

Spontaneous Emission of a Dressed Atomic System in a Strong Light Field

A. V. Bogatskaya and A. M. Popov

D. V. Skobel'syn Institute of Nuclear Physics, Moscow State University, Moscow, 119991, Russia

Keywords: Quantum Description of Interaction of Light and Matter, Photoionization of Atoms, Rydberg States, Interference Stabilization.

Abstract: New approach to the study of the spontaneous emission of an atomic system driven by a strong light field is developed. This approach is based on the accurate consideration of quantum system interaction with vacuum quantized field modes in the first order of perturbation theory, while the strong light field is considered classically. The proposed approach is applied to study the dynamics of field-driven atomic systems. Among them are Rabi oscillations in two-level system, resonant and nonresonant Raman and Rayleigh scattering, interference stabilization of Rydberg atoms. It is demonstrated that analyzing the spontaneous emission allows to study the specific features of quantum systems dressed by the field.

1 INTRODUCTION

Study of the nonperturbative atomic dynamics in strong laser fields is the core problem of nonlinear optics (Akhmanov and Nikitin, 1997). Typically this dynamics is studied in the semiclassical approach (Agostini and Di Mauro, 2004; Couairon and Mysyrowicz, 2007; Krausz and Ivanov, 2009; Chin, 2010). It means that while the atomic system is analyzed from quantum-mechanical point of view the electromagnetic field is considered still classically. In (Bogatskaya et al, 2016) the possibility to use semiclassical approach for analyzing radiative processes in high intensity fields was questioned. It was demonstrated that the application of the semiclassical approach to study emission of the quantum system driven by high intensity laser field is generally in contradiction with quantum electrodynamical calculations. Also it means that spontaneous emission from the quantum system is neglected. If laser field is strong enough the probability of spontaneous transitions is negligibly small in comparison with stimulated transitions. Nevertheless, any stimulated emission starts from a spontaneous background radiation. It means that in order to study initial stage of any nonlinear process one need to take into account spontaneous processes as well. On the other hand it is known atomic spectrum can be dramatically reconstructed by the

strong external laser field (Delone and Krainov, 1994; Fedorov, 1997). New quantum object with essentially different spectrum than the field-free atomic spectrum, (so called the dressed atom), appears to exist. The simplest example of such reconstruction is the AC Stark shift of energy levels in relatively weak laser field. Another example of the dressed atom is the so called Kramers - Henneberger atom that appears to exist in superatomic fields (Fedorov, 1997). The spontaneous emission from this dressed by the laser field atomic system can provide the unique information about its energy spectrum. To study the structure of dressed atom one needs to take into account the interaction with the electromagnetic vacuum. The interaction with vacuum modes also should be taken into account for analyzing a lot of nonlinear processes. A lot of practical applications of theory including interaction of the atomic system with both classical and vacuum field modes can be found in quantum optics (Scally and Zubairy, 1997).

The aim of this paper is to develop the approach for studying first-order spontaneous radiative processes in a quantum system driven by a strong classical laser field. This approach is based on the first order perturbation theory applied to the interaction of the atomic system dressed by the strong laser field with a lot of quantized field modes in the assumption that initially all the modes are in a vacuum state. The proposed approach is applied for

study of a number of quantum systems such as quantum dots, quantum wires, clusters in the presence of the intense laser field.

2 ATOM DYNAMICS IN A STRONG LASER FIELD IN THE PRESENCE OF QUANTIZED ELECTROMAGNETIC FIELD

We analyse the atomic system using the following Hamiltonian

$$H = H_0(\vec{r}, t) + H_f(\varepsilon) + V(\vec{r}, \varepsilon), \quad (1)$$

where $H_0 = H_{at}(\vec{r}) + W(\vec{r}, t)$; $H_{at}(\vec{r})$ is the atomic Hamiltonian, $W = -\vec{d} \cdot \vec{E}(t)$ represents the interaction of atom with classical laser field in the dipole approximation, H_f - Hamiltonian of the set of field modes excluding laser field mode, $V(\vec{r}, \varepsilon)$ stands for the interaction of the atom with the quantized electromagnetic field, $\vec{d} = e\vec{r}$ is the dipole moment, \vec{r} is the electron radius-vector and ε is the set of quantized field mode coordinates.

We are going to deal with the quantized field using perturbation theory. In the case when there is no interaction with quantized field modes one can write the well-known equation

$$i\hbar \frac{\partial \psi(\vec{r}, t)}{\partial t} = H_0(t)\psi(\vec{r}, t), \quad (2)$$

which describes the atomic dynamics in a classical laser field. Initial condition can be written as

$$\psi(\vec{r}, t=0) = \phi(\vec{r}), \quad (3)$$

where $\phi(\vec{r})$ is some stationary or unstationary state of the atomic discrete spectrum or continuum. We will suppose also that at the initial instant of time all field modes are in the vacuum state $|0\rangle$. Provided that we know the solution of equation (2), the solution of the general equation with the Hamiltonian (1)

$$i\hbar \frac{\partial \Psi(\vec{r}, \varepsilon, t)}{\partial t} = (H_0(t) + H_f + V)\Psi(\vec{r}, \varepsilon, t), \quad (4)$$

and initial condition $\Psi(\vec{r}, \varepsilon, t=0) = \phi(\vec{r}) \times |0\rangle$ can be found by means of the perturbation theory.

