Study of Inheritance and Approximation Techniques for Adaptive Multi-objective Particle Swarm Optimization

Ibtissem Bouoni[1], Nadia Smairi[2] and Kamel Zidi[3]
[1]Faculty of Science Gafsa, Gafsa, Tunisia
[2]Laboratory of Optimization Strategy and Intelligent Computer, ISG Tunis, Tunis, Tunisia
[3]University of Tabuk, Tabuk, Saudi Arabia

Keywords: Multi-objective Optimization, Particle Swarm Optimization, MO-TRIBES, Inheritance Technique, Approximation Technique, Fitness Evaluation, Time.

Abstract: In this paper, we propose to introduce inheritance and approximation techniques for the evaluation of the objective function. The main idea of the approaches is to reduce MO-TRIBES complexity. Besides, in our study, we incorporate at the beginning, an inheritance technique then an approximation technique (Approximation 1: to consider the whole swarm, Approximation 2: to consider the tribe) at the evaluation of the objective function. We conducted in our experiments eleven well-known multi-objective test functions. The results showed a good behavior of our propositions on most tested functions. Moreover, TRIBES-inheritance provided the best compared to MO-TRIBES, we concluded that MO-TRIBES with inheritance give the best time than MO-TRIBES and MO-TRIBES with approximation. It also kept the same performances with MO-TRIBES with a simple improvement for several functions.

1 INTRODUCTION

One of the problems of evolutionary algorithms is that each one of them requires setting several control parameters depending on the problem considered, MO-TRIBES, an adaptive Particle Swarm Optimization (PSO) technique, has the advantage to be considered as a black box; the specialist defines only the search space. The adaptability of MO-TRIBES shows an increase in complexity especially compared to a conventional MOPSO algorithm.

We propose in this paper to minimize MO-TRIBES complexity while keeping its performance. In fact, the evaluation of the objective function is often complicated especially in the multiobjective case. We propose to introduce inheritance and approximation techniques for the evaluation of the objective function in order to reduce MO-TRIBES complexity.

In section 2 of this paper, we introduce the existing inheritance and approximation techniques. In section 3, we define and discuss the state of art of MO-TRIBES. In section 4, we present our proposed approach and we use eleven well-known multi-objective test functions in order to find the best one from the proposed techniques. Then comparative results are described in section 5, from which conclusions are drawn in section 6.

2 STATE OF ART

2.1 Fitness Inheritance

Smith is the first who used Fitness Inheritance technique to improve the Genetic algorithm performance (Smith, Dike and Stegmann, 1995). Authors proposed two probable ways of fitness inheritance. The first consists in taking the average fitness of the two parents while the second consists of taking a weighted average of the fitness of the two parents.

Sierra and Coello in (2005) proposed an integration of inheritance techniques in a real code multi-objective PSO (MOPSO). They concluded that fitness inheritance reduces the cost without decreasing the performance.

The purpose of this paper (Montes, Dávila and Coello, 2007) is to find a trade-off between a lower number of evaluations of each solution and a good performance of the approach. A set of test problems

Bouoni I., Smairi N. and Zidi K..
Study of Inheritance and Approximation Techniques for Adaptive Multi-objective Particle Swarm Optimization.
DOI: 10.5220/0005529901460154
In *Proceedings of the 12th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2015), pages 146-154
ISBN: 978-989-758-122-9

taken from the specialized literature was used to test the capabilities of the proposed approach to save evaluations and to preserve a competitive performance.

This chapter (Becerra, Quintero and Coello, 2008), presents a review of techniques used to integrate knowledge into evolutionary algorithms, with particular emphasis on multi-objective optimization.

