# A Variational Method to Remove the Combination of Poisson and Gaussian Noises

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Abstract: In this paper, we propose a method to remove noise in digital images. Our method is based on the well-known variational approach. The novelty of proposed method consists in removing of mixed Poisson-Gaussian noise. This is the actual problem for many types of real raster images, for example, biomedical images. Our method is developed with the goal to combine two famous models: ROF for removing Gaussian noise and modified ROF for removing Poisson noise. As a result, our proposed method can be also applied to remove Gaussian or Poisson noise separately. We develop procedure to perform noise removal with automatically evaluated parameters to get the best result of denoising.

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## 1 INTRODUCTION

Digital image is a type of a signal that is obtained from a real analogous signal by discretization and quantization. Many digital devices can create digital images, such as digital camera, X-ray scanner, and so on. In practice, these devices can give unexpected effects. One of them is noise. Noise reduces image quality and efficiency of image processing.

The problem of noise removal from digital images is very actual today. In order to solve this problem, many different strong approaches were already developed.

The variational approach (Chan, 2005, Burger, 2008, Chambolle, 2009, Xu, 2014, Rankovic, 2012, Lysaker, 2006, Li, 2006, Zhu, 2012, Tran, 2012, Getreuer, 2012, Caselles, 2011, Rudin, 1992, Chen, 2013) is well-known and very promising.

This concept was pioneered by Rudin (1992). He proposed the total variation to solve many problems in image processing. Especially, he built a model for denoising of digital images. This model is referred to by ROF (Rudin, 1992, Chen, 2013).

It is known, ROF model is used to remove only Gaussian noise. However, another important type like Poisson noise is usually presented in digital images. For example, this noise appears in medical X-ray images. In order to remove this noise, Le T. (2007) developed so called modified ROF model.

Gaussian and Poisson noises are popular separately, but their combination is also important (Luisier, 2011). This combination of noises usually appears in biomedical images, for example, in electronic microscope images (Jezierska, 2011, 2012).

Nevertheless, ROF and modified ROF models ineffectively treat this combination. ROF model gives priority to Gaussian noise, but modified ROF model gives it to Poisson noise.

Our goal is to combine ROF model (for Gaussian noise) and modified ROF model (for Poisson noise) to create new model that can treat this combination effectively. Our model will treat this combination with considering proportion of noise between them.

In experiments, we used initial images and added noise into them. We performed denoising of digital images by proposed method and other methods, such as ROF model, median filter (Wang 2012) and Wiener filter (Abe 2012). In order to evaluate an image quality after denoising, we used well-known criteria MSE (Mean Square Error), PSNR (Peak Signal-to-Noise Ratio) and SSIM (Structure SIMilarity) (Wang, 2004, 2006). We give priority to PSNR, because it is most popular and used to evaluate the quality of restored signal in signal processing in general, and in image processing, especially.



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# 2 DENOISING MODEL FOR MIXED POISSON-GAUSSIAN NOISE

Let in $R^2$ space a bounded domain $\Omega \subset R^2$ be given. Let us call functions $u(x,y) \in R^2$ and $v(x,y) \in R^2$, respectively, ideal (without noise) and observed images (noisy), where $(x,y) \in \Omega$.

If the function $u$ is smooth, then its total variation is defined by

$$V_T[u] = \int_\Omega |\nabla u| \, dxdy ,$$

where $\nabla u = (u_x, u_y)$ is a gradient (nabla operator), $u_x = \partial u / \partial x$, $u_y = \partial u / \partial y$, $|\nabla u| = \sqrt{u_x^2 + u_y^2}$. In this paper, we only consider function $u$ that always has limited total variation $V_T[u] < \infty$.

## 2.1 Denoising Model

According to results (Chang, 2005, Burger, 2008, Rudin, 1992, Chen, 2013, Scherzer, 2009), image smoothness is characterised by the total variation. The total variation of noisy image is always greater than the total variation of smoothed image.

When Rudin solved the problem $V_T[u] \to \min$, he used this characteristic and assumed, that Gaussian noise variance is fixed by the additional constraint

$$\int_\Omega (u-v)^2 \, dxdy = const$$

He proposed the ROF model to remove Gaussian noise from an image

$$u^* = \arg\min_u \left( \int_\Omega |\nabla u| \, dxdy + \frac{\lambda}{2} \int_\Omega (u-v)^2 \, dxdy \right),$$

where $\lambda > 0$ is a Lagrange multiplier.

Le T. (2007) proposed another model to remove Poisson noise based on ROF model:

$$u^* = \arg\min_u \left( \int_\Omega |\nabla u| \, dxdy + \beta \int_\Omega (u - v\ln(u)) dxdy \right),$$

where $\beta$ is a regularization coefficient. We call it a modified ROF model for Poisson noise.

