A Novel Multiband Filter Design based on Ring Resonators and DSP Approach

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Abstract: This paper proposes a novel ring resonator based optical filter that has an outstanding multi narrow band response due to adopting quasi structures such as Thue-Morse sequence as the radius-pattern. This capability introduces this design approach as an effective method for the design of filters for emerging dense wavelength division multiplexing networks. The design process incorporates analysing through the transfer matrix method and the powerful discrete-time signal processing techniques. Giving an adequate overview of analysing basic optical building blocks in the Z-domain, the procedure develops to analysing any optical structure imposed by mathematical sequences. The proposal is discussed employing pole-zero diagrams, discrete-time signal processing approach including apodization techniques. The point of the discrete-time signal processing approach is that the effect of dominant optical parameters over operation is clarified through the pole-zero position. Features like number of poles, bandwidth, and position of stop-bands can be controlled using ring diameter ratio. Finally, apodization of coupling coefficients attains a filter with an FWHM of 0.3 nm.

1 INTRODUCTION

In recent years, with increasing the number of channels, devising novel multi narrow band filters is essential to enhance the current Dense Wavelength Division Multiplexing (DWDM) networks. Realization of such precise wavelength selective filters plays a key role in improving the capacity of optical networks and satisfying growing demand for efficient photonic components. Furthermore, these optical filters are capable of carrying out the role of various photonic components such as add-drop multiplexers, gain equalizers, dispersion compensators, and interleavers. One of the widely used structures for such optical components is the micro ring-resonator. This research has focused on ring-resonator based narrowband optical filter design as a preferred method to separate single or multi channels simultaneously and hence exploiting the full bandwidth potential offered by optical fiber. Important features of such a filter include narrow bandwidth, ease of integration, and high side lobe suppression. The realization of narrowband optical filters such as Quasi-periodic structures is feasible by utilizing two methods, multilayer structures and micro ring-resonators. The optical properties of Fibonacci class, ring-resonator and multilayer structures studied respectively in (Rostami *et al.*, 2005) and (Rostami *et al.*, 2004). Multi band filter design is also possible by means of aperiodic Thue-Morse class structures (Liu, 1997).

Dong *et al.* proposed a GHz-bandwidth optical filter based on second-order and fifth-order ring resonators. They have also used metal heaters situated on top of the ring to tune the wavelength of filtering. The filter demonstrates a 3 dB bandwidth f_{3dB}=1.0 GHz and 1.9 GHz for 2nd- and 5th-order rings, respectively (Dong, 2010).Park *et al.* demonstrated 3rd order micro ring-resonator filters, 100 GHz-spaced 16 channels and 50 GHz-spaced 32 channels. The radius of micro ring-resonators are 9 µm (Park, 2011). Since micro ring-resonators are capable of performing as the basic building block for the design of various optical components, we intend to hire this potential for optical filter design. However, it is possible to design 10 GHz filters using micro ring-resonators but our design target is the standard of 0.3 µm for the bandwidth.

For the first time, this paper presents a unique technique for the design of ring-resonator based multiband filters with Thue-Morse class structures. Applying Thue-Morse sequence over the radius of

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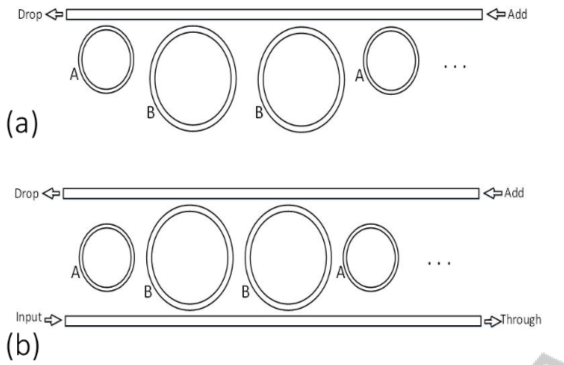

Figure 1: One dimensional micro-ring resonator chains. (a) Single channel SCISSOR chain whose system response implement multiple poles, (b) Double channel SCISSOR with separate resonances.

ring-resonators results in exceptional features such as narrow stop-bands. The structure comprises two distinct types of ring-resonators with different radiuses because of the Thue-Morse pattern from which the structure is constructed of. In manufacturing point of view different radiuses is simple to implement. It seems that using Quasi-Periodic structures give the potential of an exceptional design. We want to evaluate the performance of such structures as a filter. In this design, the difficult part is the design of a two-dimensional structure with 16 rings, with a footprint of 3.6 nm^2, which is very small but it seems that has no complexity.