There are several techniques of inheritance that we can divide into three broad families. For each particle, we can apply different types of techniques such as the inheritance shown in the following figure:

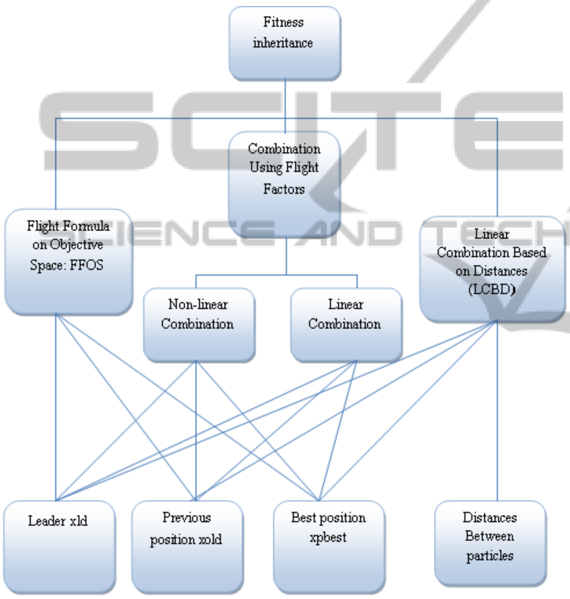

Figure 1: Fitness Inheritance.

2.2 Fitness Approximation

Ratle presented a new approach based on a real code genetic algorithm to accelerate convergence of evolutionary optimization methods (Ratle, 1998).

In paper of (Jin, 2005), a comprehensive review of the research on fitness approximation in evolutionary calculation is presented. Main problems like approximation levels, approximate model management schemes, model construction techniques are reviewed.

Lim presented (Lim, Jin, Ong, Bernhard and Sendhoff, 2006) a Trusted Evolutionary Algorithm TEA for solving optimization problems with computationally expensive fitness functions. The TEA is designed to maintain good worthiness of the substitute models in predicting fitness improvements or controlling approximation errors throughout the evolutionary search.

(Bhattacharya, 2013) discusses some of the key issues concerned with use of approximation in evolutionary algorithm, possible best practices and solutions.

3 MO-TRIBES

TRIBES is an adaptive Particle Swarm Optimization (PSO) algorithm developed by Clerc (2006). This algorithm is sufficient to delimit the space of research and indicate how to evaluate the objective function. Actually, it is enough to specify the problem to be solved. This algorithm must incorporate rules that define how, the structure of the swarm must be modified and how a particle data must behave while integrating information.

Multi-objective TRIBES was elaborated in the beginning by Cooren (2008) and later by Smairi, Bouamama, Ghedira and Siarry(2010). We consider in the continuation the Smairi aproach. In fact, this version takes the main mechanisms of TRIBES to which are added to treat multi-objective problems. In MO-TRIBES the swarm is divided into several under-swarms (tribes), of different size and evolves during the time. Every tribe is composed of a variable number of particles.

At the beginning, we start with one particle forming a tribe. After the first iteration a second particle is generated, which will, in turn, form a new tribe. In the next iteration, if the situation of both particles does not improve, every tribe creates two new particles: We form a new tribe containing four particles. However, if we are close to an optimal solution, the process is reversed and we begin to eliminate particles, even tribes (only the good tribes are capable of eliminating their worst elements).

4 OUR APPROACHES

The adaptability of MO-TRIBES shows an increase in complexity especially compared to a conventional MOPSO algorithm. Since the evaluation of the objective function is often complex especially in multi-objective case. We will propose, in this paper, to incorporate inheritance and approximation techniques for the objective function evaluation in order to decrease MO-TRIBES complexity. In their previous work, Sierra and Coello (Sierra and Coello, 2005) proposed to incorporate this technique into a MOPSO.

4.1 Fitness Inheritance

From the previous work of Sierra and Coello, we can conclude that the best inheritance technique is Linear Combination Based on Distances (LCBD) which gives better results and, at the same time, is adaptable to MO-TRIBES. In fact, LBCD does not consider the concept of speed that does not appear in Mo-Tribes. In fact, this technique is not the best but among the best ones that we have chosen because it perfectly fits Mo-Tribes.