In order to develop the denoising model for mixed noise, we also solve the problem based on the smooth characteristic of the total variation

$$V_T[u] \to \min .$$

And we also define a constrained condition. We assume that with given image $u$, the mixed noise in image is fixed too (Poisson noise is unchangeable, and Gaussian noise only depends on noise variance):

$$\int_\Omega \ln(p(v \mid u)) dxdy = const , \quad (1)$$

where $p(v \mid u)$ is a conditional probability.

Let us consider Gaussian noise. Its probability density function (*p.d.f.*) is

$$p_1(v \mid u) = \exp\left( -\frac{(v-u)^2}{2\sigma^2} \right) / (\sigma\sqrt{2\pi}) .$$

For Poisson noise the *p.d.f.* is

$$p_2(v \mid u) = \frac{\exp(-u) u^v}{v!} .$$

We have to note that intensity levels of image colours are integer (for example, the intensity interval for an 8-bit grayscale image is from 0 to 255), so we regard $u$ as an integer value, but this will ultimately be unnecessary (Le 2007).

In order to treat combination of Gaussian and Poisson noises, we assume the following linear combination

$$\ln(p(v \mid u)) = \lambda_1 \ln(p_1(v \mid u)) + \lambda_2 \ln(p_2(v \mid u)) ,$$

where $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_1 + \lambda_2 = 1$.

According to (1), we obtain the denoising problem with constrained condition as following:

$$\begin{cases} u^* = \arg\min_u \int_\Omega |\nabla u| \, dxdy \\ \int_\Omega \left( \frac{\lambda_1}{2\sigma^2} (v-u)^2 + \lambda_2 (u - v\ln(u)) \right) dxdy = \kappa , \end{cases}$$

where $\kappa$ is a constant value.

We can transform this constrained optimization problem to the unconstrained optimization problem by using Lagrange functional

$$L(u,\tau) = \int_\Omega |\nabla u| \, dxdy + \tau \left( \frac{\lambda_1}{2\sigma^2} \int_\Omega (v-u)^2 \, dxdy + \lambda_2 \int_\Omega (u - v\ln(u)) dxdy - \kappa \right)$$

to find

$$(u^*, \tau^*) = \arg\min_{u,\tau} L(u,\tau) , \quad (2)$$

where $\tau > 0$ is a Lagrange multiplier.

This is our proposed model to remove mixed Poisson-Gaussian noise from digital image. We have to notice that, if $\lambda_1 = 0$ and $\beta = \lambda_2 \tau$, we obtain modified ROF model for removing Poisson noise. If $\lambda_2 = 0$ and $\lambda = \lambda_1 / (2\sigma^2)$, then we obtain ROF model for removing Gaussian noise. In the case of $\lambda_1 > 0, \lambda_2 > 0$ we get the model for removing mixed Poisson-Gaussian noise.

## 2.2 Model Discretization

In order to solve the problem (2), we can use the Lagrange multipliers method (Zeidler, 1985, Rubinov, 2003, Gill, 1974).

However, in this paper, we will solve it by using the Euler-Lagrange equation (Zeidler, 1985).

Let function $f(x,y)$ be defined in limited domain $\Omega \subset R^2$ and be the second-order continuous differentiable one by $x$ and $y$ for $(x,y) \in \Omega$.

We consider the special convex functional $F(x,y,f,f_x,f_y)$, where $f_x = \partial f / \partial x$, $f_y = \partial f / \partial y$.

The solution of the optimization problem

$$\int_{\Omega} F(x,y,f,f_x,f_y)dxdy \to \min$$

satisfies the following Euler-Lagrange equation

$$F_f(x,y,f,f_x,f_y) - \frac{\partial}{\partial x} F_{f_x}(x,y,f,f_x,f_y) - \frac{\partial}{\partial y} F_{f_y}(x,y,f,f_x,f_y) = 0,$$

where

$$F_f = \partial F / \partial f, \; F_{f_x} = \partial F / \partial f_x, \; F_{f_y} = \partial F / \partial f_y.$$