Wave function of zero-order approximation

excluding interaction with the quantum field modes reads

$$\Psi^{(0)}(\vec{r}, \varepsilon, t) = \psi(\vec{r}, t) \times |0\rangle, \quad (5)$$

We are going to find the solution of (4) in the form:

$$\Psi(\vec{r}, \varepsilon, t) = \Psi^{(0)}(\vec{r}, \varepsilon, t) + \delta\Psi(\vec{r}, \varepsilon, t), \quad (6)$$

with $\delta\Psi \ll \Psi^{(0)}$.

Substituting (6) in (4) in the first order of smallness one obtains:

$$i\hbar \frac{\partial \delta\Psi(\vec{r}, \varepsilon, t)}{\partial t} = (H_0(t) + H_f)\delta\Psi(\vec{r}, \varepsilon, t) + V\Psi^{(0)}(\vec{r}, \varepsilon, t) \quad (7)$$

with the initial condition $\delta\Psi(\vec{r}, \varepsilon, t=0) = 0$.

In fact (7) can be formulated as Schrodinger equation for the $\delta\Psi$ with the source in the right hand. For further analysis of eq. (7) let us remind that initially we have vacuum in all field modes. Therefore in the first order of perturbation theory $\delta\Psi$ contains only one-photon excitations in a some field mode:

$$\delta\Psi(\vec{r}, \varepsilon, t) = \sum_{k, \lambda} \delta\psi_{k\lambda}(\vec{r}, t) \times \{0, 0, \dots, 1_{k\lambda}, 0, \dots, 0, 0\}, \quad (8)$$

Here $\delta\psi_{k\lambda}(\vec{r}, t)$ is the electron wave function provided that one photon with wave vector \vec{k} and polarization λ has appeared.

As the interaction of the atom with quantized field can be written in a form

$$V(\vec{r}, \varepsilon) = \sum_{k, \lambda} V_{k\lambda} = - \sum_{k\lambda} (\vec{d} \vec{e}_{k\lambda}) \varepsilon_{k\lambda}, \quad (9)$$

($\varepsilon_{k\lambda}$ is the field operator of mode $\{k, \lambda\}$ and $\vec{e}_{k\lambda}$ is the polarization vector) for a given mode with one-photon excitation one can write:

$$i\hbar \frac{\partial \delta\psi_{k\lambda}(\vec{r}, t)}{\partial t} |k, \lambda\rangle + \delta\psi_{k\lambda}(\vec{r}, t) \times (3/2)\hbar\omega_{k\lambda} |k, \lambda\rangle = (H_0(t) + h_{k\lambda}^{(f)})\delta\psi_{k\lambda}(\vec{r}, t) |k, \lambda\rangle + V_{k\lambda}\psi(\vec{r}, t) |0\rangle. \quad (10)$$

Here $h_{k\lambda}^{(f)}$ is the Hamiltonian of the field mode $|k, \lambda\rangle$. Provided that $h_{k\lambda}^{(f)} |k, \lambda\rangle = 3/2 \cdot \hbar\omega_{k\lambda} |k, \lambda\rangle$, and $\varepsilon_{k\lambda} |0\rangle = \frac{\varepsilon_{norm}}{\sqrt{2}} |k, \lambda\rangle$, ($\varepsilon_{norm} = \sqrt{4\pi\hbar\omega_{k\lambda}/L^3}$ is the auxiliary normalizing constant, L^3 is the normalization volume), the final form of the eq. (10) can be written as

$$i\hbar \frac{\partial \delta\psi_{k\lambda}(\vec{r}, t)}{\partial t} = H_0(t) \delta\psi_{k\lambda}(\vec{r}, t) - (\vec{d}\vec{e}_{k\lambda}) \times \frac{\mathcal{E}_{norm}}{\sqrt{2}} \times \psi(\vec{r}, t) \times \exp(i\omega_{k\lambda}t) \quad (11)$$

with the initial condition $\delta\psi_{k\lambda}(\vec{r}, t=0) = 0$.