We have specifically chosen this technique that takes into consideration the Euclidean distance d, pbest which denotes its best position and the leader. In this paper, we propose to simplify the calculation of the objective function and minimize the time of execution. We are going to integrate the technique chosen inheritance while basing our work on:

1. The leader xld,
2. The old position of particle xold,
3. The best position xpbest,
4. The new particle xnew,

Here is the algorithm of MO-TRIBES after modification:

```
Archive initialization
Swarm initialization
For each particle i, Determination of the state of the particle
Évaluate Objectif function
Insert leader in archives
While criterion is not verified
   Choice of the strategy of movement
   Update of the position
   While pi isn't a leader,
      For p% of particles applies inheritance
```

$$\vec{f}_i(t) = r_1 \vec{f}_i(t-1) + r_2 \vec{f}_{pbesti} + r_3 \vec{f}_{gbesti}$$

```
      End For
   End While
   Update the archive
   If n<NL
      Determination of the quality of the tribe
      Adaptation of the swarm
      Update archive Size
      Calculate NL
   End If
End While
```

Figure 2: Algorithm Mo-Tribes with Inheritance.

$$\text{With: } r_1 = \frac{d_1}{d_1 + d_2 + d_3}; \quad (1)$$

$$r_2 = \frac{d_2}{d_1 + d_2 + d_3}; \quad (2)$$

$$r_3 = \frac{d_3}{d_1 + d_2 + d_3}; \quad (3)$$

And d1 =distance (xnew; xold); d2 =distance (xnew; xpbest); d3 =distance (xnew; xld)

4.2 Fitness Approximation

A promising possibility when an evaluation is very time-consuming or expensive is not to evaluate every individual, but just estimate the quality of some of the individuals based on an approximate model of the fitness landscape.

```
Archive initialization
Swarm initialization
For each particle i, Determination of the state of the particle
Évaluate Objectif function
Insert leader in archives
While criterion is not verified
   Choice of the strategy of movement
   Update of the position
   While pi isn't a leader,
      For p% of particles applies approximation
        d(Xj, Xk) = min d(Xj, Xi=1..n)
```

$$\vec{f}_j(t) = \vec{f}_k(t)$$

```
        j=j+1
      End For
   End While
   Update the archive
     If n<NL
      Determination of the quality of the tribe
      Adaptation of the swarm
      Update archive Size
      Calculate NL
     End If
End While
```

Figure 3: Algorithm Mo-Tribes with Approximation.

Approximations techniques approximate individual fitness on the basis of the previously observed objective function values of neighboring individuals. There are many possible approximation models. In this approach we propose the technique in which the particle will take the objective value of the nearest particle without considering the leader of the swarm. Therefore, it is necessary to calculate the distance between each particle and the other particle members of the swarm, supposing that d is the Euclidean distance between two particles. We will test two versions of approximation:

- To take into consideration the totality of the swarm in the calculation of distance.
- To take into consideration each tribe like a separated swarm.

5 EXPERIMENTATIONS AND RESULTS

5.1 Test Functions

In order to compare the proposed techniques, we perform a study using eleven well-known test functions taken from the specialized literature on evolutionary algorithms (see table 6). These functions present diverse difficulties such as convexity, concavity, multimodality…etc. Moreover, we fix the maximal number of evaluations in the experimentations to 5e+4.

To study the fitness inheritance and approximation, at each iteration, we vary the number of particles on which we apply the inheritance and approximation (p%): 20%, 40%, 60% and 80%, knowing that these particles are not leaders. The purpose is to see the effects of variation of particle number inherited or approximated on convergence, diversity and time.

It should be noted that these particles are not introduced in the archive because their goals are fictitious values (not actual values).

5.2 Metrics of Comparison

For assessing the performance of the algorithms, there are many existent indicators measuring quality, diversity and convergence. We choose the combination of two binary indicators that was proposed in (Knowles,Thiele and Zitler, 2006): R indicator and hypervolume indicator. And time (The unit time measurement used is the second) is used as the metric of comparison to study the time variation.

