known rasterization-based CSG operation techniques: (Goldfeather et al., 1986) and (Stewart et al., 2002).

Structure of the Paper. In chapter 2, related work to our approach is described. In chapter 3, our CSG ray tracing approach and its implementation using NVIDIA Optix (Parker et al., 2010) is described. In chapter 4, the proposed ray tracing approach is compared to rasterization-based CSG rendering approaches (Goldfeather (Goldfeather et al., 1986) and SCS (Stewart et al., 2002)). Additionally, we describe our experiences of applying real-time CSG ray tracing to models from two different domains. The first one is a complex IC package of a Cell Broadband Engine chip. The second one is a collection of plant models described in the AVEVA PDMS 3D model format (RVM). Conclusions of the paper are presented in chapter 5.

2 RELATED WORK

An early algorithm for directly ray tracing CSG objects without extracting a surface mesh was presented by Roth (Roth, 1982). It computes ray–object intersection intervals and combines them using Boolean operations. Our approach is similar to this paper. However, we benefit from state-of-the-art ray tracing techniques and utilize today's modern parallel graphics and central processors. Later (Goldfeather et al., 1986) demonstrated how to directly render CSG models using a depth-layering Z-buffer approach. The number of render passes required in this method is of $O(n^2)$ complexity where n is the number of primitives. The required number of draw calls for this method is similarly of subquadratic order. (Wiegand, 1996) implements a GPU accelerated version of this algorithm. (Stewart et al., 2002) introduces an image-based *Sequence Convex Subtraction*(SCS) algorithm utilizing depth and stencil buffer to render CSG models. While this approach only supports convex primitives, it can render the CSG model in a constant number of passes. The drawback of this method is its large number of required draw calls to achieve the correct stencil and depth buffer values which is of subquadratic order w.r.t the number of primitives. (Guha et al., 2003) devised an algorithm which uses depth and stencil buffers to perform depth peeling (Everitt, 2001) which peels off the model layers and perform CSG operations on the fly. For every peeled layer, it tests if the pixels belong to the final result. If a result pixel was found, the pixel is masked for the next peeling steps to reduce the computation load. The number of passes for this method is still dependent on the number of primitives which causes a performance drop when being applied to massive CSG models. (Hable and Rossignac, 2005) combines depth peeling and a linear formulation of Boolean expressions to render CSG models. It depth peels the scene while performing CSG operations from front-to-back. The objects need to be drawn in k stages, where k is the scene's number of depth-layers. As massive and complex scenes are made of a tremendous number of layers (e.g. for doing CSG operations on a PCB board made of many different layers), this method will have a significant performance drop. More recently, Romeiro et al. (Romeiro et al., 2006) used a combination of spatial subdivision and ray tracing methods to render massive CSG models. They tried to construct a simple Octree containing the CSG tree which can be used for rendering the model correctly. As this method ends up with a huge number of draw calls, it can distribute the workload properly on CPU and GPU for small enough models. However, the CPU will limits the GPU rendering for handling massive models. In 2010 Hijazi et al. (Hijazi et al., 2010) interactively ray traced CSG objects defined as implicit functions using interval arithmetic. (Engel, 2014) presents a two stage procedure to render deformable meshes made by CSG operations. In the first stage, the fragments are collected into a big enough constantly allocated buffer, and in the second stage, they are sorted by depth and CSG operations are applied per-pixel. This method requires a huge amount of memory to store the whole fragments in the first stage. The size of buffer can exceed the available GPU memory, e.g. for PCB board models which contain a lot of different levels of primitives overlapping each other.

3 CSG RAY TRACING

When modeling a scene with CSG techniques, simple objects, such as boxes, cylinders, tori, etc., form more complex objects by applying Boolean set operations, where the basic operators are union, difference, and intersection. (Although we will here use solids the concept is also applicable to the combination of 2D objects.) A compound object can be described by means of a binary tree, where the leaves correspond to primitives and the inner nodes to set operations.