It is obvious that the expression

$$W_{k\lambda}(t) = \int |\delta\psi_{k\lambda}(r, t)|^2 d^3r \quad (12)$$

represents the probability to find a photon in the mode $|k, \lambda\rangle$ as a function of time. Then the total probability to emit the photon of any frequency and polarization during the transition $f \rightarrow i$ is

$$W_{fi}(t) = \sum_{k, \lambda} W_{k\lambda}(t) \quad (13)$$

As the spectrum of field modes is dense, we can replace the sum in eq. 13 by the integral:

$$2L^3 \int \frac{d^3k}{(2\pi)^3} = \frac{2L^3}{8\pi^3 c^3} \int \omega_{k\lambda}^2 d\omega_{k\lambda} \int d\Omega \quad (14)$$

After the integration over angular distribution of photons and summation over possible polarizations the probability of the spontaneous decay in the spectral interval $(\omega, \omega + d\omega)$ can be expressed in the form

$$W_{\omega} d\omega = \frac{L^3}{3\pi^2 c^3} \omega^2 d\omega \times W_{k=\omega/c, \lambda} \quad (15)$$

where $W_{k, \lambda}$ is given by (12). One should note, that the expression (14) does not depend on the normalization volume, as $W_{k, \lambda} \sim 1/L^3$.

3 RABI OSCILLATIONS IN A TWO-LEVEL SYSTEM AND TRIPLET MOLLOW

Let us restrict ourselves to the consideration of two-level system (energy levels and stationary state wave functions E_1 and φ_i , $i=1,2$ respectively) interacting with near resonant field of frequency $\omega \approx \omega_{21} = (E_2 - E_1)/\hbar$ and initially ($t=0$) being in the state $|1\rangle$.

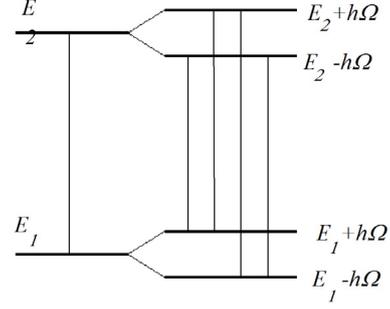


Figure 1: Dressing of two-level system in a resonant electromagnetic field.

In the case of exact resonance $\Delta\omega = \omega_{21} - \omega \equiv 0$ wave function of the system governed by the classical field with electric field $\mathcal{E} = \mathcal{E}_0 \cos \omega t$ can be represented in a form

$$\psi(\vec{r}, t) = \sum_{n=1,2} C_n(t) \varphi_n(\vec{r}) \exp(-\frac{i}{\hbar} E_n t) \quad (16)$$

with $C_1 = \cos \Omega t$ and $C_2 = -i \sin \Omega t$, where $\Omega = d_{21} \mathcal{E}_0 / 2\hbar$ is the Rabi frequency. One can easily see that in our case dressing means splitting of each level into two quasienergy states with energies $E_i \rightarrow E_i \pm \hbar\Omega$. The structure of this splitting for the case $\Omega \ll \omega_{21}$ is performed at fig.1. It means that the line of spontaneous emission corresponding to the transition $\omega_{k\lambda} = \omega = (E_2 - E_1)/\hbar$ should split up into three lines $\omega_{k\lambda} = \omega, \omega \pm 2\Omega$, so-called triplet Mollow (Mollow, 1969). To confirm this statement we will find the solution of general equation (11) in a form

$$\delta\psi_{k\lambda}(\vec{r}, t) = \sum_{n=1,2} C_n^{(k\lambda)}(t) \varphi_n(\vec{r}) \exp(-\frac{i}{\hbar} E_n t) \quad (17)$$

Then the equations for amplitudes $C_n^{(k\lambda)}$ reads

$$\begin{aligned} i\dot{C}_1^{(k\lambda)} &= -\Omega C_2^{(k\lambda)} - \Omega^* \times \\ &(\exp(i(\omega_{k\lambda} - \omega_{21} - \Omega)t) + \exp(i(\omega_{k\lambda} - \omega_{21} + \Omega)t)), \\ i\dot{C}_2^{(k\lambda)} &= -\Omega C_1^{(k\lambda)} - \Omega^* \times \\ &(-\exp(i(\omega_{k\lambda} + \omega_{21} - \Omega)t) + \exp(i(\omega_{k\lambda} + \omega_{21} + \Omega)t)). \end{aligned} \quad (18)$$

Here $\Omega^* = \frac{d_{21} \mathcal{E}_{norm}}{\hbar 2\sqrt{2}}$ is the coupling constant of the

quantum system with quantized field. First terms in (18) mean the transitions between two atomic states of the system under the resonant laser field while second terms stand for the emission of photons

$\{k, \lambda\}$. General analytical solution of eq. (18) with initial conditions $C_{1,2}^{(k\lambda)}(t=0) = 0$ we can write as

$$\begin{aligned} |C_1^{(k\lambda)}(t)|^2 &= (\Omega^*)^2 \times \left[\frac{\sin^2(\omega_{k\lambda} - \omega_{21})t/2}{(\omega_{k\lambda} - \omega_{21})^2/4} 4\cos^2 \Omega t \right. \\ &\quad \left. + \frac{\sin^2(\omega_{k\lambda} - \omega_{21} \pm 2\Omega)t/2}{(\omega_{k\lambda} - \omega_{21} \pm 2\Omega)^2/4} \right] \\ |C_2^{(k\lambda)}(t)|^2 &= (\Omega^*)^2 \times \left[\frac{\sin^2(\omega_{k\lambda} - \omega_{21})t/2}{(\omega_{k\lambda} - \omega_{21})^2/4} 4\sin^2 \Omega t \right. \\ &\quad \left. + \frac{\sin^2(\omega_{k\lambda} - \omega_{21} \pm 2\Omega)t/2}{(\omega_{k\lambda} - \omega_{21} \pm 2\Omega)^2/4} \right] \end{aligned} \quad (19)$$