5.3 Results

For both indicators, we present the summary of the results obtained. In each case, we present the mean of R indicator (table 4), hypervolume (table 3) and time (table 2 and 5) measures over 20 independent runs, the best results are shown in bold in the tables. We observe that:

- MO-TRIBES with Inheritance generally give better result than MO-TRIBES and MO-TRIBES with approximation.
- For S-ZDT2 and S-ZDT4, MO-TRIBES have the best values of the hypervolume and R-indicator.
- Moreover for the rest of functions MO-TRIBES with Inheritance has the better found front.
- For the ZDT family, except S-ZDT1, WFG family, R-DTLZ2 and SYMPART have the best time obtained by MO-TRIBES with Inheritance especially for percentages 40% and 60% of inherited particle.
- For S-ZDT1, S-DTLZ2 and R-ZDT4 the best time obtained with 20% to particles inheritance.

In addition, we conclude that TRIBES-Inheritance is very competitive as it supports both convergence and diversity. In fact, it gives the best time in comparison with MO-TRIBES and the two types of approximation (We don't show the result of hypervolume and R indicator for approximation because we conclude that the approximation does not improve the time).

For 11 test functions we conclude that the improvement is clear in terms of time, indicator R and hypervolume especially for 40% and 60% of inherited particles, the following table shows the number of functions improved compared to MO-TRIBES (see table 1).

We can conclude that this improvement results in the guidance of the particle during inheritance. However, approximation increases complexity because of the calculation of the distance between each current particle and the other particles of swarm (or the distance between each current particle and the other particles of swarm tribe of approximation 2).

The approximation can be complex compared to the actual calculation of the objective function, especially when the size of the search space (ie when the number of decision variables is limited the real calculation of objective fitness is less complex than approximation) is quite small and the number of objectives too.

Table 1: The number of functions improved compared to MO-TRIBES.

Nb of inherited particles	H	R	Time
40%	6	6	8
60%	7	7	7

Table 2: Results for the time (Mo-Tribes with Inheritance).

Test functions		MO-TRIBES with Inheritance				MO-TRIBES
		20%	40%	60%	80%	
S-ZDT1	Best	70,95	67,66	38,16	73,3	77,6
	Mean	78,02	75,5	**67,14**	83,91	81,92
	Worst	87,05	83,87	81,8	90,48	85,93
S-ZDT2	Best	8,68	9,09	10,76	11,47	40,59
	Mean	16,34	**14,16**	15,35	16,86	51,71
	Worst	28,2	21,62	25,32	21,61	63,63
S-ZDT4	Best	30,6	30,79	51,7	42,77	47,13
	Mean	44,78	**43,55**	58,24	54,94	58,22
	Worst	52,32	48,66	67,23	66,3	69,43
S-ZDT6	Best	20,74	13,59	32,68	32,24	26,18
	Mean	25,69	**18,62**	35,08	39,12	30,23
	Worst	28,34	23,32	36,99	47	36,52
DTLZ2	Best	215,32	212,07	211	220,07	210,35
	Mean	**218,8**	224,99	223,19	228,21	219,11
	Worst	225,57	232,27	230,67	240,11	224,44
R-DTLZ2	Best	43	48,72	47,81	17,2	33,41
	Mean	50,13	54,76	51,52	**20,98**	38,78
	Worst	58,34	64,73	54,34	28,69	44,84
R-ZDT4	Best	40,58	46,29	46,29	64,13	66,72
	Mean	**51,9**	58,35	56,95	66,93	72,3
	Worst	62,59	67,62	67,62	73,07	78,13
SYMPART	Best	74,25	72,25	70,83	51,83	85,73
	Mean	81,29	79,79	79,22	**71,39**	88,91
	Worst	85,87	85,86	88,37	86,55	93,08
OKA2	Best	92,07	94	94,73	96,57	89,86
	Mean	95,92	96,96	102,3	101,38	**95,11**
	Worst	99,15	100,5	119,16	105,72	100,83
WFG8	Best	7,29	11,3	10,48	14,69	22,12
	Mean	**11,24**	12,6	14,406	16,26	31,23
	Worst	16,37	14,62	16,51	17,61	47,86
WFG9	Best	144,89	137,1	167,77	152,33	168,66
	Mean	170,15	**159,41**	180,15	174,68	181,28
	Worst	191,25	187,5	204,58	205,13	189,39

Table 3: Result for Hypervolume(Mo-Tribes with Inheritance).

