A Complementarity Problem Formulation for Chance-constraine Games

Vikas Vikram Singh[1], Oualid Jouini[2] and Abdel Lisser[1]

[1]*Laboratoire de Recherche en Informatique, Université Paris Sud XI, Bât 650, 91405, Orsay, France*

[2]*Laboratoire Génie Industriel, Ecole Centrale Paris, Grande Voie des Vignes, 92290, Châtenay-Malabry, France*

Keywords: Chance-Constrained Game, Nash Equilibrium, Normal Distribution, Cauchy Distribution, Nonlinear Complementarity Problem, Linear Complementarity Problem.

Abstract: We consider a two player bimatrix game where the entries of each player's payoff matrix are independent random variables following a certain distribution. We formulate this as a chance-constrained game by considering that the payoff of each player is defined by using a chance-constraint. We consider the case of normal and Cauchy distributions. We show that a Nash equilibrium of the chance-constrained game corresponding to normal distribution can be obtained by solving an equivalent nonlinear complementarity problem. Further if the entries of the payoff matrices are also identically distributed with non-negative mean, we show that a strategy pair, where each player's strategy is the uniform distribution on his action set, is a Nash equilibrium of the chance-constrained game. We show that a Nash equilibrium of the chance-constrained game corresponding to Cauchy distribution can be obtained by solving an equivalent linear complementarity problem.

1 INTRODUCTION

It is well known that there exists a mixed strategy saddle point equilibrium for a two player zero sum matrix game (Neumann, 1928). John Nash (Nash, 1950) showed that there exists a mixed strategy equilibrium for the games with finite number of players where each player has finite number of actions. Later such equilibrium was called Nash equilibrium. For two player case the game considered in (Nash, 1950) can be represented by $m \times n$ matrices A and B. The matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ denote the payoff matrices of player 1 and player 2 respectively, and m, n denote the number of actions of player 1 and player 2 respectively. Let $I = \{1,2,\cdots,m\}$, and $J = \{1,2,\cdots,n\}$ be the action sets of player 1 and player 2 respectively. The sets I and J are also called the sets of pure strategies of player 1 and player 2 respectively. The set of mixed strategies of each player is defined by the set of all probability distributions over his action set. Let $X = \{x = (x_1,x_2,\cdots,x_m) | \sum_{i=1}^{m} x_i = 1, x_i \geq 0,\ \forall\ i \in I\}$ and $Y = \{y = (y_1,y_2,\cdots,y_n) | \sum_{j=1}^{n} y_j = 1, y_j \geq 0,\ \forall\ j \in J\}$ be the sets of mixed strategies of player 1 and player 2 respectively. For a given strategy pair (x,y), the payoffs of player 1 and player 2 are given by $x^T Ay$ and $x^T By$ respectively; T denotes the transposition. For a fixed strategy of one player, another player seeks for a strategy that gives him the highest payoff among all his other strategies. Such a strategy is called the best response strategy. The set of best response strategies of player 1 for a fixed strategy y of player 2 is given by

$$BR(y) = \left\{\bar{x} | \bar{x}^T Ay \geq x^T Ay,\ \forall\ x \in X\right\}.$$

The set of best response strategies of player 2 for a fixed strategy x of player 1 is given by

$$BR(x) = \left\{\bar{y} | x^T B\bar{y} \geq x^T By,\ \forall\ y \in Y\right\}.$$

A strategy pair (x^*, y^*) is said to be a Nash equilibrium if and only if $x^* \in BR(y^*)$ and $y^* \in BR(x^*)$. A Nash equilibrium of above bimatrix game can be obtained by solving a linear complementarity problem (LCP) (Lemke and Howson, 1964).

Both (Nash, 1950) and (Neumann, 1928) considered the games where the payoffs of the players are exact real values. In some cases the payoffs of players may be within certain ranges. In (Collins and Hu, 2008) these situations are modeled as interval valued matrix game using fuzzy theory. The computational approaches have been proposed to solve interval valued matrix game (see (Deng-Feng Li and Zhang, 2012), (Li, 2011), (Mitchell et al., 2014)). However, in many situations payoffs are random variables due to uncertainty which arises from various external factors. The wholesale electricity markets are the good examples (see (Mazadi et al., 2013), (Couchman et al., 2005), (Valenzuela and Mazumdar, 2007), (Wolf and Smeers, 1997)). One way to

Singh, V., Jouini, O. and Lisser, A.
A Complementarity Problem Formulation for Chance-constraine Games.
DOI: 10.5220/0005754800580067
In *Proceedings of 5th the International Conference on Operations Research and Enterprise Systems (ICORES 2016)*, pages 58-67
ISBN: 978-989-758-171-7
Copyright © 2016 by SCITEPRESS – Science and Technology Publications, Lda. All rights reserved

handle this type of game is by taking the expectation of random payoffs and consider the corresponding deterministic game (see (Valenzuela and Mazumdar, 2007), (Wolf and Smeers, 1997)). Some recent papers on the games with random payoffs using expected payoff criterion include (Ravat and Shanbhag, 2011), (Xu and Zhang, 2013), (Jadamba and Raciti, 2015), (DeMiguel and Xu, 2009).

The expected payoff criterion does not take a proper account of stochasticity in the cases where the observed sample payoffs are large amounts with very small probabilities. These situations are better handled by considering a satisficing payoff criterion that uses chance-constrained programming (see (Charnes and Cooper, 1963), (Cheng and Lisser, 2012), (Prékopa, 1995)). Under satisficing payoff criterion the payoff of a player is defined using a chance-constraint and for this reason these games are called chance-constrained games. There are few papers on zero sum chance-constrained games available in the literature (see (Blau, 1974), (Cassidy et al., 1972), (Charnes et al., 1968), (Song, 1992)). Recently, a chance-constrained game with finite number of players is considered in (Singh et al., 2015a), (Singh et al., 2015b) where authors showed the existence of a mixed strategy Nash equilibrium. In (Singh et al., 2015a), the case where the random payoff vector of each player follows a certain distribution is considered. In particular, the authors considered the case where the components of the payoff vector of each player are independent normal random variables, and they also consider the case where the payoff vector of each player follows a multivariate elliptically symmetric distribution. In (Singh et al., 2015b), the case where the distribution of payoff vector of each player is not known completely is considered. The authors consider a distributionally robust approach to handle these games. In application regimes some chance-constrained game models have been considered, e.g., see (Mazadi et al., 2013), (Couchman et al., 2005). In (Mazadi et al., 2013), the randomness in payoffs is due to the installation of wind generators on electricity market, and they consider the case of independent normal random variables. Later, for better representation and ease in computation the authors, in detail, considered the case where only one wind generator is installed in the electricity market. In (Couchman et al., 2005), the payoffs are random due to uncertain demand from consumers which is assumed to be normally distributed.