The first step is to restrict the focus on two types of micro ring-resonator based building blocks from which the high-order coupled micro ring-resonator chains can be constructed of. Optical building block modelling in the Z-domain gives the opportunity of applying Discrete-time Signal Processing (DSP) techniques in the optical filter design process. Once the transfer function in the Z-domain is available, designer can observe whole the system characteristics and complex parametrical dependencies of pole and zero to other parameters in the pole-zero diagram. During the design process, designer can vary optical parameters and observe the pattern in which the pole and zero moves. Varying device parameters manipulates pole-zero positions (Kaalund *et al.*, 2004). Thus, adjusting pole-zero position is a possible way of modifying spectral response in order to optimize the performance of a filter. Designer can make the stop/pass bands, narrow or wide by moving pole-zero pairs toward/outward each other, sharpen the edge roll-off of bands or even organise the value of an optical parameter in different stages according to a window function to optimize the output response, which is a well-known technique in DSP and discrete filter design. Pole-zero diagram illustrates the stability and causality of a filter, which are the essential characteristics of a system performing a signal processing duty. All of these exploit DSP approach that is a systematic approach to avoid tedious electromagnetic methods and provides a mathematical framework for easy description of discrete-modelled optical filters and their optimization. The purpose is the feasibility study of exploiting design ideas related to signal processing techniques like Z-transform and apodization. We compare and contrast the characteristics of three different Thue-Morse class structures in one and two dimension.

The ratio of distinct ring-resonator radiuses and the coupling coefficients are two key parameters that control the number and the position of pole and zero respectively. Various manipulating techniques can be implemented over these parameters like apodization to get the desired pole-zero locations and hence the most efficient response.

Coupled ring-resonator chains fall into one of following two categories. Either the overall system contains a system of distributed feedback like double-channel side-coupled integrated spaced sequence of resonators (SCISSOR) which is shown in Figure 1(b), or contains localized feedbacks like single-channel SCISSOR or double-channel SCISSOR with dual-ring which are shown in Figure 1(a) and Figure 2, respectively. In the first group, the resonances occur not only in the resonators of each stage but also resonances distribute and develop among all the structure, enabling distributed feedback. In the second case, the lack of mechanism for contra-directional coupling makes the net light propagation unidirectional, enabling a localized kind of feedback at each block (Heebner *et al.*, 2004). The pole-zero analysis of these structures reveals another distinction between the localized and distributed feedback structures according to the pole-zero diagrams. These structures either have high order repetitive and folded poles in their pole-zero diagram or have distinct single poles. An Nth order system without backward coupling is the same as the cascaded connection of N identical building blocks, so that, the overall pole-zero diagram is an N times folded version of the diagram belonging to the basic section. The situation is completely different for the structures with backward coupling, like double-channel SCISSOR, because it splits the same resonances into distinct resonances with a relative phase shift (Chamorro-Posada *et al.*, 2011). We also find the same splitting effect while coupling resonant circuits.

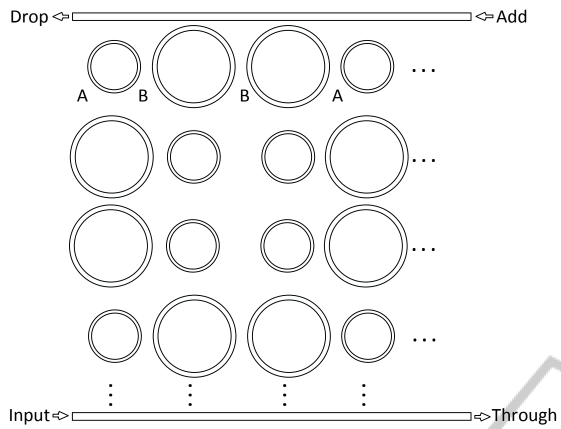

Figure 2: Two dimensional micro-ring resonator structure implementing multiple poles.