Test functions		MO-TRIBES with Inheritance				MO-TRIBES
		20%	40%	60%	80%	
S-ZDT1	Best	6.65e-3	6.36e-3	6.29e-3	5.51e-3	2.28e-2
	Mean	1.19e-2	1.75e-2	1.57e-2	**1.03e-2**	2.97e-2
	Worst	2.66e-2	2.80e-2	5.75e-2	2.36e-2	4.01e-2
S-ZDT2	Best	2.16e-3	5.24e-3	1.19e-3	2.25e-3	4.36e-3
	Mean	2.57e-2	2.98e-2	3.05e-2	3.13e-2	**2.19e-2**
	Worst	5.59e-2	5.84e-2	5.21e-2	5.40e-2	4.86e-2
S-ZDT4	Best	3.07e-2	3.61e-2	2.19e-2	3.27e-2	1.86e-2
	Mean	6.47e-2	5.49e-2	4.81e-2	4.94e-2	**3.18e-2**
	Worst	9.70e-2	8.71e-2	6.31e-2	6.06e-2	3.96e-2
S-ZDT6	Best	5.81e-2	5.29e-2	3.16e-2	2.86e-2	3.65e-2
	Mean	8.20e-2	8.71e-2	**4.80e-2**	6.03e-2	7.94e-2
	Worst	1.09e-1	1.17e-1	7.95e-2	1.781e-1	1.79e-1
S-DTLZ2	Best	1.64e-4	3.75e-4	1.08e-4	2.58e-4	5.56e-4
	Mean	**7.63e-4**	1.10e-3	1.11e-3	9.39e-4	1.49e-3
	Worst	1.61e-3	2.50e-3	1.79e-3	1.01e-3	3.22e-3
R-DTLZ2	Best	2.15e-2	1.99e-2	2.03e-2	1.95e-2	2.10e-2
	Mean	2.65e-2	2.55e-2	2.55e-2	**2.34e-2**	2.85e-2
	Worst	3.76e-2	3.17e-2	3.02e-2	2.84e-2	4.20e-2
R-ZDT4	Best	4.81e-3	2.93e-2	1.62e-2	2.65e-2	2.50e-2
	Mean	6.21e-3	3.12e-2	**2.08e-2**	3.87e-2	2.96e-2
	Worst	8.23e-3	3.42e-2	2.58e-2	4.75e-2	3.29e-2
SYMPART	Best	-3.96e-4	-4.77e-4	3.77e-4	3.44e-4	3.25e-4
	Mean	1.07e-4	**-4.33e-4**	5.32e-4	5.65e-4	5.59e-4
	Worst	5.29e-4	-2.89e-4	6.90e-4	7.76e-4	7.02e-4
OKA2	Best	-8.47e-4	-1.23e-3	-1.23e-3	-1.23e-3	2.94e-5
	Mean	-8.08e-4	-1.18e-3	-1.21e-3	**-1.23e-3**	2.0e-3
	Worst	1.81e-3	-8.75e-4	-1.19e-3	-1.20e-3	1.11e-2
WFG8	Best	-1.89e-1	-1.96e-1	-1.89e-1	-2.03e-1	-1.93e-1
	Mean	-1.86e-1	-1.89e-1	-1.88e-1	**-1.94e-1**	-1.90e-1
	Worst	-1.82e-1	-1.86e-1	-1.87e-1	-1.85e-1	-1.84e-1
WFG9	Best	-1.45e-1	-1.44e-1	-1.46e-1	-1.41e-1	-1.46e-1
	Mean	-1.42e-1	-1.41e-1	**-1.44e-1**	-1.40e-1	-1.44e-1
	Worst	-1.42e-1	-1.35e-1	-1.40e-1	-1.38e-1	-1.41e-1

Table 4: Result for R-Indicator (Mo-Tribes with Inheritance).