We use the result above to solve the problem (2). The solution of the problem (2) is given by the following Euler-Lagrange equation:

$$-\frac{\lambda_1}{\sigma^2}(v-u) + \lambda_2(1-\frac{v}{u}) - \mu \frac{\partial}{\partial x}\left(\frac{u_x}{\sqrt{u_x^2+u_y^2}}\right) - \mu \frac{\partial}{\partial y}\left(\frac{u_y}{\sqrt{u_x^2+u_y^2}}\right) = 0, \quad (3)$$

where $\mu = 1/\tau$. We can reduce (3) to

$$\frac{\lambda_1}{\sigma^2}(v-u) - \lambda_2(1-\frac{v}{u}) + \mu \frac{u_{xx}u_y^2 - 2u_x u_y u_{xy} + u_x^2 u_{yy}}{(u_x^2+u_y^2)^{3/2}} = 0, \quad (4)$$

where

$$u_{xx} = \frac{\partial^2 u}{\partial x^2}, \; u_{yy} = \frac{\partial^2 u}{\partial y^2},$$

$$u_{xy} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = u_{yx}.$$

In order to discretize the equation (4), we add an artificial time parameter and consider the function $u = u(x,y,t)$. Then the equation (4) relates to the following diffusion equation

$$u_t = \frac{\lambda_1}{\sigma^2}(v-u) - \lambda_2(1-\frac{v}{u}) + \mu \frac{u_{xx}u_y^2 - 2u_x u_y u_{xy} + u_x^2 u_{yy}}{(u_x^2+u_y^2)^{3/2}}, \quad (5)$$

where $u_t = \partial u / \partial t$.

We can write the discretized form of the equation (5) as following:

$$u_{ij}^{k+1} = u_{ij}^k + \xi\left(\frac{\lambda_1}{\sigma^2}(v_{ij} - u_{ij}^k) - \lambda_2(1-\frac{v_{ij}}{u_{ij}^k}) + \mu\varphi_{ij}^k\right), \quad (6)$$

where

$$\varphi_{ij}^k = \frac{\nabla_{xx}(u_{ij}^k)(\nabla_y(u_{ij}^k))^2}{((\nabla_x(u_{ij}^k))^2 + (\nabla_y(u_{ij}^k))^2)^{3/2}} + \frac{-2\nabla_x(u_{ij}^k)\nabla_y(u_{ij}^k)\nabla_{xy}(u_{ij}^k) + (\nabla_x(u_{ij}^k))^2\nabla_{yy}(u_{ij}^k)}{((\nabla_x(u_{ij}^k))^2 + (\nabla_y(u_{ij}^k))^2)^{3/2}},$$

$$\nabla_x(u_{ij}^k) = \frac{u_{i+1,j}^k - u_{i-1,j}^k}{2\Delta x}, \; \nabla_y(u_{ij}^k) = \frac{u_{i,j+1}^k - u_{i,j-1}^k}{2\Delta y},$$

$$\nabla_{xx}(u_{ij}^k) = \frac{u_{i+1,j}^k - 2u_{ij}^k + u_{i-1,j}^k}{(\Delta x)^2},$$

$$\nabla_{yy}(u_{ij}^k) = \frac{u_{i,j+1}^k - 2u_{ij}^k + u_{i,j-1}^k}{(\Delta y)^2},$$

$$\nabla_{xy}(u_{ij}^k) = \frac{u_{i+1,j+1}^k - u_{i+1,j-1}^k - u_{i-1,j+1}^k + u_{i-1,j-1}^k}{4\Delta x \Delta y},$$

$$u_{0j}^k = u_{1j}^k; \; u_{N_1+1,j}^k = u_{N_1,j}^k; \; u_{i0}^k = u_{i1}^k; \; u_{i,N_2+1}^k = u_{i,N_2}^k;$$

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$$i=1,...,N_1;\ j=1,...,N_2;$$

$$k=0,1,...,K;\ \Delta x=\Delta y=1;\ 0<\xi<1.$$

Here $K$ is enough great number. In this paper, we use $K=500$. For initial condition, we have

$$u^0_{ij}=v_{ij};\ i=1,...,N_1;\ j=1,...,N_2.$$

## 2.3 Finding Optimal Parameters

We can use the procedure (6) to perform image denoising. In this procedure, values of parameters $\lambda_1,\lambda_2,\mu,\sigma$ need to be given. In some cases, we have to define these parameters to perform image denoising automatically. Then parameters $\lambda_1,\lambda_2,\mu$ in process (6) need to be changed into $\lambda^k_1,\lambda^k_2,\mu^k$ for each step $k$. So we obtain new procedure that allows us to calculate values of these parameters automatically in iteration steps.

