

Interfaces in a Game-theoretic Setting for Controlling the Plasmodium Motions

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Abstract: The plasmodium is the large one-cell organism containing a mass of multinucleate protoplasm. It is an active feeding stage of *Physarum polycephalum* or *Badhamia utricularis* and it moves by protoplasmic streaming which reverses every 30-60 s. In moving, the plasmodium switches its direction or even multiplies in accordance with different biosignals attracting or repelling its motions, e.g. in accordance with pheromones of bacterial food, which attract the plasmodium, and high salt concentrations, which repel it. So, the plasmodium motions can be controlled by different topologies of attractants and repellents so that the plasmodium can be considered a programmable biological device in the form of a timed transition system, where attractants and repellents determine the set of all plasmodium transitions. Furthermore, we can define p -adic probabilities on these transitions and, using them, we can define a knowledge state of plasmodium and its game strategy in occupying attractants as payoffs for the plasmodium. As a result, we can regard the task of controlling the plasmodium motions as a game and we can design different interfaces in a game-theoretic setting for the controllers of plasmodium transitions.

1 INTRODUCTION

Conventionally, the intelligent behavior of animals is explained by their nervous system that coordinates voluntary and involuntary actions of animal's body and transmits signals between different parts of its body, which allows animals to act intentionally and efficiently. There is an approach in artificial intelligence, consisting in building computational models inspired by these nervous systems, that is called *artificial neural network*.

Nevertheless, there are one-cell organisms like *Physarum polycephalum* or *Badhamia utricularis*¹ (supergroup Amoebozoa, phylum Mycetozoa, class Myxogastria) without any nervous system and they are able at their plasmodial stage to build complex networks for solving different tasks: maze-solving (Nakagaki, Yamada, and Toth, 2000), minimum-risk path finding (Nakagaki, Yamada, and Toth, 2001), (Nakagaki et al., 2007), associative learning (Shirakawa, Gunji, and Miyake, 2011), etc. In other

words, *Physarum polycephalum* and *Badhamia utricularis* demonstrate an intelligent behavior with intentionality and efficiency, although they do not have nervous systems at all. In particular, they demonstrate the ability to memorize and anticipate repeated events (Saigusa et al., 2008). Furthermore, by means of plasmodium behavior, it is possible to simulate the behavior of some collectives such as collectives of parasites (Schumann and Akimova, 2013). Thus, the complex intelligent behavior of plasmodium is biologically unexplained still and shows the limits of our understanding what natural intelligence is.

Now, there are many attempts to involve the plasmodium into semi-electrical devices to obtain a semi-biological and semi-electrical chip in due course (Sun et al., 2009), (Tsuda, Aono, and Gunji, 2004), (Tsuda et al., 2011), (Adamatzky, 2010). The point is that the plasmodium spread by networks can be programmable and thereby it may simulate different intelligent processes. We are working on this problem, too (Adamatzky et al., 2012). In this paper, we are going to present our results in modelling the plasmodium networks as timed transition systems (Section 2). Propagations in these systems can be calculated

¹References on this new culture are contained in (Neubert et al., 1995)

by means of p -adic valued probabilities and fuzziness (Khrennikov and Schumann, 2006), (Schumann, 2008), (Schumann, 2010), see Section 3. In terms of these probabilities we can define a knowledge state of plasmodium and its game strategy in occupying attractants as payoffs for the plasmodium (Section 4). Hence, we can control the plasmodium motions as a game (Section 5). As a consequence, user interfaces for the controllers of plasmodium propagations can have a natural form of game-theoretic setting.