Test functions		MO-TRIBES with Inheritance				MO-TRIBES
		20%	40%	60%	80%	
S-ZDT1	Best	1.68e-3	2.08e-3	1.63e-2	1.47e-3	2.22e-2
	Mean	3.08e-3	4.59e-3	4.13e-3	**2.59e-3**	7.77e-3
	Worst	7.18e-3	8.66e-3	8.88e-3	6.42e-3	4.016e-2
S-ZDT2	Best	8.45e-4	2.45e-3	4.27e-4	8.17e-4	4.36e-3
	Mean	1.89e-2	2.25e-2	2.47e-2	3.07e-2	**1.52e-2**
	Worst	4.27e-2	4.35e-2	4.14e-2	4.21e-2	4.86e-2
S-ZDT4	Best	1.06e-2	1.24-2	7.55e-3	1.13e-2	1.86e-2
	Mean	2.20e-2	1.87e-2	**1.65e-2**	1.69e-2	9.91e-2
	Worst	3.28e-2	2.95e-2	2.15e-2	2.07e-2	3.96e-2
S-ZDT6	Best	2.55e-2	2.49e-2	1.42e-2	1.28e-2	3.65e-2
	Mean	3.58e-2	3.84e-2	**2.19e-2**	2.63e-2	3.47e-2
	Worst	4.73e-2	5.36e-2	3.63e-2	3.85e-2	1.79e-1
S-DTLZ2	Best	2.84e-5	7.43e-5	8.39e-5	6.63e-5	3.59e-4
	Mean	**1.55e-4**	1.61e-4	2.11e-4	1.85e-4	2.20e-4
	Worst	2.63e-4	2.32e-4	3.32e-4	2.78e-4	1.04e-4
R-DTLZ2	Best	3.56e-4	3.12e-4	3.54e-4	3.20e-4	3.82e-4
	Mean	6.69e-4	4.23e-4	4.52e-4	**3.79e-4**	6.64e-4
	Worst	2.46e-3	4.98e-4	5.97e-4	4.58e-4	2.44e-3
R-ZDT4	Best	2.40e-3	9.88e-3	5.41e-3	7.04e-3	2.50e-2
	Mean	1.93e-3	1.05e-2	**6.94e-3**	1.29e-2	9.96e-3
	Worst	2.60e-3	1.15e-2	8.66e-3	1.58e-2	3.29e-2
SYMPART	Best	8.46e-5	9.83e-5	1.27e-4	1.16e-4	1.1e-4
	Mean	**1.29e-4**	1.47e-4	1.81e-4	1.92e-4	1.90e-4
	Worst	1.8e-4	1.64e-4	2.35e-4	2.64e-4	2.38e-4
OKA2	Best	-1.06e-3	-1.06e-3	-1.06e-3	-1.06e-3	-1.06e-3
	Mean	-1.06e-3	**-1.05e-3**	**-1.05e-3**	**-1.05e-3**	4.88e-4
	Worst	-1.03e-3	-1.02e-3	-1.03e-3	-1.03e-3	6.70e-3
WFG8	Best	-2.33e-2	-2.46e-2	-2.32e-2	-2.58e-2	-2.41e-2
	Mean	-2.28e-2	-2.33e-2	-2.31e-2	**-2.41e-2**	-2.35e-2
	Worst	-2.21e-2	-2.27e-2	-2.31e-2	-2.27e-2	-2.24e-2
WFG9	Best	-1.88e-2	-1.88e-2	-1.91e-2	-1.83e-2	-1.92e-2
	Mean	-1.86e-2	-1.83e-2	**-1.89e-2**	-1.82e-2	-1.88e-2
	Worst	-1.85e-2	-1.73e-2	-1.81e-2	-1.78e-2	-1.84e-2

Table 5: Result for the Time (MO-TRIBES with Approximation 1 and Approximation 2).