### 2.3.1 Optimal Parameters $\lambda_1$ and $\lambda_2$

Let $(u,\tau)$ be a solution of the problem (2). Then we get the condition

$$\frac{\partial L(u,\tau)}{\partial u}=0.$$

This condition gives us the optimal parameters $\lambda_1,\lambda_2$:

$$\lambda_1=\frac{\int_\Omega(1-\frac{v}{u})dxdy}{\frac{1}{\sigma^2}\int_\Omega(v-u)dxdy+\int_\Omega(1-\frac{v}{u})dxdy},$$

$$\lambda_2=1-\lambda_1.$$

Its discretized form is

$$\lambda^k_1=\frac{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(1-\frac{v_{ij}}{u^k_{ij}})}{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(\frac{v_{ij}-u^k_{ij}}{\sigma^2}+1-\frac{v_{ij}}{u^k_{ij}})},$$

$$\lambda^k_2=1-\lambda^k_1,$$

where $k=0,1,...,K$.

### 2.3.2 Optimal Parameter $\mu$

In order to find an optimal parameter $\mu$, we multiply (3) by $(u-v)$ and integrate by parts over $\Omega$. Finally, we obtain the formula to find the optimal parameter $\mu$:

$$\mu=\frac{\int_\Omega(-\frac{\lambda_1}{\sigma^2}(u-v)^2-\lambda_2\frac{(u-v)^2}{u})dxdy}{\int_\Omega(\sqrt{u_x^2+u_y^2}-\frac{u_xv_x+u_yv_y}{\sqrt{u_x^2+u_y^2}})dxdy}.$$

Its discretized form is

$$\mu^k=\frac{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(-\frac{\lambda^k_1}{\sigma^2}(u^k_{ij}-v_{ij})^2-\lambda^k_2\frac{(u^k_{ij}-v_{ij})^2}{u^k_{ij}})}{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\eta^k_{ij}},$$

where

$$\eta^k_{ij}=\sqrt{(\nabla_x(u^k_{ij}))^2+(\nabla_y(u^k_{ij}))^2}-\frac{\nabla_x(u^k_{ij})\nabla_x(v_{ij})+\nabla_y(u^k_{ij})\nabla_y(v_{ij})}{\sqrt{(\nabla_x(u^k_{ij}))^2+(\nabla_y(u^k_{ij}))^2}},$$

$$\nabla_x(u^k_{ij})=\frac{u^k_{i+1,j}-u^k_{i-1,j}}{2\Delta x},\ \nabla_y(u^k_{ij})=\frac{u^k_{i,j+1}-u^k_{i,j-1}}{2\Delta y},$$

$$\nabla_x(v^k_{ij})=\frac{v^k_{i+1,j}-v^k_{i-1,j}}{2\Delta x}v,\ \nabla_y(v^k_{ij})=\frac{v^k_{i,j+1}-v^k_{i,j-1}}{2\Delta y},$$

$$u^k_{0j}=u^k_{1j};\ u^k_{N_1+1,j}=u^k_{N_1,j};\ u^k_{i0}=u^k_{i1};\ u^k_{i,N_2+1}=u^k_{i,N_2};$$

$$v_{0j}=v_{1j};\ v_{N_1+1,j}=v_{N_1,j};\ v_{i0}=v_{i1};\ v_{i,N_2+1}=v_{i,N_2};$$

$$i=1,...,N_1;\ j=1,...,N_2;\ k=0,1,...,K;\ \Delta x=\Delta y=1.$$

### 2.3.3 Optimal Parameter $\sigma$

In order to evaluate this parameter $\sigma$, we use the result of Immerker (1996):

$$\sigma=\frac{\sqrt{\pi/2}}{6(N_1-2)(N_2-2)}\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}|u_{ij}*\Lambda|,\ \text{where}$$

$$\Lambda=\begin{pmatrix}1&-2&1\\-2&4&-2\\1&-2&1\end{pmatrix}\ \text{is the mask of an image.}$$

Operator * is a convolution operator, where

$$u_{ij}*\Lambda=u_{i-1,j-1}\Lambda_{33}+u_{i,j-1}\Lambda_{32}+u_{i+1,j-1}\Lambda_{31}+u_{i-1,j}\Lambda_{23}+$$

$$u_{ij}\Lambda_{22}+u_{i+1,j}\Lambda_{21}+u_{i-1,j+1}\Lambda_{13}+u_{i,j+1}\Lambda_{12}+u_{i+1,j+1}\Lambda_{11},$$

$$i=1,...,N_1;\ j=1,...,N_2;$$

$u_{ij} = 0$ , if $i = 0$ , or $j = 0$ , or $i = N_1 + 1$ ,

or $j = N_2 + 1$ .

We have to notice, that the parameter $\sigma$ is just evaluated at first time of the iteration process.