2 TIMED TRANSITION SYSTEMS FOR PROGRAMMING THE PLASMEDIUM MOTIONS

The plasmodium is an amorphous yellowish mass with networks of protoplasmic tubes, programmed by spatial configurations of attracting and repelling stimuli. Any motion of plasmodium proceeds from one stimuli to others. As a result, we deal with a kind of natural transition systems with states presented by attractants and events presented by plasmodium transitions between attractants. We can distinguish several operations (instructions) in the plasmodium networks like: add node, remove node, add edge, remove edge (Adamatzky, 2010). Adding and removing nodes can be implemented through activation and deactivation of attractants, respectively. Adding and removing edges can be implemented by means of repellents put in proper places in the space. An activated repellent can avoid a plasmodium transition between attractants.

Adding and removing edges (in fact, adding and removing protoplasmic tubes) can change dynamically over time. To model such behavior, we have proposed to use timed transition systems as a high-level model of behavior of plasmodium. Let N be a set of nonnegative integers. Formally, a timed transition system $TTS = \langle S, E, T, s_0, l, u \rangle$ consists of the non-empty set of states S , the set of events E , the transition relations $T \subseteq S \times E \times S$, the initial state s_0 as well as a minimal delay function (a lower bound) $l : E \rightarrow N$ assigning a nonnegative integer to each event and a maximal delay function (an upper bound) $u : E \rightarrow N \cup \{\infty\}$ assigning a nonnegative integer or infinity to each event. Usually transition systems are based on actions which may be viewed as labelled events. If $\langle s, e, s' \rangle \in T$, then the idea is that TS can go from s to s' as a result of the event e occurring at s . In timed transition systems, timing constraints restrict the times at which events may occur. The timing constraints are classified into two categories: lower-

bound and upper-bound requirements. A transition system can be presented as a graph structure with nodes corresponding to states and edges corresponding to transitions. In case of plasmodium, states represent attractants whereas edges represent protoplasmic tubes (plasmodium transitions between attractants).

To program computation tasks for the plasmodium propagations, we are developing a new object-oriented programming language (Schumann and Pancercz, 2013) called the *Physarum language*, where the following three basic set descriptions are defined: (i) *TS.State* – setting states of plasmodium, including initial states; (ii) *TS.Event* – setting events transiting one states to others; (iii) *TS.Transition* – setting transitions of plasmodium. The proposed language can be used for developing programs for plasmodium motions by the spatial configuration of stimuli.

Let us consider a simple timed transition system shown as a graph structure in Figure 1 with the following timing constraints: $l(e_1) = 0, u(e_1) = \infty, l(e_2) = 0, u(e_2) = \infty, l(e_3) = 5, u(e_3) = 10$.

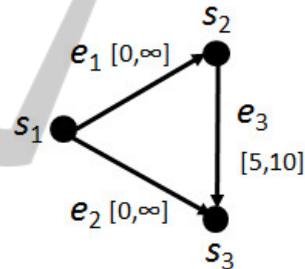


Figure 1: An example of timed transition system.

The code in the *Physarum language* has the following form:

```
#TRANSITION_SYSTEM s1=new TS.State("s1");
s1.setAsInitial;
s2=new
TS.State("s2");
s3=new TS.State("s3");
e1=new TS.Event("e1");
t1=new TS.Transition(s1,e1,s2);
e2=new TS.Event("e2");
t2=new
TS.Transition(s1,e2,s3);
e3=new
TS.Event("e3");
e3.setTimingConstraints(5,10);
t3=new
TS.Transition(s2,e3,s3);
```

The default timing constraints are 0 as a lower bound and ∞ as an upper bound.

As a result of programming the plasmodium transitions, we obtain spatial configurations of stimuli

presented in Figure 2: (a) for the time instant $t = 4$, (b) for the time instant $t = 8$, where P is plasmodium, $A_{s_1}, A_{s_2}, A_{s_3}$ are attractants, and R is a repellent. It is easy to see that the event e_3 is allowed only if actual time $t \in \{5, 6, \dots, 10\}$. Therefore, in the model in Figure 2(a), a repellent, avoiding the transition between states s_2 and s_3 as a result of the event e_3 , is present, i.e. it is activated.