The objective of this research is designing passive devices. Light propagation in passive structures inevitably accompanies various losses. Incorporating active sections, either by semiconductor optical amplifier, SOA, or erbium-doped fiber amplifier, EDFA, can compensate propagation losses. Using active sections must not cause poles to step over a threshold, not crossed in stable and causal systems. For the sake of stability, poles must always lie inside the unit circle (Oppenheim *et al.*, 1989).

The most dominant parameter determining the pole zero location in an ideal filter is coupling coefficient. The proposed architecture introduces the ratio of radiuses as the second effective factor. It is essential to consider the overall loss and gain because loss affects pole-zero diagram. For the case of active devices, gain and loss are available tools to make the characteristics desirable. In (Lenz *et al.*, 1998) an infinitesimal amount of loss used to make the filter minimum phase and in conjunction with minimum-phase systems, Hilbert transform is applicable.

The organization of the paper is as follows: section 2 overviews Z-transform modeling and obtaining transfer functions associated with each unit cell. An introduction to Thue-Morse structure also presents in this section. Section 3 concentrates on mathematical modeling of Thue-Morse class architectures using the transfer matrix method. The paper continues approaching to an enhanced design by utilizing apodization functions to present a narrow band-pass filter with an FWHM equal to 0.3 nm. Finally, Conclusions present in the last section.

2 TRANSFER MATRICES OF SOME BASIC STRUCTURES

Figure 3 shows the schematic of optical architecture's unit cell selected to investigate in this paper with ring diameter r, the index of refraction n, the power coupling ratio K, the through-port transmission ratio c and the cross-port transmission ratio $-js = -j\sqrt{K}$.

Following the procedure presented in (Madsen *et al.*, 1999), we derive the Z-transform for each unit cell. The definition of unit delay length must satisfy the following relation: a multiple of the central resonance wavelength of the ring must be equal to the product of refractive index and the unit length. The unit delay length is presented as z^{-1} in the Z-domain. The circumference of the micro ring-resonator is equal to a multiple of z^{-1}. The parameter z is equal to e^{jkL}, where k is the propagation constant and L is the unit length. The equations can also include the effects of power dissipation in the coupler and waveguide by multiplying output signals in the coupler by $\sqrt{1-\gamma}$ and by multiplying z^{-1} by $\beta = e^{-\alpha L/2}$ respectively, where γ is the coupler attenuation coefficient and α is the waveguide attenuation coefficient.

In this part, we determine the relations between the output and input signals. The following equations describing add-drop resonator borrows from (Kaalund & Peng, 2004). As shown in Figure 3 (a), the signal is applied to the input port. Coupler K_2 couples a portion of input signal to the ring and a portion of input is passed to the through port. A portion of power after traversing half of the circumference or circulating additional cycles around the ring couples by coupler K_1 to the drop port.

$$S_{drop} = -s_1 s_2 \sqrt{\beta z^{-1}} \left\{1 + c_1 c_2 \beta z^{-1} + ...\right\} S_{input} \tag{1}$$

Using the Taylor series expansion, the infinite sum in (1) simplifies to the denominator of following expression, presenting drop port transfer function

$$\frac{S_{drop}}{S_{input}} = \frac{-s_1 s_2 \sqrt{\beta z^{-1}}}{1 - c_1 c_2 \beta z^{-1}} \tag{2}$$

This architecture is single pole with a zero at the origin. The effect of the origin positioned zero is a delay in overall response without any effect on the spectrum. In order to use transfer matrix method we