3.1 Interval Evaluation

Ray tracing such a structure can be done in a straightforward way. For each ray all the intersection points of the ray with the primitives of the scene are calculated. This leads to a set of intervals that determine the ranges where a ray runs inside or outside of a primitive. These intervals can be grouped in a so-called *Roth diagram* (Roth, 1982). Here, for each primitive that was pierced by the ray, all entry and exit points are recorded in a sorted order. For each two primitives that are to be combined, the intervals in the Roth diagram are merged according to the relevant Boolean operation. The resulting Roth diagram, i.e, the resulting intervals, represent the entry and exit points of the composite object. The entry point with the closest distance to the ray origin represents the eventual hitpoint (i.e., the nearest surface point which is visible).

3.2 Optimized Hitpoint Calculation

Obviously, for scenes with millions of primitives this can result in a high number of intervals. While the amount of memory required to store these intervals may be negligible for a single ray, it can become prohibitive on GPUs (or many-core architectures in general) where hundreds of thousands of rays may be traced in parallel.

In order to avoid having to compute and store all intersection intervals along a ray, we take advantage of the structure of our scenes. Similar to depth peeling, from the camera perspective the CSG model is organized into a number of layers L_i ($i = 1, \ldots, l$). Each layer is composed of a number of positive solids $P_{i,j}$ ($j = 1, \ldots, p_i$) and negative solids $N_{i,k}$ ($k = 1, \ldots, n_i$). We will refer to the negative solids as cutouts in the following. Thus, the CSG operations for a scene S can be described as

$$S = \sum_{i=1}^{l} L_i \quad \text{with} \quad L_i = \sum_{j=1}^{p_i} P_{i,j} - \sum_{k=1}^{n_i} N_{i,k}$$

where $+$ and $-$ denote union and difference set operations, respectively. Basically, a layer can be seen as the difference of two compound (positive and negative) objects. Because of this, CSG ray tracing a single layer can be done by tracking when a ray runs within a positive or negative medium. To this end, we employ two counters (*posDepth* and *negDepth*) that are attached as custom parameters to each ray. Whenever the ray tracer finds the closest intersection of a ray with the primitives of a layer, a hit program is called which is illustrated in algorithm 1.

Algorithm 1: RayTraceLayer(*ray*).

ray: current ray hitting a primitive

if entering primitive **then**
  delta := $+1$;
else
  delta := -1;
end if

if positive primitive hit **then**
  ray.posDepth += delta;
else
  ray.negDepth += delta;
end if

if (*ray.posDepth* > 0) && (*ray.negDepth* $<= 0$) **then**
  ReportHit(); // final hit in layer found
else
  ContinueRay(*ray*); // still inside a negative medium
end if

Each time a ray enters or leaves a positive or negative primitive, the counters are increased or decreased, respectively. In case we are inside a positive medium, but not inside a negative medium, we found the correct hitpoint corresponding to a layer. Otherwise ray traversal continues using the updated counters.

To find the final global hitpoint in the scene, a primary ray is sequentially tested against all layers, and from the set of layer hitpoints the nearest one is accepted as final position for shading. In the scene graph used by the ray tracing engines, the layers of a scene can be stored as independent sub-trees, which can be intersected separately.

3.3 Implementation

The heart of a ray tracer is its acceleration structure. We have used the Bounding Volume Hierarchy (BVH) (MacDonald and Booth, 1990) acceleration structure as it provides better performance for our CSG models than other available acceleration structures. To exploit multi-threaded acceleration structure construction in NVIDIA Optix (Parker et al., 2010), its scenegraph is constructed in correspondence to the binary tree description of the CSG operations. As we described in the last section, the ray definition needs to be extended to store depth of positive and negative primitives intersections. Nvidia Optix (Parker et al., 2010) provides a *rtcDeclare* macro to attach additional information to a ray definition. When a ray–primitive intersection is reported, the shading function is executed. The shader is implemented according to algorithm 1. The primitives supported by our implementation are shown in figure 4.

The ray–primitive intersection test is calculated with 32-bit floating point operations for all primitives robustly except the circular torus where intersection test equation is quartic; while this equation type can be solved in constant time (Abramowitz, 1974), achieving a robust solution demands double-precision calculation. Furthermore, the accumulation buffer (Haeberli and Akeley, 1990) is used to remove artifacts from the final result image when the camera is fixed.