In this paper, we consider the case where the entries of the payoff matrices are independent random variables following the same distribution (possibly with different parameters). For a given strategy pair (x,y), the payoff of each player is a random variable which is a linear combination of the independent random variables. We consider the distributions that are closed under a linear combination of the independent random variables. The normal and Cauchy distributions satisfy this property. We consider each distribution separately. We show that a Nash equilibrium of the chance-constrained game corresponding to normal distribution can be obtained by solving an equivalent nonlinear complementarity problem (NCP). Further we consider a special case where the entries of the payoff matrices are also identically distributed with non-negative mean. We show that a strategy pair, where each player's strategy is a uniform distribution over his action set, is a Nash equilibrium. We show that a Nash equilibrium of the chance-constrained game corresponding to Cauchy distribution can be obtained by solving an equivalent LCP.

Now, we describe the structure of rest of the paper. Section 2 contains the definition of a chance-constrained game. Section 3 contains the complementarity problem formulation of chance-constrained game. We conclude the paper in Section 4

2 THE MODEL

We consider two player bimatrix game where the entries of the payoff matrices are random variables. We denote the random payoff matrices of player 1 and player 2 by A^w and B^w respectively, where w denotes the uncertainty parameter. Let $(\Omega, \mathcal{F}, P)$ be a probability space. Then, for each $i \in I$, $j \in J$, $a_{ij}^w : \Omega \to \mathbb{R}$, and $b_{ij}^w : \Omega \to \mathbb{R}$. For each $(x,y) \in X \times Y$, the payoffs $x^T A^w y$ and $x^T B^w y$ of player 1 and player 2 respectively would be random variables. We assume that each player uses satisficing payoff criterion, where the payoff of each player is defined using a chance-constraint. At strategy pair (x,y), each player is interested in the highest level of his payoff that he can attain with at least a specified level of confidence. The confidence level of each player is given a priori. We assume that the confidence level of one player is known to another player. Let $\alpha_1 \in [0,1]$ and $\alpha_2 \in [0,1]$ be the confidence levels of player 1 and player 2 respectively. Let $\alpha = (\alpha_1, \alpha_2)$ be a confidence level vector. For a given strategy pair (x,y) and a given confidence level vector α, the payoff of player 1 is given by

$$u_1^{\alpha_1}(x,y) = \sup\{u | P(x^T A^w y \geq u) \geq \alpha_1\}, \quad (1)$$

and the payoff of player 2 is given by

$$u_2^{\alpha_2}(x,y) = \sup\{v | P(x^T B^w y \geq v) \geq \alpha_2\}. \quad (2)$$

We assume that the probability distributions of the entries of the payoff matrix of one player are known to another player. Then, for a given α the payoff function of one player defined above is known to another player. That is, for a given α the chance constrained game is a non-cooperative game with complete information. For a given α, the set of best response strategies of player 1 against the fixed strategy y of player 2 is given by

$$BR^{\alpha_1}(y) = \left\{\bar{x} \in X | u_1^{\alpha_1}(\bar{x}, y) \geq u_1^{\alpha_1}(x,y),\ \forall\, x \in X\right\},$$

and the set of best response strategies of player 2 against the fixed strategy x of player 1 is given by

$$BR^{\alpha_2}(x) = \left\{\bar{y} \in Y | u_2^{\alpha_2}(x, \bar{y}) \geq u_2^{\alpha_2}(x,y),\ \forall\, y \in Y\right\}.$$

Next, we give the definition of Nash equilibrium.

Definition 2.1 (Nash equilibrium)**.** *For a given confidence level vector* α*, a strategy pair* (x^*, y^*) *is said to be a Nash equilibrium of the chance-constrained game if the following inequalities hold:*

$$u_1^{\alpha_1}(x^*, y^*) \geq u_1^{\alpha_1}(x, y^*),\ \forall\, x \in X,$$
$$u_2^{\alpha_2}(x^*, y^*) \geq u_2^{\alpha_2}(x^*, y).\ \forall\, y \in Y.$$

3 COMPLEMENTARITY PROBLEM FOR CHANCE-CONSTRAINED GAME

In this section, we consider the case where the entries of payoff matrix A^w of player 1 are independent random variables following a certain distribution, and the entries of payoff matrix B^w of player 2 are independent random variables following a certain distribution. Then, at strategy pair (x,y) the payoff of each player is a linear combination of the independent random variables. We are interested in those probability distributions that are closed under a linear combination of the independent random variables. That is, if $Y_1, Y_2, \cdots, Y_k$ are independent random variables following the same distribution (possibly with different parameters), for any $b \in \mathbb{R}^k$, the distribution of $\sum_{i=1}^k b_i Y_i$ is same as Y_i up to parameters. The normal and Cauchy distributions satisfy the above property (Johnson et al., 1994). We discuss each probability distribution mentioned above separately. For the case of normal distribution we show that a Nash equilibrium of the chance-constrained game can be obtained by solving an equivalent NCP, and for the case of Cauchy distribution we show that a Nash equilibrium of the chance-constrained game can be obtained by solving an equivalent LCP.

3.1 Payoffs Following Normal Distribution

We assume that all the components of matrix A^w are independent normal random variables, where the mean and variance of a_{ij}^w, $i \in I$, $j \in J$, are $\mu_{1,ij}$ and $\sigma_{1,ij}^2$ respectively, and all the components of matrix B^w are independent normal random variables, where the mean and variance of b_{ij}^w, $i \in I$, $j \in J$, are $\mu_{2,ij}$ and $\sigma_{2,ij}^2$ respectively. For a given strategy pair (x,y), $x^T A^w y$ follows a normal distribution with mean $\mu_1(x,y) = \sum_{i \in I, j \in J} \mu_{1,ij} x_i y_j$ and variance $\sigma_1^2(x,y) = \sum_{i \in I, j \in J} x_i^2 y_j^2 \sigma_{1,ij}^2$, and $x^T B^w y$ follows a normal distribution with mean $\mu_2(x,y) = \sum_{i \in I, j \in J} \mu_{2,ij} x_i y_j$ and variance $\sigma_2^2(x,y) = \sum_{i \in I, j \in J} x_i^2 y_j^2 \sigma_{2,ij}^2$. Then, $Z_1^N = \frac{x^T A^w y - \mu_1(x,y)}{\sigma_1(x,y)}$ and $Z_2^N = \frac{x^T B^w y - \mu_2(x,y)}{\sigma_2(x,y)}$ follow a standard normal distribution. Let $F_{Z_1^N}^{-1}(\cdot)$ and $F_{Z_2^N}^{-1}(\cdot)$ be the quantile functions of a standard normal distribution. From (1), for a given strategy pair (x,y) and a given confidence level α_1, the payoff of player 1 is given by

$$\begin{aligned} u_1^{\alpha_1}(x,y) &= \sup\{u | P(x^T A^w y \geq u) \geq \alpha_1\} \\ &= \sup\{u | P(x^T A^w y \leq u) \leq 1 - \alpha_1\} \\ &= \sup\left\{u | F_{Z_1^N}\left(\frac{u - \mu_1(x,y)}{\sigma_1(x,y)}\right) \leq 1 - \alpha_1\right\} \\ &= \sup\left\{u | u \leq \mu_1(x,y) + \sigma_1(x,y) F_{Z_1^N}^{-1}(1 - \alpha_1)\right\}. \end{aligned}$$