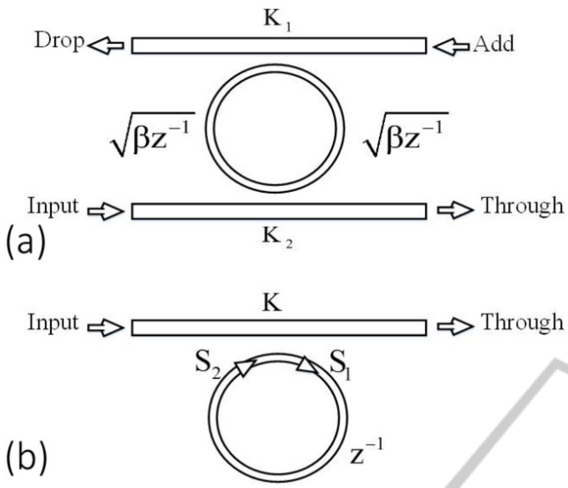

Figure 3: Schematic of the unit cells. (a) A ring resonator with two directional couplers K_1 and K_2. $\sqrt{\beta z^{-1}}$ is the amplitude transmission of the signal.

need to determine the relations between different ports by following the above procedure and then it is possible to write the transfer matrix of the unit cell. The final transfer matrix is given by

$$\begin{pmatrix} S_{input} \\ S_{drop} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} S_{through} \\ S_{add} \end{pmatrix} \tag{3}$$

Where

$$T_{11} = \frac{1 - c_1 c_2 \beta z^{-1}}{c_1 - c_2 \beta z^{-1}} \tag{4}$$

$$T_{12} = -T_{21} = \frac{s_1 s_2 \sqrt{\beta z^{-1}}}{c_1 - c_2 \beta z^{-1}} \tag{5}$$

$$T_{22} = \frac{c_1 c_2 - \beta z^{-1}}{c_1 - c_2 \beta z^{-1}} \tag{6}$$

Figure 3(b) shows a single coupler ring resonator (SCRR) structure, with a single pole and zero. According to the transfer function pole-zero pairs are inversely dependent on each other in SCRR. The input signal enters the input port and in the coupler a portion of power couples to the ring resonator and the rest of power passes through the coupler, the portion of signal coupled in the ring-resonator undergoes a phase delay after traversing the ring circumference.

$$S_2 = S_1.\beta z^{-1} \tag{7}$$

Associating a power-coupling ratio of K to the coupler, the input output relations of the coupler are given by

$$S_1 = \sqrt{1-\gamma}\left(c.S_2 + js.S_{in}\right) \tag{8}$$

$$S_{through} = \sqrt{1-\gamma}\left(c.S_{input} + js.S_2\right) \tag{9}$$

Where all the parameters have the similar definition to the previous unit cell. Using (7-9), the following transfer function presents the input output relation

$$\frac{S_{through}}{S_{in}} = \sqrt{1-\gamma}\left(\frac{c - \sqrt{1-\gamma}.\beta z^{-1}}{1 - c.\sqrt{1-\gamma}.\beta z^{-1}}\right) \tag{10}$$

In this part, the Thue-Morse sequence is introduced (Brlek, 1989). The Thue-Morse sequence is generated by a map, P, which satisfies the following identity relation for all A and E. This equation is called Thue-Morse morphism

$$P(AB) = P(A)P(B) \tag{11}$$

Where A and B are the distinct ring-resonators with different radiuses. The Thue-Morse morphism is defined as

$$\begin{aligned} P(A) &= AB \\ P(B) &= BA \end{aligned} \tag{12}$$

Following identity rule we have

$$\begin{aligned}
&P(A) = AB \\
&P(B) = BA \\
&P^2(A) = P(P(A)) = AB\ BA \\
&P^2(B) = P(P(B)) = BA\ AB \\
&P^3(A) = P(P^2(A)) = ABBA\ BAAB \\
&P^3(B) = P(P^2(B)) = BAAB\ ABBA \\
&P^4(A) = P(P^3(A)) = ABBABAAB\ BAABABBA
\end{aligned} \tag{13}$$

Which has a recursion equation as

$$P^n(X) = P^{n-1}(X)\ P^{n-1}(\overline{X}) \tag{14}$$

The Thue-Morse morphism also introduced in two dimension, in the form of matrices as

$$\begin{aligned} P(A) &= \begin{pmatrix} A & B \\ B & A \end{pmatrix} \\ P(B) &= \begin{pmatrix} B & A \\ A & B \end{pmatrix} \end{aligned} \tag{15}$$

Figure 2 is built upon such a morphism and represents $P^2(A)$ in two-dimension, while figure 1 follows the one-dimensional pattern. We are going to make use of these sequences to design Thue-Morse structures in one and two dimension.