## 2.4 Image Quality Evaluation

In order to evaluate image quality after denoising, we use criteria MSE (Mean Square Error), PSNR (Peak Signal-to-Noise Ratio) and SSIM (Structure SIMilarity) (Wang 2004, 2006):

$$Q_{MSE} = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (u_{ij} - v_{ij})^2 ,$$

$$Q_{PSNR} = 10 \lg \left( \frac{N_1 N_2 L^2}{\sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (u_{ij} - v_{ij})^2} \right),$$

$$Q_{SSIM} = \frac{(2\bar{u}\,\bar{v} + C_1)(2\sigma_{uv} + C_2)}{(\bar{u}^2 + \bar{v}^2 + C_1)(\sigma_u^2 + \sigma_v^2 + C_2)} ,$$

where

$$\bar{u} = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} u_{ij} , \ \bar{v} = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} v_{ij} .$$

$$\sigma_u^2 = \frac{1}{N_1 N_2 - 1} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (u_{ij} - \bar{u})^2 ,$$

$$\sigma_v^2 = \frac{1}{N_1 N_2 - 1} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (v_{ij} - \bar{v})^2 ,$$

$$\sigma_{uv} = \frac{1}{N_1 N_2 - 1} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (u_{ij} - \bar{u})(v_{ij} - \bar{v}) ,$$

$$C_1 = (K_1 L)^2, C_2 = (K_2 L)^2; K_1 << 1; K_2 << 1 .$$

For example, $K_1 = K_2 = 10^{-6}$ , $L$ is image intensity, where, for example, $L = 2^8 - 1 = 255$ for an 8-bit greyscale images.

The greater value $Q_{PSNR}$ , the better image quality. If $Q_{PSNR}$ is between 20 and 25, then an image quality is acceptable, for example, for the wireless transmission (Thomos 2006).

$Q_{SSIM}$ is used to evaluate image quality by comparing similarity of both images. Its value is between -1 and 1. The greater value $Q_{SSIM}$ , the better image quality.

$Q_{MSE}$ is a criteria to evaluate the difference between two images. $Q_{MSE}$ is mean-squared error. The lower value $Q_{MSE}$ , the better restoration result. The value of $Q_{MSE}$ also relates to the value of $Q_{PSNR}$ .

## 2.5 Image Sample Initialization

In experiment with artificial image, we use an image with the size of 300x300 pixels. This image has six vertical bars (Fig. 1a). Bar grey level intensities are 110, 130 and 160, respectively, where numbers of pixels are same (30.000). We zoom, crop and show the part of the original image under processing (Fig. 1b – 1f).

First, we create the noisy image by adding Gaussian noise (Fig. 1c) and second, create noisy image by adding Poisson noise (Fig. 1d).

We want to generate a noisy image, the quality of which is very low, but we cannot control the Poisson noise intensity. So, we just only control the variance of Gaussian noise. In order to calculate proportion between intensities of Gaussian and Poisson noises, we calculate the variance of Poisson noise. The value of variance of Gaussian noise is calculated via Poisson noise variance. Let the variance of Gaussian noise be four times greater than the variance of Poisson noise.

First, let us consider Poisson noise. Its distribution is $p_2(v \mid u)$ , value of the variance of Poisson noise is $\sigma_2 = \sqrt{u_{ij}}$ , respectively, with $u_{ij}$ at every pixel $(i, j)$ of image, where $i = 1,..., N_1; j = 1,..., N_2$ . We denote this Poisson noisy image as $v^{(2)}$ . Obviously, intensity value of $v^{(2)}$ ought to be between 0 and 255. If the intensity value of some pixels are out of this interval, they need to be reset to intensity value of respective pixel of the original image $u$ , that means $v^{(2)}{}_{ij} = u_{ij}$ .

In this case, number of them is zero. The variance of Poisson noise can be calculated as average value $\sigma_2 = (\sqrt{110} + \sqrt{130} + \sqrt{160}) = 11.5130$ , because this image has three intensity levels and their numbers of pixels are identical.

Now, we consider Gaussian noise. Its variance need to be 46.052 (because we explained above, variance of Gaussian noise is four times over variance of Poisson noise). We denote this Gaussian noisy image as $v^{(1)}$ . As above case, intensity value of $v^{(1)}$ also need to be between 0 and 255. In this case, there are 1063 pixels out of this interval, respectively 1.2% of all image pixels.

We create resulting noisy image (Fig. 1e) by combining first noisy and second noisy images with proportion 0.6 for Gaussian noisy image $v^{(1)}$ and 0.4 for Poisson noisy image $v^{(2)}$.

This means $v = 0.6v^{(1)} + 0.4v^{(2)}$. Hence:

$$\lambda_1 / \lambda_2 = 46.052 \cdot 0.6 / 11.513 \cdot 0.4 =$$

$$27.63 / 4.6 = 6/1.$$

As a result: $\lambda_1 = 6/7 = 0.8571$, $\lambda_2 = 1/7 = 0.1429$.