It represents emission of photons near the frequencies $\omega_{k\lambda} = \omega, \omega \pm 2\Omega$ and subsequent Rabi oscillations of the atomic population probabilities. The probability to emit photon $\{k, \lambda\}$ is $W_{k\lambda}(t) = |C_1^{(k\lambda)}|^2 + |C_2^{(k\lambda)}|^2$ and is given at fig.2 for two different laser pulse durations. As we supposed earlier, the initial line splits into three lines that are known as triplet Mollow. The intensity of the central line is twice larger in comparison with satellites at frequencies $\omega_{k\lambda} = \omega \pm 2\Omega$. As the duration of the pulse increases the splitting of the initial line into the triplet Mollow becomes more and more pronounced (see solid and dashed curves at fig.2).

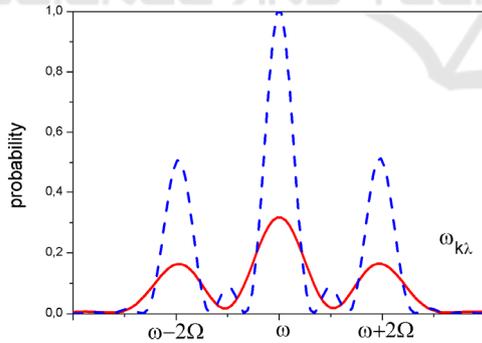


Figure 2: Triplet Mollow for short (solid) and long (dash) laser pulses. Pulse durations are 10^4 and $2 \cdot 10^4$ at.un.

4 NONRESONANT SPONTANEOUS RAMAN AND RAYLEIGH SCATTERING

In this chapter we will study Raman and Rayleigh scattering of laser radiation by atomic system. To obtain the general expression for probabilities of

Raman and Rayleigh scattering let us consider the atomic dynamics in classical laser field also in the first order of perturbation theory. Then

$$\begin{aligned} \psi(\vec{r}, t) &= \varphi_i(\vec{r}) \exp\left(-\frac{i}{\hbar} E_i t\right) \\ &+ \sum_{n \neq i} C_n^{(1)}(t) \varphi_n(\vec{r}) \exp\left(-\frac{i}{\hbar} E_n t\right), \end{aligned} \quad (20)$$

where

$$C_n^{(1)}(t) = \frac{d_{ni} E_0}{2\hbar} \left(\frac{\exp(i(\omega_{ni} - \omega)t)}{(\omega_{ni} - \omega)} + \frac{\exp(i(\omega_{ni} + \omega)t)}{(\omega_{ni} + \omega)} \right) \quad (21)$$

Substituting (20) and (21) into (11) and assuming that $|i\rangle$ is the initial atomic state one derives the equation for probability amplitude to find the atom in $|f\rangle$ and the photon in the mode $\{k, \lambda\}$:

$$\begin{aligned} i\hbar \dot{C}_f^{(k\lambda)} &= - \sum_n C_n^{(k\lambda)} (\vec{d}_{fn} \vec{E}(t)) \exp(i\omega_{fn} t) \\ &- \frac{\epsilon_{norm}}{\sqrt{2}} \sum_{n \neq i} C_n^{(1)} (\vec{e}_{k\lambda} \vec{d}_{fn}) \exp(i(\omega_{fn} + \omega_{k\lambda})t). \end{aligned} \quad (22)$$

Second term in the right part of (22) stands for the emission of photon $\{k, \lambda\}$ while the first one describes the evolution of atomic wave function in the classical field after the emission of photon and here we will neglect such evolution. From (22) one derives:

$$\begin{aligned} |C_f^{(k\lambda)}(t \rightarrow \infty)|^2 &= \epsilon_{norm}^2 \left| \sum_{n \neq i} \frac{(\vec{d}_{fn} \vec{e}_{k\lambda})(\vec{d}_{ni} \vec{e}_0)}{2\sqrt{2}\hbar} \frac{1}{\omega_{ni} - \omega} \right|^2 \\ &\times 2\pi\delta(E_f - E_i - \hbar\omega + \hbar\omega_{k\lambda}) \times t. \end{aligned} \quad (23)$$