4 RESULTS

Our original motivation for applying CSG ray tracing to CAD data stems from two massive models: A complex electronic circuit that is contained in an IC package of a Cell Broadband Engine chip (Gjonaj et al., 2006), and a collection of plant models. To compare the performance of the CSG ray tracer, we have proposed three test cases to compare the ray tracing approach to well-known rasterization image-based CSG rendering techniques (Goldfeather et al., 1986) and (Stewart et al., 2002). OpenCSG (Kirsch and Döllner, 2005) is a library for image-based CSG rendering which provides optimized implementations of these two methods. OpenCSG 1.4 (which is released September 2014) is used for implementing and evaluating the test cases. For the sake of simplicity, the primitives rendered with OpenCSG are tesselated with fixed number of triangles which result in not pixel-accurate images in most of cases; especially when the primitives are close to the camera.

4.1 Comparison with Rasterization-based Approaches

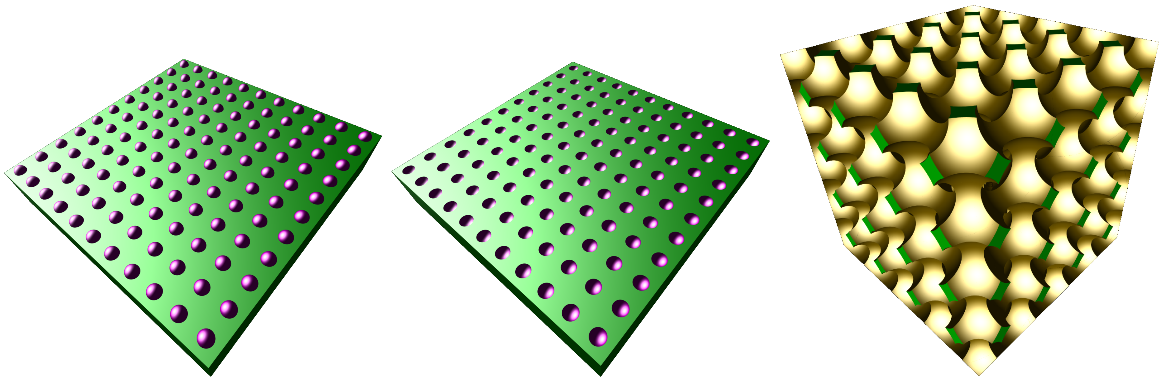
To clarify our approach's benefits, it is necessary to compare it against state-of-the-art real-time CSG rendering algorithms. Goldfeather (Goldfeather et al., 1986) and SCS (Stewart et al., 2002) are two of the well-known CSG rendering methods. OpenCSG as a library for CSG model rendering provides optimized implementations of these two methods. Three different test cases are defined to compare performance of our proposed approach in comparison to (Goldfeather et al., 1986) and SCS (Stewart et al., 2002). The test cases are defined as follows:

- **Unions test model**: Consists of a box with n spheres placed on top of it (Figure 2a).
- **Subtraction test model**: Consists of a box with n spheres placed on top of it which are reduced from the supporting box (Figure 2b).
- **Overlapping subtraction test model**: The model consists of a cubic box from which a cubic grid of spheres is subtracted (Figure 2c).

The test system used for comparison has an Intel Core i5-6400 CPU, one NVIDIA Geforce GTX 960, and 16 GB of memory.

The rasterization-based and ray tracing based evaluation results are shown in figure 3. Figure 3a shows the number of union operations and resulting frames per second for rendering the union test case. It shows that the rasterization-based rendering methods are faster for few number of primitives. However, their performance vanishes with increasing number of union operations. The ray tracing approach's performance is reducing with lower rate in comparison to the rasterization-based approaches w.r.t. the number of CSG operations. Furthermore, It can be seen that the ray tracing approach keeps 40 frames per second while rendering 1 million pixel-accurate spheres.