3 MATHEMATICAL MODELING, SIMULATION, AND RESULTS

The procedure employs transfer matrix method outlined in (Madsen & Zhao, 1999) to determine the overall ring-resonator chain response. The point of the transfer matrix method is that the transmission of signal into the drop and through ports can be obtained through multiplication of transfer matrices of each stage. An N-coupled array of double channel SCISSOR comprises N ring-resonators, connected via T_0, describing straight waveguide segments. The matrices are multiplied together to obtain the transfer matrix

$$\begin{pmatrix} S^1_{input} \\ S^1_{drop} \end{pmatrix} = T_A.T_0.T_B.T_0.T_B \ldots T_A \begin{pmatrix} S^N_{through} \\ S^N_{add} \end{pmatrix} = TM \begin{pmatrix} S^N_{through} \\ S^N_{add} \end{pmatrix} \qquad (16)$$

Where N shows the size of the array and is equal to 2^n, and n is the order of Thue-Morse sequence. Using final transfer matrix, TM, the transmission of signal into the through port is as $S^N_{through} / S^1_{input} = 1/TM_{11}$. In addition, by assuming the signal S^N_{add} zero, the drop port signal is $S^1_{drop} / S^1_{input} = TM_{21} / TM_{11}$.

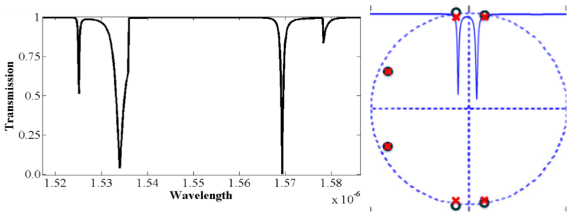

Figure 4: Fourth order double-channel SCISSOR filter with coupling coefficient 0.4, the radius r_a=20 µm and. $r_b = 2r_a$.

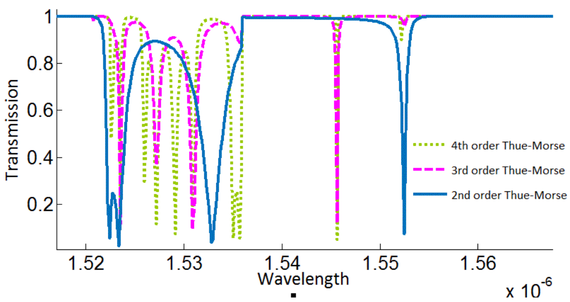

Figure 5: Design of double-channel SCISSOR filters for various class factors of Thue-Morse chains. The power coupling factor is 0.4 at both couplers, r_a=20 µm, $r_b = 2r_a$, refractive index is equal to 1.44, and rings are separated by the straight waveguide with the length of r_a and the effect of loss is neglected.

The transfer functions for high order chains are quite long, however to visualize the vital information contained in the transfer function we present them in the form of pole-zero diagrams. The distributed feedback identity of the double-channel SCISSOR is obvious in Figure 4 although all the stages are the same but their resonances split with a relative shift. The potential of realizing multi-narrow stop bands is evident from simulation results for higher order chains. The transmission spectra of the SCISSOR structure, shown in Figure 5, reveals a multi-channel response with narrow bandwidth, corresponding with requirements of optical networks.