This is the probability of Stokes component of the spontaneous Raman scattering corresponding to the final state $|f\rangle$ and emission of a photon

$$\hbar\omega_{k\lambda} = \hbar\omega - (E_f - E_i) \quad (24)$$

If the final state coincides with the initial one $|f\rangle = |i\rangle$ we derive the expression for the Rayleigh scattering

$$\begin{aligned} |C_i^{(k\lambda)}(t \rightarrow \infty)|^2 &= \epsilon_{norm}^2 \left| \sum_{n \neq i} \frac{(\vec{d}_{in} \vec{e}_{k\lambda})(\vec{d}_{ni} \vec{e}_0)}{2\sqrt{2}\hbar} \frac{1}{\omega_{ni} - \omega} \right|^2 \\ &\times 2\pi\delta(\hbar\omega_{k\lambda} - \hbar\omega) \times t, \end{aligned} \quad (25)$$

when the frequency of spontaneous photon is the same as for laser radiation. Not far from resonance when laser frequency is $\omega \approx \omega_{ni}$ with definite value

of n , the transition takes place through the only intermediate state and summation over all intermediate states in expressions (23) and (25) should be omitted:

$$\left| C_{f(i)}^{(k\lambda)}(t \rightarrow \infty) \right|^2 = \varepsilon_{norm}^2 \left| \frac{(\vec{d}_{f(i)n} \vec{e}_{k\lambda})(\vec{d}_m \vec{e}_0)}{2\sqrt{2}\hbar} \frac{1}{\omega_{ni} - \omega} \right|^2 \times 2\pi\delta(E_{f(i)} - E_i - \hbar\omega + \hbar\omega_{k\lambda}) \times t. \quad (26)$$

5 FOUR-LEVEL SYSTEM DYNAMICS IN A PRESENCE OF QUANTIZED ELECTROMAGNETIC FIELD

To provide more insight into atomic dynamics in discrete spectrum we will study the spontaneous emission in four-level system $|0\rangle, |1\rangle, |2\rangle, |3\rangle$ with even parities for $|0\rangle, |2\rangle$ and odd ones for $|1\rangle, |3\rangle$. The wave function of the system

$$\psi(\vec{r}, t) = \sum_{n=0}^3 C_n(t) \varphi_n(\vec{r}) \exp\left(-\frac{i}{\hbar} E_n t\right) \quad (27)$$

is obtained from the set of equations

$$i\hbar \dot{C}_f = - \sum_n C_n (d_{fn} \varepsilon_0) \cos \omega t \exp(i\omega_{fn} t) \quad (28)$$

with initial condition $C_0(t=0) = 1, C_{i=1,2,3}(t=0) = 0$. Then the equation (11) is also equivalent to a set of equations for amplitudes $C_f^{(k\lambda)}(t)$ for different $\{k, \lambda\}$ modes:

$$i\hbar \dot{C}_f^{(k\lambda)} = - \sum_n C_n^{(k\lambda)} (d_{fn} \varepsilon_0) \cos \omega t \exp(i\omega_{fn} t) - (\vec{d}_{e_{k\lambda}}) \frac{\varepsilon_{norm}}{\sqrt{2}} \sum_{n=0}^3 C_n(t) \varphi_n(\vec{r}) \exp\left(-\frac{i}{\hbar} E_n t\right) \exp(i\omega_{k\lambda} t). \quad (29)$$

The positions of energy levels were chosen as follows $\omega_{10} = 0.25, \omega_{21} = 0.2, \omega_{32} = 0.1$ and hence $\omega_{30} = 0.55$. Hereafter all values will be given in atomic units. Nonzero values of the dipole matrix elements were chosen equal to each other $d_{10} = d_{30} = d_{32} = d_{21} = 1$. The duration of laser pulse was $\tau = 4 \times 10^4$, laser intensity was 10^{-3} , that means the Rabi frequency is $\Omega = 5 \times 10^{-4}$. The spectrum of spontaneous emission for various spectral intervals

and different detuning of the laser frequency from the frequency of transition $|0\rangle \rightarrow |3\rangle$ ($\omega_{30} = 0.55$) are presented at fig.3. First we should mention that in the laser frequency range $\omega - \omega_{30} > 0.005$ for definite laser pulse parameters both Raman and Rayleigh processes $|0\rangle \rightarrow |3\rangle \rightarrow |2\rangle$ and $|0\rangle \rightarrow |3\rangle \rightarrow |0\rangle$ correspondingly have nonresonant character and can be studied using the expression (23). Near the resonance the situation is changing dramatically due to dressing of the upper $|3\rangle$ and ground $|0\rangle$ levels by external laser field. As far as levels $|0\rangle$ and $|3\rangle$ split into two sublevels the lines of Raman also splits into two lines (fig.3a) while the Rayleigh scattering line is divided into three lines, that corresponds to the Mollow triplet. In the case of exact resonance the value of this splitting is equal to 2Ω (curves 1). For the near-resonance case $\delta = \omega - \omega_{30} = 0.001$ the splitting of states becomes asymmetrical and the position of doublet Raman and triplet Rayleigh lines changes (curves 2). Further increment of the detuning (curves 3) results in the formation the ordinary Raman and Rayleigh lines with frequencies $\omega - \omega_{30}$ and ω respectively.