Values of $Q_{MSE}$, $Q_{PSNR}$ and $Q_{SSIM}$ of noisy image are respectively 718.8782, 19.5643, and 0.1036. The value of $Q_{PSNR}$ of noisy image is lower than 20. That means quality of noisy image is very low and it cannot be used, for example, for wireless transmission.

## 2.6 Experiments

In order to test our model, we consider the following cases for above sample image: parameters are $\lambda_1$=0.2, $\lambda_2$=0.8 (worse restoration); $\lambda_1$=$\lambda_2$=0.5 (better restoration); $\lambda_1$=0.8571, $\lambda_2$=0.1429 (best restoration) and with automatically evaluated $\lambda_1$=0.8102, $\lambda_2$=0.1898. Result of denoising with $\lambda_1$=0.8571, $\lambda_2$=0.1429 is given on Fig. 1f.

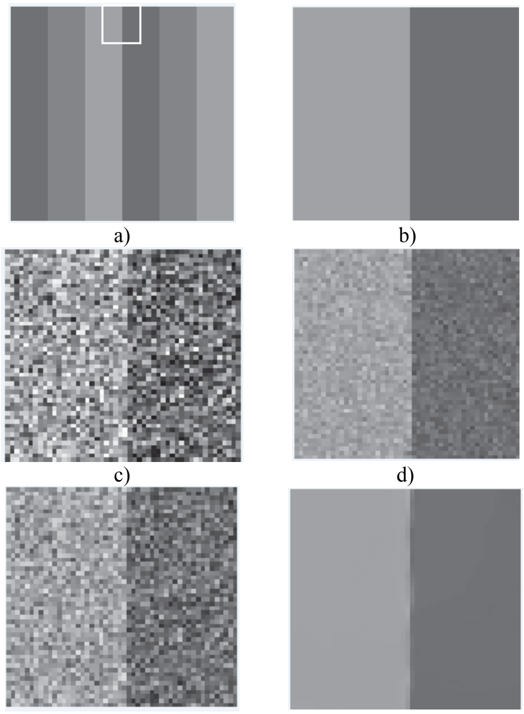


Figure 1: Result of noise initialization and denoising: a) original image, b) cropped image, c) with Gaussian noise, d) with Poisson noise, e) with mixed noise, f) after denoising.

We also compare the result of our model with other noise removal methods: ROF, Wiener filter, median filter. Results are given in Table 1.

Table 1 shows, the proposed method with automatically evaluated parameters effectively removes noise and gives us the high quality images to use them, for example, for wireless transmission.

Fig. 2 shows vertical cut of grey level intensities of original image, noisy image and denoised image.

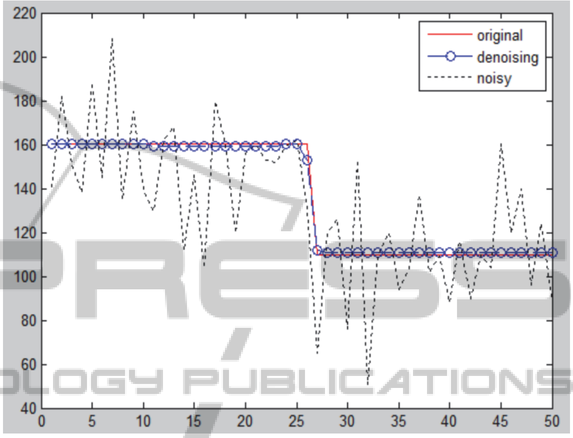


Figure 2: Intensity of original, denoised and noisy images.

Table 1: Quality comparison of noise removal methods for the artificial image.

| | $Q_{PSNR}$ | $Q_{SSIM}$ | $Q_{MSE}$ |
|---|---|---|---|
| Noisy | 19.5643 | 0.1036 | 718.8782 |
| ROF | 35.1284 | 0.9130 | 19.9635 |
| Median | 31.4844 | 0.7797 | 46.1996 |
| Wiener | 30.1502 | 0.6018 | 62.8146 |
| Proposed method with $\lambda_1$=0.2, $\lambda_2$=0.8, $\mu$= 0.1140, $\sigma$= 46.0520 | 29.1325 | 0.5933 | 79.4014 |
| Proposed method with $\lambda_1$=$\lambda_2$=0.5, $\mu$= 0.1429, $\sigma$= 46.0520 | 37.0462 | 0.9453 | 12.8370 |
| Proposed method with $\lambda_1$=0.8571, $\lambda_2$=0.1429, $\mu$= 0.4738, $\sigma$= 46.0520 | 42.8237 | 0.9902 | 3.3940 |
| Proposed method with evaluated parameters $\lambda_1$=0.8102, $\lambda_2$=0.1898, $\mu$= 0.3846, $\sigma$= 45.4523 | 42.7795 | 0.9900 | 3.4287 |

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We also use another example to test our model in the case of processing a real image. In this case, we use an image of human skull (Nick 2009) with the size 300x300 pixel (Fig. 3a). We add Gaussian noise (Fig. 3b), Poisson noise (Fig. 3c) and combine two these images to make final noisy image (Fig. 3d).