Figure 3b shows the rendering performance curve (frames per second) of different rendering approaches for models with different number of subtraction operations. Similar to the union test case, for a small num-



(a) model made by union operations (b) model made by subtraction operations (c) model made by overlapping subtraction operations

Figure 2: Evaluation of CSG models. Three CSG models were designed to compare the proposed CSG ray tracing approach against (Goldfeather et al., 1986) and (Stewart et al., 2002) rasterization-based CSG rendering algorithms. The first model (left figure) is the union of a number of spheres and a supporting box underneath. The second model (middle figure) is similar to the first model but the spheres are subtracted from the supporting box. The last model (right figure) is constructed by subtracting a 3d grid of spheres from a box. These cases are used to evaluate every aspect of CSG operations. The first one is used to evaluate rendering of unions of primitives, the second one evaluates rendering of subtracted primitives, and the last one is used to evaluate rendering of overlapping subtracted primitives. The evaluation results are shown in figure 3.

ber of subtractions the performance of rasterization-based rendering approaches is higher than ray tracing. Similarly it is abruptly dropping by increasing number of subtraction operations. A box with 200,000 holes inside can be ray traced approximately 20 times per second.

Figure 3c shows the rendering performance for the model made by subtracting different number of overlapping holes. Since a lot of overlapping primitives exist in this case, enabling hardware occlusion query (Bittner et al., 2004) can have a significant impact on the Goldfeather and SCS rendering modes because the primitives which do not contribute to the final frame buffer will be removed from CSG rendering draw calls. Our measurements show that the occlusion query improves the performance by factor of 2-10 for Goldfeather and 2-30 for SCS.

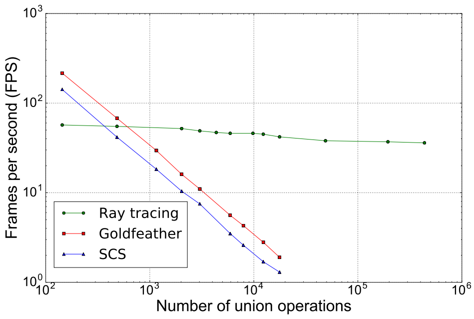
In this case ray tracing performance curve is much lower than rasterization-based methods. On our test machine ray tracing performance outperforms the rasterization-based methods when more than 5 thousand CSG operations are performed. Although number of frames per second is still larger than for rasterization-based methods beyond 5k operations, the absolute performance is not interactive anymore (below 1 frames per second). The ray tracing performance drops because a lot of subtracted primitives are overlapping, the rays stop at every ray–primitive intersection and a new ray is shot against the next subtracted primitive. The acceleration structure has to be traversed from root for every new ray shot which will decreases the performance significantly.

4.2 Electric Circuit Model

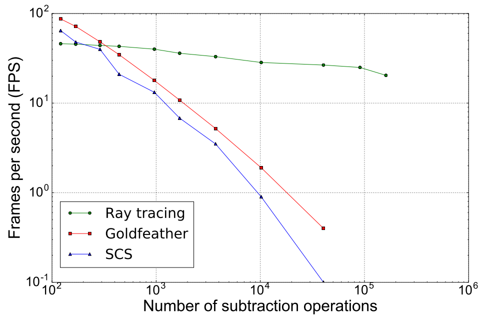
A more practically relevant example, is an electric circuit model consists of a number of stacked circuit layers that are interconnected. In order to describe the circuits, the model makes use of three different primitives: extruded polygons, oriented boxes, and cylinders (no triangles were used). Boxes are typically used for conductive circuit paths, while cylinders model so-called vias, which connect circuit layers. Essentially, each model layer is an extruded 2D design. This allows for storing the geometry data in a very efficient way. For example, cylinders are always oriented perpendicular to the layers, and only require a center, vertical extrusion and radius as parameters. Likewise, an extruded polygon is described by a set of 2D points, plus its vertical position and extrusion length. The extrusion vector is always perpendicular to the 2D area.

Figure 5 shows two layers of the circuit model. It can be seen how circular cutouts cut holes into an extruded polygon. Conductive traces are formed with boxes and cylinders. Since for all primitives the intersection with a ray is directly calculated, all curved surfaces are pixel-accurately displayed without the need to use explicitly defined triangles.