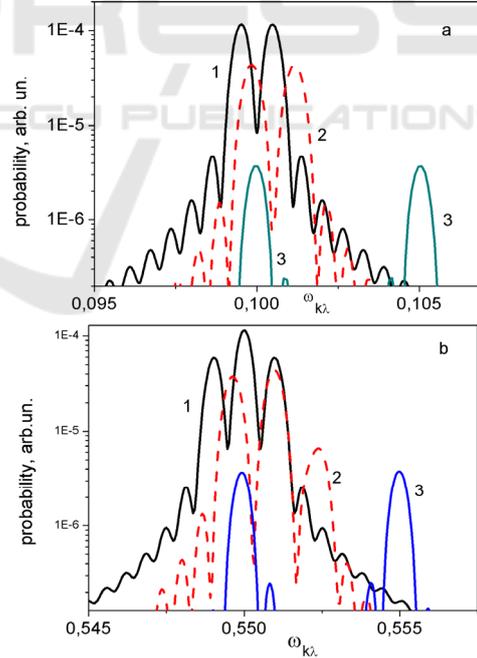


Figure 3: Spectral lines of Raman (a) and Rayleigh (b) processes in dependence on laser frequency detuning ($\delta = \omega - \omega_{30}$) from the atomic transition frequency (1 - $\delta = 0$, 2 - $\delta = 0.001$, 3 - $\delta = 0.005$). Pulse duration is 40000, Rabi frequency $\Omega = 5 \times 10^{-4}$.

Rest lines with frequency ω_{32} and ω_{30} can be interpreted as spontaneous transitions $|3\rangle \rightarrow |2\rangle$ and $|3\rangle \rightarrow |0\rangle$ resulting from the nonresonant excitation of level $|3\rangle$ by laser radiation.

6 STABILIZATION IN A PRESENCE OF QUANTIZED ELECTROMAGNETIC FIELD

The interference stabilization (IS) of Rydberg atoms was first predicted in (Fedorov and Movsesian, 1988), analyzed in detail in subsequent works (Fedorov et al, 1996; Fedorov and Tikhonova, 1998), as well as in (Fedorov, 1997). According to (Fedorov, 1988), IS occurs due to destructive interference of the amplitudes of transitions to a continuum from excited Rydberg states coherently repopulated by Raman Λ -type transitions during laser excitation. It is known that the widths of these transitions are determined by Fermi's golden rule

$$\Gamma = 2\pi \left| \bar{d}_{nE} \bar{\epsilon}_0 / 2 \right|^2 \quad (30)$$

Here, \bar{d}_{nE} is the matrix element of the dipole moment operator for the $|E_n\rangle \rightarrow |E = E_n + \hbar\omega\rangle$ transition and ϵ_0 is the electromagnetic field strength amplitude of the wave. According to (Fedorov, 1997), the IS threshold is determined by the overlap of ionization widths of Rydberg states and can be written in the atomic system of units in the form:

$$\epsilon_0 / \omega^{5/3} > 1 \quad (31)$$

or by passing to intensities: $I > I^* \approx \omega^{10/3}$. For example, for the emission frequency of a Ti:Sa laser, we obtain from (31) the threshold intensity $I^* \approx 2.5 \times 10^{12}$ W/cm².

The general expression for the quasi-energy spectrum of a field-dressed atom in the simplest model of two close nondegenerate levels and a nondegenerate continuum (1D) was obtained in (Fedorov, 1988):

$$\gamma_{\pm} = \frac{1}{2} \left(E_1 + E_2 - i\Gamma \pm \sqrt{(E_2 - E_1)^2 - \Gamma^2} \right) \quad (32)$$

Here, $E_{1,2}$ are unperturbed energy levels and Γ is the ionization width calculated from (30) and assumed

the same for both levels. The imaginary part of (32) determines the ionization width of shifted levels. A structure of broadened quasienergy levels is seen to change drastically at $\Gamma = E_2 - E_1$, which confirms the threshold character of the IS. In the region $\Gamma > E_2 - E_1$ the shift results in a form of a narrowing quasienergy level located approximately at $(E_1 + E_2)/2$ embedded into a widening one. In terms of the time evolution of populations this means that the decay (ionization) has a two-exponential character. I.e., there are short- and long-living parts of populations. Existence of the latter is a clear manifestation of stabilization.

It follows from (32) that in the strong-field limit $\Gamma > E_2 - E_1$ the decay rate of the long living stabilized part of population decrease with increasing intensity as:

$$\Gamma_{strong} = -2 \text{Im} \gamma_+ = \frac{(E_2 - E_1)^2}{2\Gamma} \sim 1/I \quad (33)$$

In this chapter we are going to embed spontaneous radiation in the stabilization phenomenon. We employ with three-level system where there is a ground state $|0\rangle$ and levels $|1\rangle, |2\rangle$ which are supposed to be Rydberg levels of the opposite parity with respect to the ground state. The energies of levels are chosen $E_0 = -0.5$ for $|0\rangle$, $E_1 = -5 \times 10^{-3}$ and $E_2 = -4 \times 10^{-3}$ for levels $|1\rangle, |2\rangle$ respectively (we are working within the atomic system of units).