A single-channel SCISSOR as depicted in Figure 1(a) comprises a cascaded sequence of all-pass optical filters. Because of lack of any mechanism for backward coupling, input signals are only transmitted in forward direction pausing at each building block due to localized feedback (Chamorro-Posada *et al.*, 2011). Single-channel SCISSOR shall not perform the role of an ideal all-pass filter because of the waveguide and coupling dissipation. Its transmission response is not flat and contains some narrow notches caused by pole-zero pairs out of symmetry around the unit circle. As shown in Figure 6, with increasing the class factor of Thue-Morse structure the bandwidth of stop band increases and the overall transmission spectra undergoes an amplitude decrease. Repetitive pole and zero severely affects stop band bandwidth; folded pole-zero pairs emphasize the effect of early pair and broad the stop-band. As mentioned before this case has no mechanism for contra-directional coupling and no splitting effect is present, so each pole folds over the previous one and the result is an exaggeration in the transmission response of a single ring-resonator optical filter. With increasing the number of rings coupled to the channel, the increasing effect of dissipation results in the amplitude decrease; the more the number of rings coupled to the channel, the more the dissipation, however optical amplifiers can compensate this attenuation. Since we have assumed the circumference of micro ring-resonator B twice the unit delay length there are two pole-zero pairs for stage B. The designer can opt to select the radius B an arbitrary multiple of radius A. The advantage of this technique is that high-order filters can be implemented with fewer rings; on the other hand, larger rings make fabrication process easier.

A dual-ring double-channel SCISSOR as depicted in Figure 2 is the same as the cascaded connection of n-coupled ring resonator optical waveguide (CROW) blocks. Even number of rings in each block provides the mechanism for unidirectional coupling across the channels, in contrast to the double channel SCISSOR

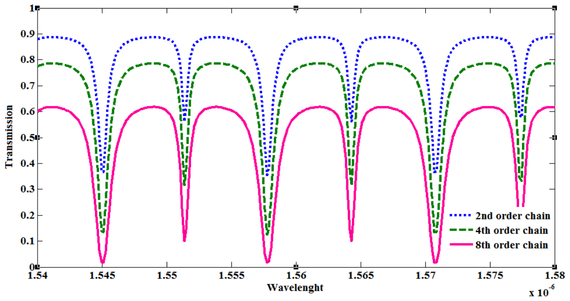

Figure 6: Various orders of single-channel SCISSOR filter imposed by Thue-Morse sequence ($n_{waveguide}$ =1.5, n_{ring} =3, c =0.8, l =20µm, α =50(µm)$^{-1}$, r_a =10µm, $r_b = r_a$, γ =0.1).

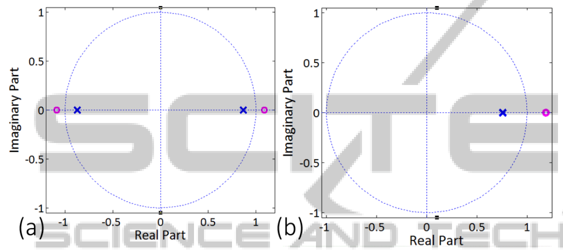

Figure 7: Pole-zero diagram corresponding to distinct basic building blocks of single channel SCISSOR (a) all pass filter with circumference equal to twice the unit delay length (b) single waveguide ring-resonator with circumference equal to the unit delay length.

case. The final pole-zero diagram is the same as each basic building block but with higher order individual pole-zero pairs, depending on the order of system. Zeros in the corresponding pole-zero diagram for each CROW block lie on the unit circle for lossless case and hence critical coupling will occur, on the other hand considering loss moves all the zeros outside the unit circle and critical coupling will not happen. For an active CROW with overall gain all the zeros lie inside the circle, resulting a minimum phase system without any critical coupling. As it is evident from Figure 8, each pole-zero pair cuts a notch in transmission spectra. The bandwidth of each notch corresponds to the angular distribution of the pole and zero and the separation between them determines the ripple-edge roll-off speed, smaller separation makes the notches sharper. There is a trade-off between these parameters, faster roll-off makes the poles closer to the unit circle and the bandwidth of notch narrower (Darmawan *et al.*, 2007). The first and second order two-dimensional structure reveals sharp ripples and narrow stop bands in the transmission spectra. Figure 8 features a band-pass filter and Figure 9 resembles a superior filter of that. The folding poles and zeros severely suppress the low frequencies and higher frequencies face more deep notches, with steeper edge roll-off, this means a better band-pass shape.