Test functions		MO-TRIBES with Approximation 1				MO-TRIBES with Approximation 2				Mo-TRIBES
		20%	40%	60%	80%	20%	40%	60%	80%	
S-ZDT1	Best	95,57	116,27	131,27	162,16	88.66	96.37	97.26	125.91	77,6
	Mean	103,45	124,41	137,05	174,78	102,8	107,21	106,57	133,23	**81,92**
	Worst	108,26	130,25	145,5	182,34	115.94	118.7	115.82	147.58	85,93
S-ZDT2	Best	43,22	56,95	73,51	109,84	70.5	51.68	70.32	100.08	40,59
	Mean	**48,73**	60,74	82,38	125,32	83,41	64,19	76,04	131,59	51,71
	Worst	58,25	68,94	91,34	148,61	94.88	84.22	90.08	149.25	63,63
S-ZDT4	Best	45,77	55,52	73,23	110,32	41.95	52.79	75.08	100.63	47,13
	Mean	59	58,39	83,13	123,76	**52,54**	65,5	81,89	117,84	58,22
	Worst	64,87	63,36	97,44	133,6	67.33	78.91	91.97	133.5	69,43
S-ZDT6	Best	44,95	56,91	76,05	103,37	44.77	56.47	73.34	94	26,18
	Mean	53,01	61,68	80,45	114,45	51,68	59,4	78,6	104,5	**30,23**
	Worst	58,83	66,3	83,44	121,13	56.5	62.52	87.33	109.45	36,52
R-ZDT4	Best	52,68	84,34	80,91	110,98	85.44	53.57	67.3	98.41	66,72
	Mean	76,94	101,01	89,14	133,56	90,34	**61,76**	71,46	76,415	72,7
	Worst	89,16	114,26	93,16	159,24	95.86	79.63	74.16	107.37	78,13
OKA2	Best	105,37	123,15	134,8	155,24	107.58	122.25	122.34	146.19	89,86
	Mean	112,63	127,13	138,8	163,2	117,61	129,44	130,26	156,21	**95,11**
	Worst	120,26	135,63	142,88	169,72	126.8	134.41	139.1	159.88	100,83
Sympart	Best	95,57	91,08	96,15	106,93	83.37	103.29	89.62	133.44	85,73
	Mean	103,45	100,98	108,03	113,01	92,75	115,43	104,11	147,56	**88,91**
	Worst	109,68	109,91	121,73	117,44	103.69	125.55	115.83	155.74	93,08
WFG8	Best	44,79	51,15	69,73	96,54	39.54	48.2	62.84	91.88	22,12
	Mean	54,59	66,63	86,4	113,08	41,93	52,06	66,13	96,84	**31,23**
	Worst	65,33	79,36	97,07	119,96	43.97	53.36	72.04	107.65	47,86
WFG9	Best	146,02	130,5	105,48	111,96	231.27	241.14	241.82	266.41	168,66
	Mean	171,19	143,19	**135,09**	146,73	240,84	249,82	255,47	281,92	181,28
	Worst	196,71	161,47	168,97	165,24	247.89	258.33	263	302.85	189,39
S-DTLZ2	Best	236,39	232,85	253,63	269,73	227.72	236.24	256.27	279.44	210,35
	Mean	244,71	244,36	267,85	279,79	240,05	248,02	266,59	290,35	**219,11**
	Worst	253,61	252,36	279,72	291,76	249.57	257.39	281.17	299.66	224,44
R-DTLZ2	Best	79,33	100,77	91,2	121,5	44.38	110.77	96.9	112.87	33,41
	Mean	90	113,23	100,43	127,83	48,01	129,46	106,57	129,57	**40,83**
	Worst	99,44	128,28	115,54	142,94	51.86	137.46	119.08	145.97	53,88

Table 6: Proprieties of the test functions.