In frames of previous consideration we solve numerically the following system of equations:

$$\begin{aligned} i\dot{C}_n &= -\frac{i}{2\hbar} \sum_{n'=1}^2 \Gamma_{nn'} C_{n'} \exp(i\omega_{nn'} t), \\ i\dot{C}_0 &= 0, \\ i\dot{C}_n^{(k\lambda)} &= -(\bar{d}_{n0} \bar{\epsilon}_{k\lambda}) \times \\ &\quad \times \frac{\mathcal{E}^{norm}}{\sqrt{2\hbar}} \times C_0(t) \exp(i(\omega_{n0} + \omega_{k\lambda}) t), \\ i\dot{C}_0^{(k\lambda)} &= -\frac{\mathcal{E}^{norm}}{\sqrt{2\hbar}} \times \\ &\quad \times \sum_{n'=1}^2 (\bar{d}_{0n'} \bar{\epsilon}_{k\lambda}) C_{n'}(t) \exp(i(\omega_{k\lambda} - \omega_{n'0}) t), \end{aligned} \quad (34)$$

$$\text{where } \Gamma_{nn'} = \sum_{n''} 2\pi d_{nE} d_{En''} \epsilon_0^4 / 4 = \frac{2\pi}{\hbar} \left| \frac{d_{nE} \epsilon_0}{2} \right|^2.$$

The first equation is obtained using the method of adiabatic elimination of the continuum (Fedorov, 1997), as well as the approximation of equal

Rydberg-continuum dipole matrix elements for all Rydberg levels efficiently involved in the process of ionization. In our simulations we assume matrix elements to be calculated in quasiclassical approximation $d_{nE} = 1/(n^{1.5}\omega^{5/3})$, where principal quantum number n is supposed to be 10 and ω equals 0.057 that corresponds to the radiation of Ti-Sa laser pulse. Fig. 4 shows the spectrum of spontaneous emission to the ground state for different values of laser intensity. The situation described at fig. 6a corresponds to the initial population amplitudes of Rydberg levels $C_1(0) = 1/\sqrt{2}$, $C_2(0) = -1/\sqrt{2}$ (antisymmetrized combination). In the absence of laser pulse one can see two independent emission profiles from Rydberg levels, but with the increase of radiation intensity due to the interference of amplitudes of Raman Λ -type transitions Rydberg levels reconstructed significantly. As a result emission lines gradually merge forming new transition with the energy $-(E_0 - (E_1 + E_2)/2)$ (curve 3 fig. 4a). Such initial condition provides pretty resistant atomic system with respect to ionization process. The fraction of

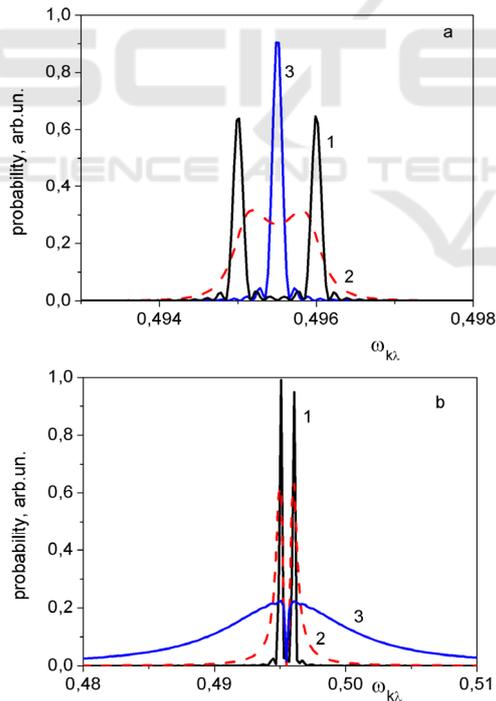


Figure 4: Reconstruction of the spectrum of the spontaneous emission of an atom in the regime of interference stabilization. Amplitudes of initially populated Rydberg states are opposite (a) and the same (b). Intensities of Ti:Sa laser ($\omega = 0.057$) are (1 - 0, 2 - 2.5×10^{-5} , 3 - 2.5×10^{-4}). Pulse duration is 40000.

population trapped in Rydberg states in this case is very close to unity (fig. 5). Another situation is developing for the symmetrized combination of population amplitudes $C_1(0) = 1/\sqrt{2}$, $C_2(0) = 1/\sqrt{2}$. In this case for rather high intensity of laser field when $\Gamma \gg E_2 - E_1$ we can clearly observe a two-exponential dynamics of population decay (fig. 5) showing short- and long-living parts of populations.

Such type of temporal dynamics is pronounced in spontaneous emission spectra. Indeed, blue curve on fig 4b represents the wide line of emission from a rapidly disintegrating background, while the narrow depletion in the center of this line results from long-time decay of the small stable fraction of population. The life times of these long- and short-living fractions can be estimated as $2\Gamma/(E_2 - E_1)^2$ and $1/\Gamma$ correspondingly.