That is,

$$u_1^{\alpha_1}(x,y) = \sum_{i \in I, j \in J} \mu_{1,ij} x_i y_j + \left(\sum_{i \in I, j \in J} x_i^2 y_j^2 \sigma_{1,ij}^2\right)^{1/2} F_{Z_1^N}^{-1}(1 - \alpha_1). \tag{3}$$

Similarly, from (2) for a given strategy pair (x,y) and a given confidence level α_2, the payoff of player 2 is given by

$$u_2^{\alpha_2}(x,y) = \sum_{i \in I, j \in J} \mu_{2,ij} x_i y_j + \left(\sum_{i \in I, j \in J} x_i^2 y_j^2 \sigma_{2,ij}^2\right)^{1/2} F_{Z_2^N}^{-1}(1 - \alpha_2). \tag{4}$$

Theorem 3.1. *Consider a bimatrix game* (A^w, B^w)*. If all the components of matrix* A^w *are independent normal random variables, and all the components of matrix* B^w *are also independent normal random variables, there exists a Nash equilibrium for the chance-constrained game in mixed strategies for all* $\alpha \in [0.5, 1]^2$.

Proof. The proof follows from (Singh et al., 2015a) □

3.1.1 Nonlinear Complementarity Problem Formulation

The payoff function of player 1 defined by (3) can be written as follows:

$$u_1^{\alpha_1}(x,y) = x^T\mu_1(y) + ||\Sigma_1^{1/2}(y)x||F^{-1}_{Z_1^N}(1-\alpha_1), \quad (5)$$

where $||\cdot||$ is the Euclidean norm, and $\mu_1(y) = \big(\mu_{1,i}(y)\big)_{i\in I}$ is an $m\times 1$ vector where $\mu_{1,i}(y) = \sum_{j\in J}\mu_{1,ij}y_j$, and $\Sigma_1(y)$ is an $m\times m$ diagonal matrix whose ith diagonal entry $\Sigma_{1,ii}(y) = \sum_{j\in J}\sigma^2_{1,ij}y_j^2$. Similarly, the payoff function of player 2 defined by (4) can be written as follows:

$$u_2^{\alpha_2}(x,y) = y^T\mu_2(x) + ||\Sigma_2^{1/2}(x)y||F^{-1}_{Z_2^N}(1-\alpha_2), \quad (6)$$

where $\mu_2(x) = \big(\mu_{2,j}(x)\big)_{j\in J}$ is an $n\times 1$ vector where $\mu_{2,j}(x) = \sum_{i\in I}\mu_{2,ij}x_i$, and $\Sigma_2(x)$ is an $n\times n$ diagonal matrix whose jth diagonal entry $\Sigma_{2,jj}(x) = \sum_{i\in I}\sigma^2_{2,ij}x_i^2$. For fixed $y\in Y$ and $\alpha_1\in[0.5,1]$, the payoff function $u_1^{\alpha_1}(\cdot,y)$ of player 1 defined by (5) is a concave function of x because $F^{-1}_{Z_1^N}(1-\alpha_1)\le 0$ for all $\alpha_1\in[0.5,1]$. Similarly, for fixed $x\in X$ and $\alpha_2\in[0.5,1]$, the payoff function $u_2^{\alpha_2}(x,\cdot)$ of player 2 defined by (6) is a concave function of y.

Then, for a fixed $y\in Y$ and $\alpha_1\in[0.5,1]$, a best response strategy of player 1 can be obtained by solving the convex quadratic program [QP1] given below:

$$[\text{QP1}]\quad \max_x \quad x^T\mu_1(y) + ||\Sigma_1^{1/2}(y)x||F^{-1}_{Z_1^N}(1-\alpha_1)$$
$$\text{s.t}$$
$$(i)\ \sum_{i\in I}x_i = 1$$
$$(ii)\ x_i\ge 0,\ i\in I.$$

It is easy to see that a feasible solution of [QP1] satisfies the linear independence constraint qualification. Then, Karush-Kuhn-Tucker (KKT) conditions of [QP1] will be necessary and sufficient conditions for optimal solution (for details see (Nocedal and Wright, 2006), (Bazaraa et al., 2006)). For a given vector ν, $\nu\ge 0$ means $\nu_k\ge 0$, for all k. The equality constraint of [QP1] can be replaced by two equivalent inequality constraints, and the free Lagrange multiplier corresponding to equality constraint can be replaced by the difference of two nonnegative variables. By using these transformations, the best response strategy of player 1 can be obtained by solving the following KKT conditions of [QP1]:

$$\begin{cases} 0\le x\perp -\mu_1(y) - \dfrac{\Sigma_1(y)x\cdot c_{\alpha_1}}{||\Sigma_1^{1/2}(y)x||} - \lambda_1 e_m + \lambda_2 e_m \ge 0,\\ 0\le\lambda_1\perp \sum_{i\in I}x_i - 1\ge 0,\\ 0\le\lambda_2\perp 1-\sum_{i\in I}x_i\ge 0, \end{cases} \quad (7)$$

where e_m is the $m\times 1$ vector of ones, and $c_{\alpha_1} = F^{-1}_{Z_1^N}(1-\alpha_1)$, and $\perp$ means that element-wise equality must hold at one or both sides. For fixed $x\in X$ and $\alpha_2\in[0.5,1]$, a best response strategy of player 2 can be obtained by solving the convex quadratic program [QP2] given below:

$$[\text{QP2}]\quad \max_y \quad y^T\mu_2(x) + ||\Sigma_2^{1/2}(x)y||F^{-1}_{Z_2^N}(1-\alpha_2)$$
$$\text{s.t}$$
$$(i)\ \sum_{j\in J}y_j = 1$$
$$(ii)\ y_j\ge 0,\ j\in J.$$

From the similar arguments used in previous case, the best response strategy of player 2 can be obtained by solving the following KKT conditions of [QP2]:

$$\begin{cases} 0\le y\perp -\mu_2(x) - \dfrac{\Sigma_2(x)y\cdot c_{\alpha_2}}{||\Sigma_2^{1/2}(x)y||} - \lambda_3 e_n + \lambda_4 e_n \ge 0,\\ 0\le\lambda_3\perp \sum_{j\in J}y_j - 1\ge 0,\\ 0\le\lambda_4\perp 1-\sum_{j\in J}y_j\ge 0, \end{cases} \quad (8)$$

where $c_{\alpha_2} = F^{-1}_{Z_2^N}(1-\alpha_2)$.