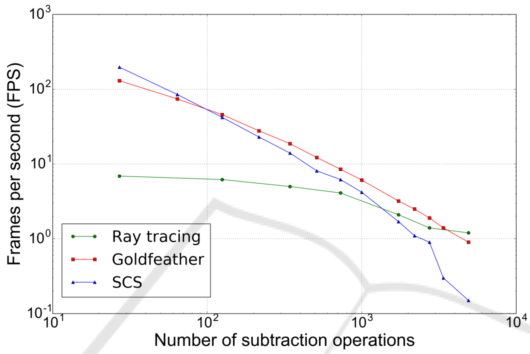
For this scene we implemented all functions for intersecting, shading, and traversal as custom programs in NVIDIA's OptiX (Parker et al., 2010) framework. The test scene shown in Figure 5 consists of more than 88,000 primitives. On an NVIDIA GeForce GTX 980 it can be rendered at more than



(a) Evaluation of union CSG operations



(b) Evaluation of subtraction CSG operations



(c) Evaluation of overlapping subtraction CSG operations

Figure 3: Evaluation of CSG model rendering using different algorithms. The proposed ray tracing approach, and two other rasterization-based CSG rendering methods (Goldfeather (Goldfeather et al., 1986) and SCS (Stewart et al., 2002)) are compared for rendering the models shown in Figure 2. Figure 3a evaluates the performance of rendering unions of primitives. The performance of the rasterization-based rendering methods reduced significantly with increasing number of CSG operations while the ray tracing approach performance is reducing slowly. The evaluation shows that the ray tracing approach is well-fit to massive model rendering. Figure 3b evaluates the performance of primitive subtraction rendering. It similarly shows that the ray tracing approach has significantly better performance for rendering a large number of subtracting primitives. Figure 3c diagram evaluates rendering of CSG models having overlapping subtraction primitives. Calculating pixel color covering a lot of overlapping subtracting primitives result in a lot of ray–primitive intersections which requires traversing the ray tracer's acceleration structure multiple times per pixel. Therefore the performance of the ray tracer drops significantly while the impact on the stencil-buffer approaches is more moderate.

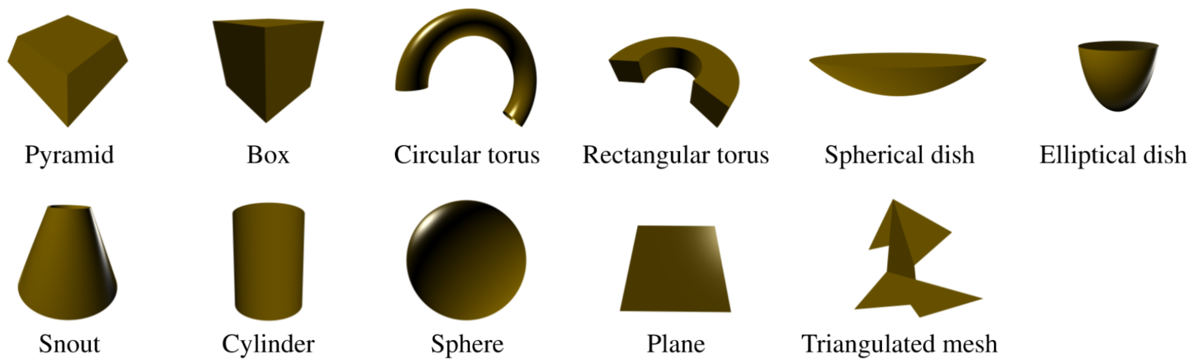


Figure 4: RVM primitives used in the plant model.

50 frames per second at full HD resolution.

4.3 Plant Model

This model (see Figure 1) was exported from the AVEVA PDMS 3D CAD software, which is tailored

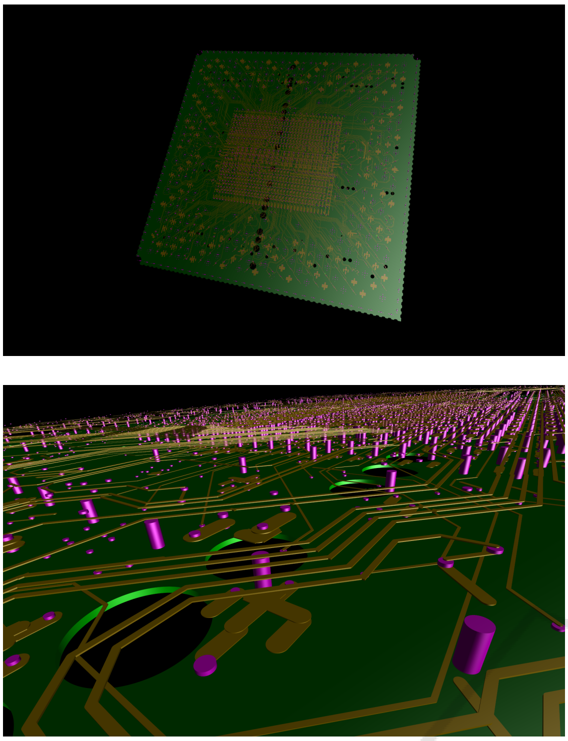


Figure 5: Two views of a Cell IC package. Top: Overview of the layers. Bottom: Closeup view showing some cutouts. Note the pixel-exact curved surfaces.