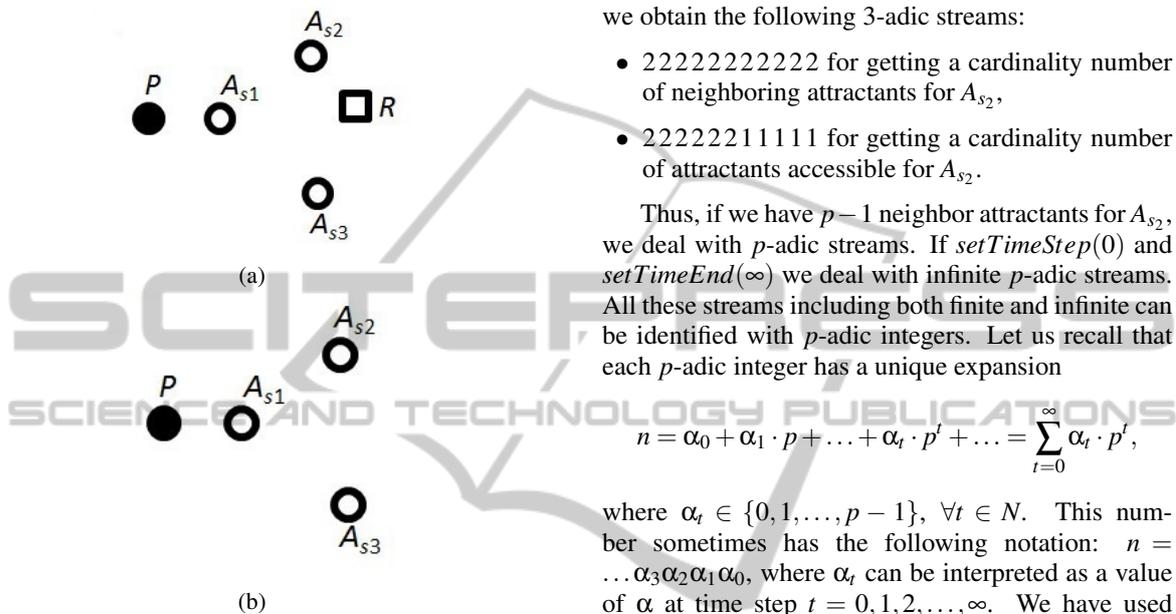


Figure 2: Spatial configurations of stimuli for the plasmodium motions.

3 P-ADIC VALUED PROBABILITIES AND FUZZINESS

We have supplemented our language with instructions enabling us to determine (in the simulation stage) possible properties of experiments in terms of the probability space:

- *setTimeStep* – setting a time step from which the experiment starts, $t = 0, 1, 2, \dots, n$,
- *setTimeEnd* – setting a time end when the experiment stops, $t = 0, 1, 2, \dots, \infty$,
- *getNeighCard* – getting a cardinality number of neighboring attractants for a given attractant at the given *setTimeStep* and *setTimeEnd*,
- *getAccessCard* – getting a cardinality number of attractants accessible for a given attractant by protoplasmic tubes at the given *setTimeStep* and *setTimeEnd*.

Instructions for the simulation stage are preceded with $\$$. Let us consider a simple timed transition system given earlier.

If we add the following instructions to the code:

```

$setTimeStep(0);
$setTimeEnd(10);
$getNeighCard(s2);
$getAccessCard(s2);
    
```

we obtain the following 3-adic streams:

- 22222222222 for getting a cardinality number of neighboring attractants for A_{s_2} ,
- 22222211111 for getting a cardinality number of attractants accessible for A_{s_2} .