Nonlinear Complementarity Problem: By combining the KKT conditions given by (7) and (8), a Nash equilibrium (x,y) can be obtained by solving the following NCP:

$$0\le\zeta\perp G(\zeta)\ge 0, \quad (9)$$

where $\zeta, G(\zeta)\in\mathbb{R}^{m+n+4}$ are given below:

$$\zeta^T = (x^T, y^T, \lambda_1, \lambda_2, \lambda_3, \lambda_4),$$

$$G(\zeta) = \begin{pmatrix} -\mu_1(y) - \frac{\Sigma_1(y)x\cdot c_{\alpha_1}}{||\Sigma_1^{1/2}(y)x||} - \lambda_1 e_m + \lambda_2 e_m \\ -\mu_2(x) - \frac{\Sigma_2(x)y\cdot c_{\alpha_2}}{||\Sigma_2^{1/2}(x)y||} - \lambda_3 e_n + \lambda_4 e_n \\ \sum_{i\in I}x_i - 1 \\ 1-\sum_{i\in I}x_i \\ \sum_{j\in J}y_j - 1 \\ 1-\sum_{j\in J}y_j \end{pmatrix}.$$

For given k, l, $\mathbf{0}_{k\times l}$ is a $k\times l$ zero matrix and $\mathbf{0}_k$ is a $k\times 1$ zero vector. Define,

$$Q=\begin{pmatrix}\mathbf{0}_{m\times m} & -\mu_{\mathbf{1}} & -e_m & e_m & \mathbf{0}_m & \mathbf{0}_m\\ -\mu_{\mathbf{2}}^{\mathbf{T}} & \mathbf{0}_{n\times n} & \mathbf{0}_n & \mathbf{0}_n & -e_n & e_n\\ e_m^T & \mathbf{0}_n^T & 0 & 0 & 0 & 0\\ -e_m^T & \mathbf{0}_n^T & 0 & 0 & 0 & 0\\ \mathbf{0}_m^T & e_n^T & 0 & 0 & 0 & 0\\ \mathbf{0}_m^T & -e_n^T & 0 & 0 & 0 & 0\end{pmatrix},$$

where $\mu_{\mathbf{1}}=(\mu_{1,ij})_{i\in I,j\in J}$, $\mu_{\mathbf{2}}=(\mu_{2,ij})_{i\in I,j\in J}$ are $m\times n$ matrices. Define,

$$R(\zeta)=\begin{pmatrix}\frac{-c_{\alpha_1}\cdot\Sigma_1(y)}{\|\Sigma_1^{1/2}(y)x\|} & \mathbf{0}_{m\times n} & \mathbf{0}_{m\times 4}\\ \mathbf{0}_{n\times m} & \frac{-c_{\alpha_2}\cdot\Sigma_2(x)}{\|\Sigma_2^{1/2}(x)y\|} & \mathbf{0}_{n\times 4}\\ \mathbf{0}_{4\times m} & \mathbf{0}_{4\times n} & \mathbf{0}_{4\times 4}\end{pmatrix},\ r=\begin{pmatrix}\mathbf{0}_m\\ \mathbf{0}_n\\ -1\\ 1\\ -1\\ 1\end{pmatrix}.$$

Then, $G(\zeta)=\big(Q+R(\zeta)\big)\zeta+r$.

Theorem 3.2. *Consider a bimatrix game (A^w,B^w) where all the components of matrix A^w are independent normal random variables, and all the components of matrix B^w are also independent normal random variables. Let $\zeta^{*T}=(x^{*T},y^{*T},\lambda_1^*,\lambda_2^*,\lambda_3^*,\lambda_4^*)$ be a vector. Then, the strategy part (x^*,y^*) of ζ^* is a Nash equilibrium of the chance-constrained game for a given $\alpha\in[0.5,1]^2$ if and only if ζ^* is a solution of NCP* (9).

Proof. Let $\alpha\in[0.5,1]^2$, then (x^*,y^*) is a Nash equilibrium of the chance-constrained game if and only if x^* is an optimal solution of [QP1] for fixed y^* and y^* is an optimal solution of [QP2] for fixed x^*. Since, [QP1] and [QP2] are convex optimization problems and linear independence constraint qualification holds at all feasible points, then the KKT conditions (7) and (8) are both necessary and sufficient conditions for optimality. Then, the proof follows by combining the KKT conditions (7), (8). □

For computational purpose freely available solvers for complementarity problems can be used, e.g., see (Schmelzer, 2012), (Ferris and Munson, 2000), (Munson, 2000).

3.1.2 Special Case

Here we consider the case where the components of payoff matrices A^w and B^w are independent as well as identically distributed. We assume that the components of matrix A^w are independent and identically distributed (i.i.d.) normal random variables with mean $\mu_1\geq 0$ and variance σ_1^2, and the components of matrix B^w are i.i.d. normal random variables with mean $\mu_2\geq 0$ and variance σ_2^2.

Theorem 3.3. *Consider a bimatrix game (A^w,B^w) where all the components of matrix A^w are i.i.d. normal random variables with mean $\mu_1\geq 0$ and variance σ_1^2, and all the components of matrix B^w are also i.i.d. normal random variables with mean $\mu_2\geq 0$ and variance σ_2^2. The strategy pair (x^*,y^*), where,*

$$x_i^*=\frac{1}{m},\ \forall\ i\in I,\ \ y_j^*=\frac{1}{n},\forall\ j\in J, \qquad (10)$$

is a Nash equilibrium of the chance-constrained game for all $\alpha\in[0.5,1]^2$.

Proof. Fix $\alpha\in[0.5,1]^2$. From Theorem 3.2, it is enough to show that there exist $(\lambda_1^*,\lambda_2^*,\lambda_3^*,\lambda_4^*)$ which together with (x^*,y^*) defined by (10) is a solution of NCP (9). For all (x,y), we have $\mu_1(y)=\mu_1 e_m$ and $\mu_2(x)=\mu_2 e_n$ because $\mu_{1,ij}=\mu_1$ and $\mu_{2,ij}=\mu_2$ for all $i\in I$, $j\in J$, and $\Sigma_1(y)=\sigma_1^2\|y\|^2 I_{m\times m}$ and $\Sigma_2(x)=\sigma_2^2\|x\|^2 I_{n\times n}$ because $\sigma_{1,ij}^2=\sigma_1^2$ and $\sigma_{2,ij}^2=\sigma_2^2$ for all $i\in I$, $j\in J$; $I_{k\times k}$ is a $k\times k$ identity matrix. Using above expressions, we have

$$G(\zeta)=\begin{pmatrix}-\mu_1 e_m-\frac{\sigma_1\|y\|x\cdot c_{\alpha_1}}{\|x\|}-(\lambda_1-\lambda_2)e_m\\ -\mu_2 e_n-\frac{\sigma_2\|x\|y\cdot c_{\alpha_2}}{\|y\|}-(\lambda_3-\lambda_4)e_n\\ \sum_{i\in I}x_i-1\\ 1-\sum_{i\in I}x_i\\ \sum_{j\in J}y_j-1\\ 1-\sum_{j\in J}y_j\end{pmatrix}.$$