Test functions	number of objective function	Number of parameter	Geometry
OKA2	2	3	Concave
SYMPART	2	30	Concave
S_ZDT1	2	30	Convex
S_ZDT2	2	30	Concave
S_ZDT4	2	30	Convex
R_ZDT4	2	10	Concave
S_ZDT6	2	30	Concave
S_DTLZ2	3	30	Concave
R_DTLZ2	3	30	Concave
WFG8	3	24	Concave
WFG9	3	24	Concave

Noting that the archive size for 2 objective functions is 100, moreover 150 for 3 objective functions.

6 CONCLUSIONS

We have incorporated a fitness inheritance and approximation techniques into MO-TRIBES proposed previously by the authors. We studied the proposed approaches using several well-known multi-objective test functions.

We concluded that fitness inheritance give the best time than MO-TRIBES and MO-TRIBES with approximation. It also kept the same performances with MO-TRIBES with a simple improvement for several functions. As part of our ongoing work, we are going to study another inheritance technique and we try to improve time and performance in the same code, indeed we can propose to test other functions having a larger number of objectives to study the effect of the size of the search space for uses these techniques.

REFERENCES

Becerra, R, L., Quintero, L., V., S., Coello, C., A., C., 2008. *Knowledge Incorporation in Multi-objective Evolutionary Algorithms*, Multi-Objective Evolutionary Algorithms for Knowledge Discovery from Databases Studies in Computational Intelligence, Volume 98, 23-46.

Bhattacharya, M., 2013. Evolutionary Approaches to Expensive Optimisation. *IJARAI International Journal of Advanced Research in Artificial Intelligence,* Volume 2, No 3.

Clerc, M., 2006. *Particle Swarm Optimization*. International Scientific and Technical Encyclopedia, John Wiley & sons.

Cooren, Y., 2008. *Perfectionnement d'un algorithme adaptatif d'optimisation par essaim particulaire. Applications en génie médicale et en électronique*. PhD thesis, Université Paris 12.

Jin, Y., 2005. A Comprehensive Survey of Fitness Approximation in Evolutionary Computation, *Soft Computing-A Fusion of Foundations, Methodologies and Applications,* Volume 9, No 1, 3-12.

Knowles, J., Thiele, L., Zitler, L., 2006. A tutorial on the Performance Assessement of Stochastic Multi-objective Optimizers. *Computer Engineering and Networks Laboratory*, ETH Zurich, Switzerland.

Lim, D., Ong, Y, S., Jin, Y., Bernhard, Sendhoff., 2006. Trusted Evolutionary Algorithm. *Congress on Evolutionary Computation CEC'2006,* Vancouver, Canada, IEEE Service Center.

Montes, E,M., Dávila, L,M., Coello, C,A,C., 2007., A Preliminary Study of Fitness Inheritance in Evolutionary Constrained Optimization, *Nature Inspired Cooperative Strategies for Optimization (NICSO 2007) Studies in Computational Intelligence,* Volume 129, 2008, 1-14.

Ratle, A., 1998. Accelerating the convergence of Evolutionary Algorithms by Fitness Landscape Approximation. International Conference on Parallel Problem Solving from Nature (PPSN V), LNCS 3242, 786-793, Morgan Kaufmann Publishers.

Sierra, M, R., Coello, C., A., C., 2005. Fitness Inheritance in Multi-Objective Particle Swarm Optimization. *In IEEE Swarm Intelligence Symposium,* Pasadena, California, USA, IEEE Service Center, 116-123.

Smairi, N., Bouamama, S., Ghedira,K., Siarry, P., 2010. A new proposal for a multi-objective technique using tribes and tabu search. *ICINCO*, Portugal, 86-91.

Smith, R, E., Dike, B, A., Stegmann, S, A., 1995. Fitness Inheritance in Genetic Algorithms, *In SAC '95*, 345-350. ACM Press.