Thus, if we have $p - 1$ neighbor attractants for A_{s_2} , we deal with p -adic streams. If *setTimeStep*(0) and *setTimeEnd*(∞) we deal with infinite p -adic streams. All these streams including both finite and infinite can be identified with p -adic integers. Let us recall that each p -adic integer has a unique expansion

$$n = \alpha_0 + \alpha_1 \cdot p + \dots + \alpha_t \cdot p^t + \dots = \sum_{t=0}^{\infty} \alpha_t \cdot p^t,$$

where $\alpha_t \in \{0, 1, \dots, p - 1\}$, $\forall t \in \mathbb{N}$. This number sometimes has the following notation: $n = \dots \alpha_3 \alpha_2 \alpha_1 \alpha_0$, where α_t can be interpreted as a value of α at time step $t = 0, 1, 2, \dots, \infty$. We have used the latter notation in our example. The set of p -adic integers is denoted by \mathbf{Z}_p . For more details about p -adic numbers, please see (Koblitz, 1984). Now, p -adic analysis is used in many applications including quantum mechanics (Vladimirov and Volovich, 1989), (Volovich, 1987).

The set \mathbf{Z}_p cannot be linearly ordered, but there are many possibilities to define a partial ordering relation. For example, we can assume that (i) for any finite p -adic integers $\sigma, \tau \in \mathbb{N}$, we have $\sigma \leq \tau$ in \mathbb{N} iff $\sigma \leq \tau$ in \mathbf{Z}_p ; (ii) each finite p -adic integer $n = \dots \alpha_3 \alpha_2 \alpha_1 \alpha_0$ (i.e. such that $\alpha_i = 0$ for any $i > j$) is less than any infinite number τ , i.e. $\sigma < \tau$ for any $\sigma \in \mathbb{N}$ and $\tau \in \mathbf{Z}_p \setminus \mathbb{N}$; (iii) each infinite p -adic integer σ is less, than p -adic integer τ iff $\sigma_t \leq \tau_t$ for all $t = 0, 1, 2, \dots$. Let us denote this ordering relation by $O_{\mathbf{Z}_p}$. We can see that there exist p -adic integers, which are incompatible by $O_{\mathbf{Z}_p}$. For example, let $p = 2$ and let σ represents the p -adic integer $-1/3 = \dots 10101 \dots 101$ and τ the p -adic integer $-2/3 = \dots 01010 \dots 010$. Then the p -adic streams σ and τ are incompatible. Now we can define *sup* and *inf* digit by digit. Then if $\sigma \leq \tau$, so $\text{inf}(\sigma, \tau) = \sigma$ and $\text{sup}(\sigma, \tau) = \tau$. The greatest p -adic integer according to our definition is $-1 = \dots xxxxx$, where $x = p - 1$, and the smallest is $0 = \dots 00000$.

Let us define the Boolean operations on attractants A_{s_i}, A_{s_j}, \dots so that

$$\begin{aligned} & \text{getNeighCard}(si \cap sj) := \\ & \quad \inf(\text{getNeighCard}(si), \text{getNeighCard}(sj); \\ & \quad \text{getAccessCard}(si \cap sj) := \\ & \quad \inf(\text{getAccessCard}(si), \text{getAccessCard}(sj); \\ & \quad \text{getNeighCard}(si \cup sj) := \\ & \quad \sup(\text{getNeighCard}(si), \text{getNeighCard}(sj); \\ & \quad \text{getAccessCard}(si \cup sj) \\ & := \inf(\text{getAccessCard}(si), \text{getAccessCard}(sj); \\ & \text{getNeighCard}(\neg si) := -1 - \text{getNeighCard}(si); \\ & \text{getAccessCard}(\neg si) := -1 - \text{getAccessCard}(si). \end{aligned}$$

Let Ω^* denote all attractants both activated and deactivated at each $t = 0, 1, 2, \dots, \infty$. It is a union of all attractants A_{s_i}, A_{s_j}, \dots at each time step. Its subsets will be denoted by $A^*, B^* \subseteq \Omega^*$.