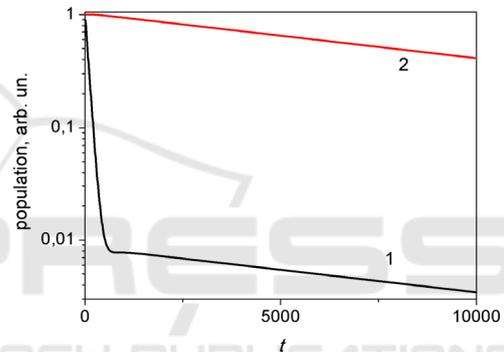


Figure 5: Temporal dependencies of the trapped population for the initially populated symmetrized (1) and antisymmetrized (2) states (see the text). Laser intensity is 2.5×10^{-4} .

7 CONCLUSIONS

To conclude, general approach to analyze the spontaneous emission of an atomic system driven by a strong laser field is developed. It based on the first order of perturbation theory for the interaction with quantized vacuum field modes while the interaction with the intense classical laser field is considered numerically or analytically beyond the perturbation theory. Several problems (Rabi oscillations and formation of the Mollow triplet, spontaneous Raman and Rayleigh scattering, ionization suppression in the regime of interference stabilization) were studied. It was demonstrated that the spontaneous emission can be effectively used to study the reconstruction of the energy spectrum by the laser field, and different types of dressing were analyzed.

We would like to mention that our approach can be used to study the spectrum and dynamics under external fluence of artificial atoms, such as quantum dots or quantum wires, and the coupling of these atoms with photons (Michler et al, 2000; Santori et al 2002; Faraon et al, 2008) or with crystalline lattice via phonons (Förstner et al, 2003; Machnikowski and Jacak, 2004; Ahn, et al, 2005). Developed approach can be of significant interest for the study of relaxation processes in a lot of modern nanoelectronic devices devoted for information receiving and processing (Hoang et al, 2012; Jöns et al, 2015).

Hoang, T. B., Beetz, J., Lerner, et al, 2012. *Opt. Express*, 20, 21758.
 Krausz, F. and Ivanov, M., 2009. *Rev. Mod. Phys.*, 81, 163.
 Machnikowski, P. and Jacak, L., 2004. *Phys. Rev. B* 69, 193302.
 Michler, P. *et al.*, 2000. *Science*, 290, 2282.
 Mollow, B. R., 1969. *Phys. Rev.* 188, 1969.
 Santori, C., Fattal, D., Vucković, J., Solomon, G. S. and Yamamoto, Y. 2002., *Nature*, 419, 594.
 Scully, M. and Zubairy, S. 1997. *Quantum Optics*, Cambridge University Press.

ACKNOWLEDGEMENTS

This work was supported by the Russian Foundation for Basic Research (projects no. 15-02-00373, 16-32-00123).

REFERENCES

- Agostini, P. and Di Mauro, L. F., 2004. *Rep. Prog. Phys.*, 67, 813.
 Ahn, K. J., Förstner, J., and Knorr, A., 2005. *Phys. Rev. B* 71, 153309.
 Akhmanov, S. A. and Nikitin, S. Yu., 1997. *Physical Optics*, Oxford.
 Bogatskaya, A. V., Volkova, E. A., Kharin, V. Yu. and Popov, A. M., 2016. *Laser Phys. Lett.*, 13, 0453014.
 Chin, S. L., 2010. *Femtosecond Laser Filamentation (Springer Series on Atomic, Optical, and Plasma Physics vol 55)* (New York: Springer).
 Couairon, A. and Mysyrowicz, A., 2007. *Phys. Rep.* 441 47.
 Delone, N. B., Krainov, V. P., 1994. *Multiphoton Processes in Atoms* (Berlin: Springer-Verlag).
 Delone, N. B., Goreslavsky S. P., Krainov, V. P., 1983. *J. Phys. B: At. Mol. Phys.* 16, 2369.
 Delone, N. B., Goreslavsky S. P., Krainov, V.P., 1989. *J. Phys. B: At. Mol. Phys.* 22, 2941.
 Faraon, A. *et al.*, 2008. *Nature Phys.* 4 859.
 Fedorov, M. V. and Movsesian, A. M., 1988. *J. Phys. B*, 21, L155.
 Fedorov, M. V., Tehranchi, M.-M., and Fedorov, S. M., 1996. *J. Phys. B*, 29, 2907.
 Fedorov, M. V. and Tikhonova, O. V., 1998. *Phys. Rev. A* 58, 1322.
 Fedorov, M. V., 1997. *Atomic and Free Electrons in a Strong Light Field*, World Scientific.
 Förstner, C., Weber, C., Danckwerts J., and Knorr, A., 2003. *Phys. Rev. Lett.* 91 127401.
 Jöns K. D., Rengstl, U., Oster, M., et al, 2015. *J. Phys. D* 48, 085101.