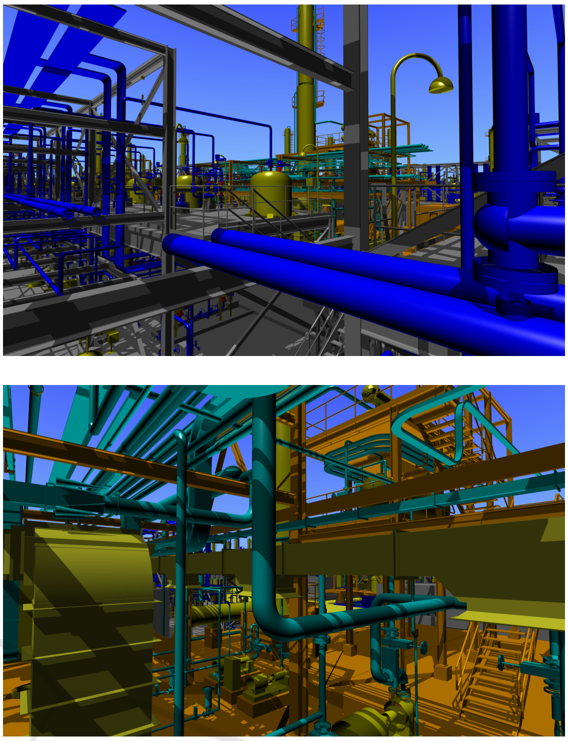


Figure 6: Close up view of the plant model. The intricate structure is modeled with the primitives described in the AVEVA RVM CAD format.

to the design of plant projects. The resulting data is stored in RVM format (AVEVA, 2001). RVM describes geometry as a tree of named groups containing 11 types of primitives (including solids and planar primitives). Figure 4 shows the different RVM primitives used to describe the plant model. In this version of the model only positive primitives are included, therefore only union operations need to be performed, i.e., simply the closest ray intersection of all primitives has to be determined.

The provided CAD data contains four different plant datasets of varying size. As a stress test we replicated them randomly 8,000 times. This resulted in a scene consisting of more than 100,000,000 non-planar primitives. Including acceleration structures the whole scene amounts to more than 50 GB in size. As this amount of data is too big to fit into GPU memory, we ported our routines over to Intel's Embree (Wald et al., 2014) CPU ray tracing framework. On a PC fitted with 64 GB of main memory and two Intel Xeon E5-2650 v2 eight-core CPUs (clocked at 2.60 GHz) the complete scene can be visualized at 10–20 frames per second including shadows.

Figure 6 presents two views illustrating the high amount of detail found in the scene. Notice the intricate structure of the pipes and tubes. Again, curved surfaces appear pixel-exact.

5 CONCLUSIONS

In this paper, we demonstrated the application of real-time ray tracing of massive CSG models made of (higher-order) primitives using available state-of-the-art ray tracing engines. This approach is of high practical relevance since in many scientific and engineering disciplines CAD models are designed which consist of simple (solid or planar) primitives that can be directly displayed by a ray tracer without having to resort to triangulation. Apart from a straightforward implementation, real-time CSG ray tracing has the advantage of on-the-fly compositing (without any pre-processing), low storage requirements, and pixel-accurate curved surfaces. Our results show that our approach which is benefiting from today's advanced rendering techniques has higher performance for rendering massive CSG model compared to well-known approaches e.g. Goldfeather (Goldfeather et al., 1986) and SCS (Stewart et al., 2002). Additionally, we applied our approach to two practical models: PCB board, and factory which are made by CSG operations. We showed that 100 Mio of non-planar higher order primitives (second-order and forth-order) can be ray traced on 16 cores CPU at 10 frames per second.

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