Let us define p -adic fuzziness as follows: a p -adic fuzzy measure is a set function $F_{Z_p}(\cdot)$ defined for sets $A^*, B^* \subseteq \Omega^*$, it runs over the set Z_p and satisfies the following properties:

- $F_{Z_p}(\Omega^*) = -1$ and $F_{Z_p}(\emptyset^*) = 0$.
- If $A^* \subseteq \Omega^*$ and $B^* \subseteq \Omega^*$ are disjoint, i.e. $\inf(F_{Z_p}(A^*), F_{Z_p}(B^*)) = 0$, then $F_{Z_p}(A^* \cup B^*) = F_{Z_p}(A^*) + F_{Z_p}(B^*)$. Otherwise, $F_{Z_p}(A^* \cup B^*) = F_{Z_p}(A^*) + F_{Z_p}(B^*) - \inf(F_{Z_p}(A^*), F_{Z_p}(B^*)) = \sup(F_{Z_p}(A^*), F_{Z_p}(B^*))$.
- If $A^*, B^* \subseteq \Omega^*$, then $F_{Z_p}(A^* \cap B^*) = \inf(F_{Z_p}(A^*), F_{Z_p}(B^*))$.
- $F_{Z_p}(\neg A^*) = -1 - F_{Z_p}(A^*)$ for all $A^* \subseteq \Omega^*$, where $\neg A^* = \Omega^* \setminus A^*$.

A p -adic probability measure is a set function $P_{Z_p}(\cdot)$ defined for sets $A^*, B^* \subseteq \Omega^*$ thus:

- $P_{Z_p}(A^*) = -F_{Z_p}(A^*) \in Z_p$
- $P_{Z_p}(A^* | B^*) \in Q_p$ is characterized by the following constraint:

$$P_{Z_p}(A^* | B^*) = \frac{P_{Z_p}(A^* \cap B^*)}{P_{Z_p}(B^*)} = \frac{F_{Z_p}(A^* \cap B^*)}{F_{Z_p}(B^*)},$$

where $P_{Z_p}(B^*) \neq 0$, $P_{Z_p}(A^* \cap B^*) = \inf(P_{Z_p}(A^*), P_{Z_p}(B^*))$.

The measure $P_{Z_p}(\cdot)$ runs over the set Q_p of all p -adic numbers (not only integers). Notice that while Z_p is the ring of p -adic integers, Q_p is the field of p -adic numbers.

4 STATES OF KNOWLEDGE AND STRATEGIES OF PLASMODIUM

Using p -adic valued fuzziness and probabilities, we can define games of plasmodia. So, in the given topology of attractants, active zones of plasmodia (initial states) can be considered players. Suppose, we have a set of N players, call them $i = 1, \dots, N$. Agent i 's knowledge structure is a function \mathbf{P}_i which assigns to each attractant $\omega \in \Omega^*$ a non-empty subset of Ω^* , so that each thing ω belongs to one or more elements of each \mathbf{P}_i , i.e. Ω^* is contained in a union of \mathbf{P}_i , but \mathbf{P}_i are not mutually disjoint. Then $\mathbf{P}_i(\omega)$ is called i 's knowledge state at the attractant ω . This means that if the actual state is ω , the individual only knows that the actual state is in $\mathbf{P}_i(\omega)$.

We can interpret $\mathbf{P}_i(\omega)$ probabilistically as follows: $\mathbf{P}^i(\omega) = \{\omega' : P_{Z_p}^i(\omega' | \omega) > 0\}$. Evidently that $P_{Z_p}^i(\omega | \omega) > 0$ for all $\omega \in \Omega^*$, therefore for all $\omega \in \Omega^*$, $\omega \in \mathbf{P}_i(\omega)$.