Consider the Lagrange multipliers $(\lambda_1^*,\lambda_2^*,\lambda_3^*,\lambda_4^*)$ as follows:

$$\lambda_1^*=-\frac{\sigma_1\cdot c_{\alpha_1}}{\sqrt{mn}},\ \lambda_2^*=\mu_1,\ \lambda_3^*=-\frac{\sigma_2\cdot c_{\alpha_2}}{\sqrt{mn}},\ \lambda_4^*=\mu_2.$$

Since, $\mu_1\geq 0$, $\mu_2\geq 0$, and for $\alpha\in[0.5,1]^2$, $c_{\alpha_1}\leq 0$ and $c_{\alpha_2}\leq 0$, then, $\lambda_1^*\geq 0$, $\lambda_2^*\geq 0$, $\lambda_3^*\geq 0$, $\lambda_4^*\geq 0$. It is easy to check that $(x^*,y^*,\lambda_1^*,\lambda_2^*,\lambda_3^*,\lambda_4^*)$ is a solution of NCP (9). That is, (x^*,y^*) defined by (10) is a Nash equilibrium of chance-constrained game.

□

3.2 Payoffs Following Cauchy Distribution

We assume that all the components of matrix A^w are independent Cauchy random variables, where the

SCITEPRESS SCIENCE AND TECHNOLOGY PUBLICATIONS

location and scale parameters of a^w_{ij}, $i \in I$, $j \in J$, are $\mu_{1,ij}$ and $\sigma_{1,ij}$ respectively, and all the components of matrix B^w are independent Cauchy random variables, where the location and scale parameters of b^w_{ij}, $i \in I$, $j \in J$, are $\mu_{2,ij}$ and $\sigma_{2,ij}$ respectively. Since, a linear combination of the independent Cauchy random variables is a Cauchy random variable (Johnson et al., 1994), then, for a given strategy pair (x,y), the payoff $x^T A^w y$ of player 1 follows a Cauchy distribution with location parameter $\mu_1(x,y) = \sum_{i\in I, j\in J} x_i y_j \mu_{1,ij}$ and scale parameter $\sigma_1(x,y) = \sum_{i\in I, j\in J} x_i y_j \sigma_{1,ij}$, and the payoff $x^T B^w y$ of player 2 follows a Cauchy distribution with location parameter $\mu_2(x,y) = \sum_{i\in I, j\in J} x_i y_j \mu_{2,ij}$ and scale parameter $\sigma_2(x,y) = \sum_{i\in I, j\in J} x_i y_j \sigma_{2,ij}$. Then, $Z^C_1 = \frac{x^T A^w y - \mu_1(x,y)}{\sigma_1(x,y)}$ and $Z^C_2 = \frac{x^T B^w y - \mu_2(x,y)}{\sigma_2(x,y)}$ follow a standard Cauchy distribution. Let $F^{-1}_{Z^C_1}(\cdot)$ and $F^{-1}_{Z^C_2}(\cdot)$ be the quantile functions of a standard Cauchy distribution. For more details about Cauchy distribution see (Johnson et al., 1994). Similar to the normal distribution case, for a given strategy pair (x,y) and a given α the payoff of player 1 is given by

$$
\begin{aligned}
u^{\alpha_1}_1(x,y) &= \sup\left\{ u \middle| F_{Z^C_1}\left(\frac{u-\mu_1(x,y)}{\sigma_1(x,y)}\right) \le 1-\alpha_1 \right\} \\
&= \sup\left\{ u \middle| u \le \mu_1(x,y) + \sigma_1(x,y) F^{-1}_{Z^C_1}(1-\alpha_1) \right\}.
\end{aligned}
$$

That is,

$$
u^{\alpha_1}_1(x,y) = \sum_{i\in I, j\in J} x_i y_j \left(\mu_{1,ij} + \sigma_{1,ij} F^{-1}_{Z^C_1}(1-\alpha_1) \right). \tag{11}
$$

Similarly, the payoff of player 2 is given by

$$
u^{\alpha_2}_2(x,y) = \sum_{i\in I, j\in J} x_i y_j \left(\mu_{2,ij} + \sigma_{2,ij} F^{-1}_{Z^C_2}(1-\alpha_2) \right). \tag{12}
$$

The quantile function of a standard Cauchy distribution is not finite at 0 and 1. Therefore, we consider the case of $\alpha \in (0,1)^2$, so that payoff functions defined by (11) and (12) have finite values. Define, a matrix $\tilde{A}(\alpha_1) = (\tilde{a}_{ij}(\alpha_1))_{i\in I, j\in J}$, where

$$
\tilde{a}_{ij}(\alpha_1) = \mu_{1,ij} + \sigma_{1,ij} F^{-1}_{Z^C_1}(1-\alpha_1), \tag{13}
$$

and a matrix $\tilde{B}(\alpha_2) = \left(\tilde{b}_{ij}(\alpha_2)\right)_{i\in I, j\in J}$, where

$$
\tilde{b}_{ij}(\alpha_2) = \mu_{2,ij} + \sigma_{2,ij} F^{-1}_{Z^C_2}(1-\alpha_2). \tag{14}
$$

Then, we can write (11) as

$$
u^{\alpha_1}_1(x,y) = x^T \tilde{A}(\alpha_1) y,
$$

and we can write (12) as

$$
u^{\alpha_2}_2(x,y) = x^T \tilde{B}(\alpha_2) y.
$$

Then, for a given $\alpha \in (0,1)^2$, the chance-constrained game is equivalent to the bimatrix game $\left(\tilde{A}(\alpha_1), \tilde{B}(\alpha_2)\right)$.