Variance of Gaussian noise is 4 times over variance of Poisson noise and proportion of combination is 0.5 and 0.5, and $\lambda_1$=0.75, $\lambda_2$=0.25. Result of denoising is shown in Fig. 3e, Fig. 3f. In this case, the variance of Poisson noise is 10.0603, the variance of Gaussian noise is 40.2412. The number of pixels intensities of which is out of interval 0 and 255 for Poisson noise is 5 (respectively 0.0056%), and for Gaussian noise is 5780 (respectively 6.42%).

The Table 2 shows the result of denoising for real image in both case: given parameters and automatically evaluated parameters.

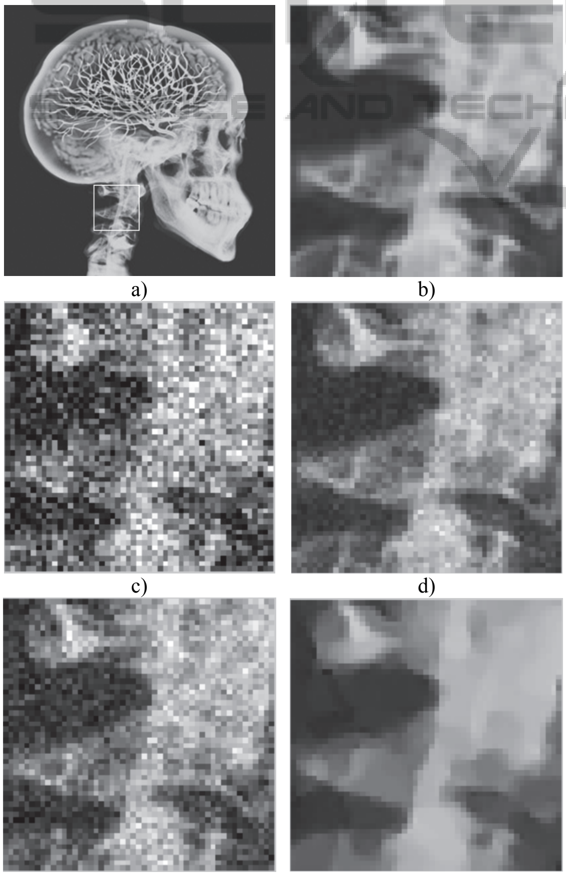

Figure 3: Denoising of real image: a) original image, b) cropped image, c) with Gaussian noise, d) with Poisson noise, e) with mixed noise, f) after denoising.

We have to notice, that in the case of the real image, the value of $Q_{PSNR}$ of denoising for given ideal parameters is better, than the value of $Q_{PSNR}$ of denoising for automatically evaluated parameters, but the value of $Q_{SSIM}$ is inversed.

Table 2: Quality comparison of noise removal methods for real image of human skull.

| | $Q_{PSNR}$ | $Q_{SSIM}$ | $Q_{MSE}$ |
|---|---|---|---|
| Noisy | 21.4168 | 0.4246 | 427.9526 |
| Proposed method with<br>$\lambda_1$=0.75,<br>$\lambda_2$=0.25,<br>$\mu$ = 0.1,<br>$\sigma$ = 40.2412. | 27.2808 | 0.8157 | 121.6189 |
| Proposed method with evaluated parameters<br>$\lambda_1$=0.8095,<br>$\lambda_2$=0.1905,<br>$\mu$ = 0.0970,<br>$\sigma$ = 38.2310. | 27.2567 | 0.8383 | 122.2941 |

# 3 CONCLUSIONS

In this paper, we proposed the approach to remove combination of Poisson and Gaussian noises (mixed noise). This method is based on variational approach.

The result of denoising depends on parameters, especially on coefficients of linear combination $\lambda_1$ and $\lambda_2$. We can specify values of parameters or these values can be automatically evaluated. In order to apply this model to real image, we need to use the proposed method with automatically evaluated parameters.

The proposed method can be applied to remove separate Gaussian or Poisson noise (respectively ROF model and modified ROF model for Poisson noise), or mixed Poisson-Gaussian noise as well.

We also can use this variational approach to remove other kinds of noise, such as noise of magnetic resonance images (MRI), ultrasonogram, etc.