Now we consider the relation $A^* \subseteq \mathbf{P}_i(\omega)$, where $A^* \subseteq \Omega^*$, as the statement that at ω agent i accepts the performance A^* :

$$K_i A^* = \{\omega : A^* \subseteq \mathbf{P}_i(\omega)\}.$$

Let B_i^* mean 'Attractants, which can be occupied by agent i '. After several steps, we expect fusions of all protoplasmic tubes so that all attractants are occupying by all agents. Does it mean that we observe a union of B_i^* ? No, it does not. We face just the situation that since a time step $t = k$ the sets B_i^* are intersected. Let C_i^* mean 'Attractants accessible for the attractant N_i by protoplasmic tubes'. Assume, $\omega \in B_i^*$ and $\omega' \in C_i^*$. Evidently, $P_{Z_p}^i(\omega' | \omega) > 0$. As a consequence, we assume according to our definitions that each agent i knows ω at ω' and knows ω' at ω , i.e. agent i accepts the performance B_i^* at ω' and i accepts the performance C_i^* at ω .

Let $\text{getAccessSet}(i, k)$ be a set of all attractants such that i knows about them at the given $\text{setTimeStep}(t_0)$ and $\text{setTimeEnd}(t_k)$. A strategy of a player i is a mapping $\text{strat}_{i,k} : \text{getAccessSet}(i, k) \rightarrow \Omega^*$ such that for any history knowledge $\text{getAccessSet}(i, k)$ it is true that $\text{strat}_{i,k}$ belongs to the set of attractants accessible at k .

5 GAME-THEORETIC INTERFACES FOR PLASMODIUM

It is known due to the experiments performed by Andrew Adamatzky and Martin Grube that if there are only two agents of the plasmodium game, where the first agent is presented by a usual *Physarum polycephalum* plasmodium and the second agent by its modification called a *Badhamia utricularis* plasmodium, then both start to compete with each other. In particular, the *Physarum polycephalum* plasmodium grows faster and could grow into branches of *Badhamia utricularis*, while the *Badhamia utricularis* plasmodium could grow over *Physarum polycephalum* veins. So, we face an interesting form of zero-sum games.

The user interface for this game is designed on the basis of the following game steps:

- first, the system of *Physarum* language generates locations of attractants and repellents;
- second, we can chose n plasmodia/agents of *Physarum polycephalum* and m plasmodia/agents of *Badhamia utricularis*;
- third, we obtain the task, for example to reach as many as possible attractants or to construct the longest path consisting of occupied attractants, etc.;
- fourth, we can chose initial points for *Physarum polycephalum* transitions and initial points for *Badhamia utricularis* transitions;
- fifth, we start to move step by step;
- sixth, we define who wins, either *Physarum polycephalum* or *Badhamia utricularis*.

Thus, the plasmodium game has the form of cycle of Figure 3.

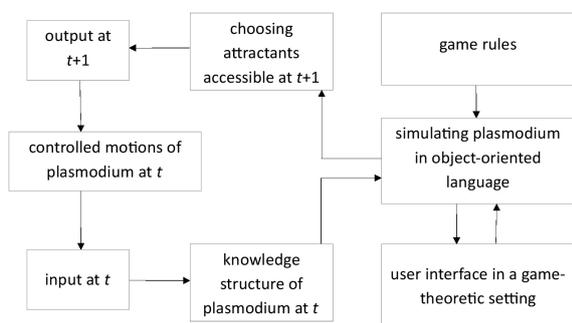


Figure 3: The operative cycle of game-theoretic controller of plasmodium motions.

In this game, we have two players (the first plays for the *Physarum polycephalum* plasmodia, the second for the *Badhamia utricularis* plasmodia). The system places attractants and repellents automatically. Then the players choose which attractants are occupied before the game and which rules of the game hold (to reach as many as possible attractants or to construct the longest path consisting of occupied attractants, etc.). Then the system shows who wins and who loses.

6 CONCLUSION

The plasmodium motion is an intelligent way of constructing expanding networks for solving complex tasks. This motion has the form of transitions determined by locations of attractants and repellents. On these transitions, it is possible to define p-adic probabilities which are used for defining a knowledge state of plasmodium and its game strategy in occupying attractants as payoffs for the plasmodium. Consequently, the task of controlling the plasmodium motions is considered a game.

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