Theorem 3.4. *Consider a bimatrix game (A^w, B^w). If all components of matrix A^w are independent Cauchy random variables, and all components of matrix B^w are also independent Cauchy random variables, there exists a Nash equilibrium for the chance-constrained game in mixed strategies for all $\alpha \in (0,1)^2$.*

Proof. For each $\alpha \in (0,1)^2$ the chance-constrained game is equivalent to the bimatrix game $\left(\tilde{A}(\alpha_1), \tilde{B}(\alpha_2)\right)$. Hence, the existence of a Nash equilibrium in mixed strategies follows from (Nash, 1950). □

Remark 3.5. *For case of i.i.d. Cauchy random variables each strategy pair (x,y) is a Nash equilibrium because from* (11), (12) *the payoff functions of both the players are constant.*

3.2.1 Linear Complementarity Problem

For a given matrix $N = [N_{ij}]$, $N > 0$ means that $N_{ij} > 0$ for all i, j. Let E be the $m \times n$ matrix with all 1's. Let k be the large enough such that $kE^T - (\tilde{B}(\alpha_2))^T > 0$ and $kE - \tilde{A}(\alpha_1) > 0$. Then, from (Lemke and Howson, 1964), (Lemke, 1965) it follows that for a given α, a Nash equilibrium of the chance-constrained game can be obtained by following LCP:

$$
0 \le z \perp Mz + q \ge 0, \tag{15}
$$

where

$$
z = \begin{pmatrix} x \\ y \end{pmatrix}, \quad M = \begin{pmatrix} \mathbf{0}_{m\times m} & kE - \tilde{A}(\alpha_1) \\ kE^T - (\tilde{B}(\alpha_2))^T & \mathbf{0}_{n\times n} \end{pmatrix},
$$

$$
q = \begin{pmatrix} -e_m \\ -e_n \end{pmatrix}.
$$

Theorem 3.6. *Consider a bimatrix game (A^w, B^w) where all the components of matrix A^w are independent Cauchy random variables, and all the components of matrix B^w are also independent Cauchy random variables, then,*

1. *For a $\alpha \in (0,1)^2$, if (x^*, y^*) is a Nash equilibrium of the chance-constrained game, $z^{*T} = \left(\frac{x^{*T}}{k - x^{*T}\tilde{B}(\alpha_2)y^*}, \frac{y^{*T}}{k - x^{*T}\tilde{A}(\alpha_1)y^*} \right)$ is a solution of LCP* (15) *at α.*
2. *For a $\alpha \in (0,1)^2$, if $\bar{z}^T = (\bar{x}^T, \bar{y}^T)$ is a solution of LCP* (15), *$(x^*, y^*) = \left(\frac{\bar{x}}{\sum_{i\in I}\bar{x}_i}, \frac{\bar{y}}{\sum_{i\in I}\bar{y}_j} \right)$ is a Nash equilibrium of the chance-constrained game at α.*

Proof. For a $\alpha \in (0,1)^2$, the chance-constrained game corresponding to Cauchy distribution is equivalent to a bimatrix game $\left(\tilde{A}(\alpha_1), \tilde{B}(\alpha_2)\right)$, where $\tilde{A}(\alpha_1)$ and $\tilde{B}(\alpha_2)$ is defined by (13) and (14) respectively. Then, the proof follows from (Lemke and Howson, 1964). □

3.2.2 Numerical Results

We consider few instances of random bimatrix game of different sizes. We compute the Nash equilibria of corresponding chance-constrained game by using Lemke-Howson algorithm (Lemke and Howson, 1964). We use the MATLAB code of Lemke-Howson algorithm given in (Katzwer, 2013).

(i) 3×3 *random bimatrix game:* We consider five instances of random bimatrix game of size 3×3. The datasets consisting of location parameters $\mu_1 = [\mu_{1,ij}]$, $\mu_2 = [\mu_{2,ij}]$, and scale parameters $\sigma_1 = [\sigma_{1,ij}]$, $\sigma_2 = [\sigma_{2,ij}]$ of independent Cauchy random variables that characterizes chance-constrained game are follows:

1. $\mu_1 = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix}, \ \sigma_1 = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 1 & 2 \end{pmatrix},$

$$\mu_2 = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}.$$

2. $\mu_1 = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 2 & 1 & 3 \end{pmatrix}, \ \sigma_1 = \begin{pmatrix} 2 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix},$

$$\mu_2 = \begin{pmatrix} 2 & 2 & 1 \\ 3 & 2 & 3 \\ 2 & 1 & 2 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{pmatrix}.$$

3. $\mu_1 = \begin{pmatrix} 2 & 1 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{pmatrix}, \ \sigma_1 = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix},$

$$\mu_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 3 & 1 & 1 \end{pmatrix}.$$

4. $\mu_1 = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \ \sigma_1 = \begin{pmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix},$

$$\mu_2 = \begin{pmatrix} 4 & 1 & 3 \\ 3 & 2 & 4 \\ 2 & 1 & 3 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 5 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 2 & 3 \end{pmatrix}.$$

5. $\mu_1 = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix}, \ \sigma_1 = \begin{pmatrix} 2 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix},$

$$\mu_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 5 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 2 & 3 \end{pmatrix}.$$

The entries of μ_1, σ_1, μ_2, σ_2 defined above are the location and scale parameters of corresponding independent Cauchy random variables. For example, in dataset 1 random payoff a_{11} is a Cauchy random variable with location parameter 1 and scale parameter 1. The Table 1 summarizes the Nash equilibria of chance-constrained game corresponding to datasets given for five instances of 3×3 random bimatrix game.

Table 1: Nash equilibria for various values of α.

No.	α		Nash Equilibrium	
	α_1	α_2	x^*	y^*
1	0.4	0.4	(0, 0, 1)	(0, 0, 1)
	0.5	0.5	(0, 1, 0)	(1, 0, 0)
	0.7	0.7	(0, 1, 0)	(0, 1, 0)
2	0.4	0.4	(1, 0, 0)	(0, 1, 0)
	0.5	0.5	(0, 1, 0)	(1, 0, 0)
	0.7	0.7	(0, 0, 1)	(1, 0, 0)
3	0.4	0.4	(1, 0, 0)	(0, 0, 1)
	0.5	0.5	(1, 0, 0)	(0, 0, 1)
	0.7	0.7	(1, 0, 0)	(0, 0, 1)
4	0.4	0.4	(1, 0, 0)	(1, 0, 0)
	0.5	0.5	(1, 0, 0)	(1, 0, 0)
	0.7	0.7	(0, 0, 1)	(0, 0, 1)
5	0.4	0.4	$\left(0, \frac{791}{1000}, \frac{209}{1000}\right)$	$\left(\frac{616}{1000}, 0, \frac{384}{1000}\right)$
	0.5	0.5	$\left(0, \frac{1}{2}, \frac{1}{2}\right)$	$\left(\frac{2}{3}, 0, \frac{1}{3}\right)$
	0.7	0.7	(0, 0, 1)	(1, 0, 0)

(ii) 5×5 *random bimatrix game:* We consider two instances of random bimatrix game of size 5×5. The location parameters μ_1, μ_2, and scale parameters σ_1, σ_2 of independent Cauchy random variables are as follows:

1. $\mu_1 = \begin{pmatrix} 1 & 2 & 1 & 1 & 3 \\ 2 & 3 & 1 & 1 & 2 \\ 1 & 2 & 3 & 2 & 3 \\ 2 & 1 & 3 & 4 & 2 \\ 1 & 2 & 4 & 5 & 2 \end{pmatrix}, \sigma_1 = \begin{pmatrix} 2 & 2 & 3 & 2 & 1 \\ 1 & 2 & 3 & 2 & 1 \\ 1 & 2 & 3 & 3 & 1 \\ 2 & 1 & 3 & 4 & 2 \\ 3 & 1 & 2 & 5 & 2 \end{pmatrix},$

$$\mu_2 = \begin{pmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 2 & 2 & 1 & 3 \\ 1 & 2 & 3 & 1 & 2 \\ 2 & 1 & 4 & 2 & 1 \\ 1 & 1 & 2 & 1 & 3 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 5 & 2 & 3 & 2 & 3 \\ 2 & 4 & 3 & 2 & 1 \\ 1 & 3 & 4 & 2 & 3 \\ 2 & 1 & 3 & 5 & 1 \\ 2 & 1 & 2 & 3 & 4 \end{pmatrix}.$$

2. $$\mu_1 = \begin{pmatrix} 1 & 2 & 2 & 4 & 3 \\ 2 & 1 & 3 & 2 & 2 \\ 1 & 2 & 4 & 2 & 1 \\ 2 & 2 & 3 & 4 & 1 \\ 1 & 2 & 4 & 5 & 2 \end{pmatrix}, \sigma_1 = \begin{pmatrix} 2 & 3 & 1 & 2 & 1 \\ 1 & 1 & 3 & 1 & 2 \\ 3 & 1 & 3 & 3 & 1 \\ 2 & 2 & 5 & 4 & 2 \\ 3 & 1 & 3 & 5 & 2 \end{pmatrix},$$

$$\mu_2 = \begin{pmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 1 & 2 & 1 & 4 \\ 2 & 1 & 3 & 4 & 2 \\ 3 & 2 & 4 & 2 & 1 \\ 2 & 4 & 2 & 1 & 3 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 5 & 2 & 4 & 2 & 1 \\ 2 & 4 & 3 & 2 & 1 \\ 4 & 3 & 3 & 2 & 3 \\ 2 & 1 & 3 & 5 & 3 \\ 1 & 3 & 4 & 3 & 4 \end{pmatrix}.$$

The Table 2 summarizes the Nash equilibria of chance-constrained game corresponding to datasets given for two instances of 5×5 random bimatrix game.

Table 2: Nash equilibria for various values of α.

No.	α		Nash Equilibrium	
	α_1	α_2	x^*	y^*
1	0.4	0.4	$\left(0,0,\frac{555}{1000},0,\frac{445}{1000}\right)$	$\left(0,0,\frac{1}{2},0,\frac{1}{2}\right)$
	0.5	0.5	$\left(0,0,\frac{1}{2},0,\frac{1}{2}\right)$	$\left(0,0,\frac{1}{2},0,\frac{1}{2}\right)$
	0.7	0.7	(0,0,0,0,1)	(0,0,1,0,0)
2	0.4	0.4	$\left(0,0,\frac{663}{1000},0,\frac{337}{1000}\right)$	(0,0,1,0,0)
	0.5	0.5	$\left(0,0,\frac{1}{2},0,\frac{1}{2}\right)$	(0,1,0,0,0)
	0.7	0.7	$\left(0,0,\frac{446}{1000},0,\frac{554}{1000}\right)$	(0,1,0,0,0)

(*iii*) 7×7 *random bimatrix game:* We consider two instances of random bimatrix game of size 7×7. The location parameters μ_1, μ_2, and scale parameters σ_1, σ_2 of independent Cauchy random variables are as follows:

1. $$\mu_1 = \begin{pmatrix} 1&2&2&4&3&2&1 \\ 1&1&2&1&3&2&2 \\ 3&2&1&2&4&2&1 \\ 2&4&2&2&3&4&1 \\ 1&2&4&5&2&2&3 \\ 1&3&4&3&2&2&3 \\ 2&1&4&2&3&2&1 \end{pmatrix}, \sigma_1 = \begin{pmatrix} 2&3&1&2&1&1&2 \\ 1&1&3&1&2&2&4 \\ 2&1&3&1&3&3&1 \\ 2&2&5&4&2&1&3 \\ 2&1&3&1&3&5&2 \\ 1&2&3&1&2&3&2 \\ 2&1&4&2&3&1&2 \end{pmatrix},$$

$$\mu_2 = \begin{pmatrix} 1&2&3&2&3&2&1 \\ 1&2&3&1&2&1&4 \\ 2&1&2&1&3&4&2 \\ 1&2&3&2&4&2&1 \\ 2&3&1&4&2&1&3 \\ 1&2&3&2&1&3&4 \\ 2&3&1&2&3&4&2 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 5&2&4&2&1&2&3 \\ 1&2&2&4&3&2&1 \\ 2&3&4&3&3&2&3 \\ 2&3&2&1&3&5&3 \\ 2&1&2&3&4&3&4 \\ 1&2&2&3&1&3&1 \\ 2&4&1&2&3&1&2 \end{pmatrix}.$$

2. $$\mu_1 = \begin{pmatrix} 1&2&3&1&3&4&1 \\ 2&1&2&1&2&4&2 \\ 1&2&1&5&3&2&1 \\ 1&3&2&2&3&2&1 \\ 2&3&4&5&2&1&3 \\ 1&3&2&1&2&4&3 \\ 2&1&3&2&1&2&1 \end{pmatrix}, \sigma_1 = \begin{pmatrix} 1&3&1&2&1&2&2 \\ 2&1&3&1&2&2&4 \\ 2&1&3&2&3&4&1 \\ 2&2&3&4&2&1&3 \\ 2&4&3&1&3&2&2 \\ 1&2&3&2&2&4&2 \\ 2&3&4&1&3&1&2 \end{pmatrix},$$

$$\mu_2 = \begin{pmatrix} 2&1&3&4&3&2&1 \\ 1&2&3&3&2&1&4 \\ 2&1&2&1&3&4&2 \\ 1&2&3&2&4&2&1 \\ 2&3&2&4&2&1&3 \\ 1&2&1&2&5&3&4 \\ 2&3&1&2&1&4&2 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 5&2&4&3&1&2&3 \\ 1&2&3&4&3&2&3 \\ 1&3&4&2&1&2&3 \\ 2&3&2&2&3&4&3 \\ 2&1&2&2&4&1&4 \\ 2&3&2&3&4&3&1 \\ 2&4&3&2&3&1&2 \end{pmatrix}.$$

The Table 3 summarizes the Nash equilibria of chance-constrained game corresponding to datasets given for two instances of 7×7 random bimatrix game.

Table 3: Nash equilibria for various values of α.