# REFERENCES

Abe C., Shimamura T., 2012, 'Iterative Edge-Preserving adaptive Wiener filter for image denoising', *ICCEE*, vol. 4, no. 4, pp. 503-506.

Burger M., 2008, *Level set and PDE based reconstruction methods in imaging*, Springer.

Caselles V., Chambolle A., Novaga M., 2011, *Handbook of mathematical methods in imaging*, Springer.

SCITEPRESS SCIENCE AND TECHNOLOGY PUBLICATIONS

Chan T. F., Shen J., 2005, *Image processing and analysis: Variational, PDE, Wavelet, and stochastic methods*, SIAM.

Chambolle A., 2009, 'An introduction to total variation for image analysis', *Theoretical foundations and numerical methods for sparse recovery*, vol. 9, pp. 263-340.

Chen K., 2013, 'Introduction to variational image processing models and application', *International journal of computer mathematics*, vol. 90, no. 1, pp. 1-8.

Getreuer P., 2012, 'Rudin-Osher-Fatemi total variation denoising using split Bregman'. *IPOL 2012*, 'http://www.ipol.im/pub/art/2012/g-tvd/'.

Gill P. E., Murray W., 1974, *Numerical methods for constrained optimization*, Academic Press Inc.

Immerker J., 1996, 'Fast noise variance estimation', *Computer vision and image understanding*, vol. 64, no.2, pp. 300-302.

Jezierska A., 2012, 'Poisson-Gaussian noise parameter estimation in fluorescence microscopy imaging', *IEEE International Symposium on Biomedical Imaging 9th*, pp. 1663-1666.

Jezierska A., 2011, 'An EM approach for Poisson-Gaussian noise modelling', *EUSIPCO 19th*, vol. 62, is. 1, pp. 13-30.

Le T., Chartrand R., Asaki T. J., 2007, 'A variational approach to reconstructing images corrupted by Poisson noise', *Journal of mathematical imaging and vision*, vol. 27, is. 3, pp. 257-263.

Li F., Shen C., Pi L., 2006, 'A new diffusion-based variational model for image denoising and segmentation', *Journal mathematical imaging and vision*, vol. 26, is. 1-2, pp. 115-125.

Luisier F., Blu T., Unser M., 2011, 'Image denoising in mixed Poisson-Gaussian noise', *IEEE transaction on Image processing*, vol. 20, no. 3, pp. 696-708.

Lysaker M., Tai X., 2006, 'Iterative image restoration combining total variation minimization and a second-order functional', *International journal of computer vision*, vol. 66, pp. 5-18.

Nick V., 2009, Getty images, 'http://well.blogs.nytimes.com/2009/09/16/what-sort-of-exercise-can-make-you-smarter/'.

Rankovic N., Tuba M., 2012, 'Improved adaptive median filter for denoising ultrasound images', *Advances in computer science,* WSEAS ECC'12, pp. 169-174.

Rubinov A., Yang X., 2003, *Applied Optimization: Lagrange-type functions in constrained non-convex optimization*, Springer.

Rudin L. I., Osher S., Fatemi E., 1992, 'Nonlinear total variation based noise removal algorithms', *Physica D*. vol. 60, pp. 259-268.

Scherzer O., 2009, Variational *methods in Imaging*, Springer.

Thomos N., Boulgouris N. V., Strintzis M. G., 2006, 'Optimized Transmission of JPEG2000 streams over Wireless channels', *IEEE transactions on image processing*, vol. 15, no.1, pp .54-67.

Tran M. P., Peteri R., Bergounioux M., 2012, 'Denoising 3D medical images using a second order variational model and wavelet shrinkage', *Image analysis and recognition*, vol. 7325, pp. 138-145.

Wang C., Li T., 2012, 'An improved adaptive median filter for Image denoising', *ICCEE*, vol. 53, no. 2.64, pp. 393-398.

Wang Z., 2004, 'Image quality assessment: From error visibility to structural similarity', *IEEE transaction on Image processing*, vol. 13, no. 4, pp. 600-612.

Wang Z., Bovik A. C., 2006, *Modern image quality assessment*, Morgan & Claypool Publisher.

Xu J., Feng X., Hao Y., 2014, 'A coupled variational model for image denoising using a duality strategy and split Bregman', *Multidimensional systems and signal processing*, vol. 25, pp. 83-94.

Zhu Y., 2012, 'Noise reduction with low dose CT data based on a modified ROF model', *Optics express*, vol. 20, no. 16, pp. 17987-18004.

Zeidler E., 1985, *Nonlinear functional analysis and its applications: Variational methods and optimization*, Springer.

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