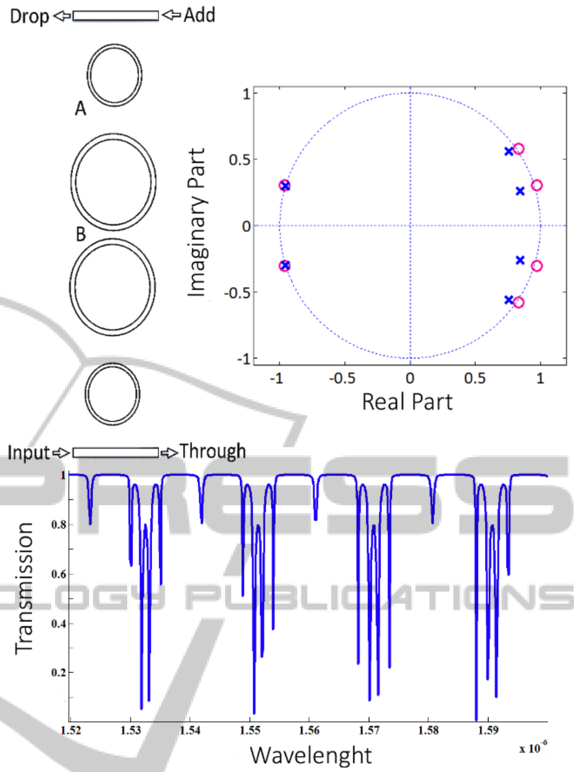

Figure 8: First stage of second order Thue-Morse imposed dual-ring double-channel SCISSOR structure, pole-zero diagram with their corresponding transmission spectra (n =1.44, l_a =20 µm, l_b =40 µm,).

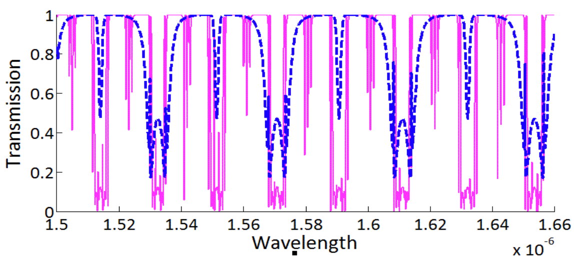

Figure 9: Transmission spectra for the first [blue] and second [pink] order Thue-Morse imposed dual-ring double-channel SCISSOR structure.

4 APODIZATION

All the investigated structures up to now have the same value of coupling coefficient. In digital filter design, a usual method to optimize filter performance is apodization or windowing. In optical filter design, apodization means changing coupling coefficient values from one cell to another based on a windowing function. In this section, we will employ this method. The following apodization function expresses Hamming window

$$\begin{aligned} & c(\mathrm{i}) = \alpha - \beta \cos(\frac{2\pi \mathrm{i}}{N}) \\ & \mathrm{i} = 0,1,\mathrm{N}-1 \\ & \alpha = 0.54 \\ & \beta = 0.45 \end{aligned} \tag{17}$$

Where N is the size of the structure. As shown in Figure 7, the apodized structure can realize the function of a very narrow filter with an FWHM equal to 0.3 nm. This behavior achieved at a cost of increased ripples near low frequencies. It must be considered that an apodized filter performs as an architecture with reduced number of rings (Capmany *et al.*, 2007).

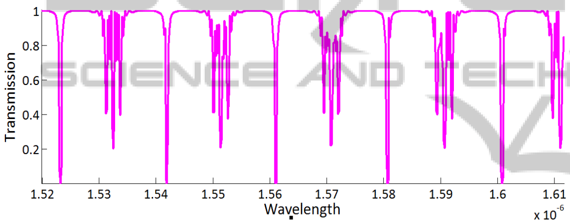

Figure 10: Transmission spectra for the second order, Thue-Morse imposed dual-ring double-channel SCISSOR structure with coupling coefficients apodized through the Hamming window.

5 CONCLUSIONS

In this paper, we investigated the ways to attain higher performance filters with respect to the conventional ring-resonator based filters. Multiband response emerged by using Thue-Morse class ring-resonators. We studied Thue-Morse based optical structures in the Z-domain and presented the transmission spectra along with pole-zero diagrams to provide the framework of an optimal filter design. The proposed filter enhanced after employing hamming function, demonstrating that coupling coefficient and radius engineering can lead to an optimum design. It would be interesting to use this approach in ultrahigh order filter design.

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