No.	α		Nash Equilibrium	
	α_1	α_2	x^*	y^*
1	0.4	0.4	$\left(0,0,\frac{2}{3},\frac{1}{3},0,0,0\right)$	$\left(0,0,0,0,\frac{505}{1000},\frac{495}{1000},0\right)$
	0.5	0.5	$\left(0,0,\frac{2}{3},\frac{1}{3},0,0,0\right)$	$\left(0,0,0,0,\frac{2}{3},\frac{1}{3},0\right)$
	0.7	0.7	(1,0,0,0,0,0,0)	(0,0,0,0,1,0,0)
2	0.4	0.4	$\left(\frac{1}{5},0,\frac{13}{25},0,\frac{7}{25},0,0\right)$	$\left(0,0,\frac{13}{50},0,\frac{675}{1000},\frac{65}{1000},0\right)$
	0.5	0.5	$\left(\frac{1}{2},0,\frac{1}{2},0,0,0,0\right)$	(0,0,0,0,1,0,0)
	0.7	0.7	(1,0,0,0,0,0,0)	(0,0,0,0,1,0,0)

4 CONCLUSIONS

We formulate the bimatrix game with random payoffs as a chance-constrained game. We consider the case where the entries of payoff matrices are independent random variables following a certain distribution. In particular, we discuss the case of normal and Cauchy distributions. We show that the chance-constrained game corresponding to normal distribution can be formulated as an equivalent NCP. Further if the entries of payoff matrices are also identically distributed with non-negative mean, a uniform strategy pair is a Nash equilibrium. We show that the chance-constrained game corresponding to Cauchy distribution can be formulated as an equivalent LCP. Recently, the electricity markets over the past few years have been transformed from nationalized monopolies into competitive markets with privately owned participants. The uncertainties in electricity markets are present due to various external factors. These situations can be modeled as chance-constrained games and the approaches developed in this paper can be applied to compute the Nash equilibrium.

REFERENCES

Bazaraa, M., Sherali, H., and Shetty, C. (2006). *Nonlinear Programming Theory and Algorithms*. John Wiley and Sons, Inc., U.S.A, Third ed.

Blau, R. A. (1974). Random-payoff two person zero-sum games. *Operations Research*, 22(6):1243–1251.

Cassidy, R. G., Field, C. A., and Kirby, M. J. L. (1972). Solution of a satisficing model for random payoff games. *Management Science*, 19(3):266–271.

Charnes, A. and Cooper, W. W. (1963). Deterministic equivalents for optimizing and satisficing under chance constraints. *Operations Research*, 11(1):18–39.

Charnes, A., Kirby, M. J. L., and Raike, W. M. (1968). Zero-zero chance-constrained games. *Theory of Probability and its Applications*, 13(4):628–646.

Cheng, J. and Lisser, A. (2012). A second-order cone programming approach for linear programs with joint probabilistic constraints. *Operations Research Letters*, 40(5):325–328.

Collins, W. D. and Hu, C. (2008). Studying interval valued matrix games with fuzzy logic. *Soft Computing*, 12:147–155.

Couchman, P., Kouvaritakis, B., Cannon, M., and Prashad, F. (2005). Gaming strategy for electric power with random demand. *IEEE Transactions on Power Systems*, 20(3):1283–1292.

DeMiguel, V. and Xu, H. (2009). A stochastic multiple leader stackelberg model: analysis, computation, and application. *Operations Research*, 57(5):1220–1235.

Deng-Feng Li, J.-X. N. and Zhang, M.-J. (2012). Interval programming models for matrix games with interval payoffs. *Optimization Methods and Software*, 27(1):1–16.

Ferris, M. C. and Munson, T. S. (2000). Complementarity problems in GAMS and the PATH solver. *Journal of Economic Dynamics and Control*, 24:165–188.

Jadamba, B. and Raciti, F. (2015). Variational inequality approach to stochastic nash equilibrium problems with an application to cournot oligopoly. *Journal of Optimization Theory and Application*, 165(3):1050–1070.

Johnson, N. L., Kotz, S., and Balakrishnan, N. (1994). *Continuous Univariate Distributions*, volume 1. John Wiley and Sons Inc., 2nd edition.

Katzwer, R. (2013). *Lemke-Howson Algorithm for 2-Player Games*. File ID: #44279 Version: 1.3.

Lemke, C. and Howson, J. (1964). Equilibrium points of bimatrix games. *SIAM Journal*, 12:413–423.

Lemke, C. E. (1965). Bimatrix equilibrium points and mathematical programming. *Management Science*, 11(7):681–689.

Li, D.-F. (2011). Linear programming approach to solve interval-valued matrix games. *Journal of Omega*, 39(6):655–666.

Mazadi, M., Rosehart, W. D., Zareipour, H., Malik, O. P., and Oloomi, M. (2013). Impact of wind integration on electricity markets: A chance-constrained Nash Cournot model. *International Transactions on Electrical Energy Systems*, 23(1):83–96.

Mitchell, C., Hu, C., Chen, B., Nooner, M., and Young, P. (2014). A computational study of interval-valued matrix games. In *International Conference on Computational Science and Computational Intelligence*.

Munson, T. S. (2000). *Algorithms and Environments for Complementarity*. PhD thesis, University of Wisconsin - Madison.

Nash, J. F. (1950). Equilibrium points in n-person games. *Proceedings of the National Academy of Sciences*, 36(1):48–49.

Neumann, J. V. (1928). Zur theorie der gesellschaftsspiele. *Math. Annalen*, 100(1):295–320.

Nocedal, J. and Wright, S. J. (2006). *Numerical Optimization*. Springer Science + Business Media LLC, New York, 2 edition.

Prékopa, A. (1995). *Stochastic Programming*. Springer, Netherlands.

Ravat, U. and Shanbhag, U. V. (2011). On the characterization of solution sets of smooth and nonsmooth convex stochastic Nash games. *Siam Journal of Optimization*, 21(3):1168–1199.

Schmelzer, S. (2012). COMPASS: A free solver for mixed complementarity problems. Master's thesis, Universität Wien.

Singh, V. V., Jouini, O., and Lisser, A. (2015a). Existence of nash equilibrium for chance-constrained games. http://www.optimization-online.org/DB_FILE/2015/06/4977.pdf.

Singh, V. V., Jouini, O., and Lisser, A. (2015b). Existence of nash equilibrium for distributionally robust chance-constrained games. http://www.optimization-online.org/DB_FILE/2015/09/5120.pdf.

Song, T. (1992). *Systems and Management Science by Extremal Methods*, chapter On random payoff matrix games, pages 291–308. Springer Science + Business Media, LLC.

Valenzuela, J. and Mazumdar, M. (2007). Cournot prices considering generator availability and demand uncertainty. *IEEE Transactions on Power Systems*, 22(1):116–125.

Wolf, D. D. and Smeers, Y. (1997). A stochastic version of a Stackelberg-Nash-Cournot equilibrium model. *Management Science*, 43(2):190–197.

Xu, H. and Zhang, D. (2013). Stochastic nash equilibrium problems: sample average approximation and applications. *Computational Optimization and Applications*, 55(3):597–645.

SCITEPRESS
SCIENCE AND TECHNOLOGY PUBLICATIONS