Newsvendor Model in Rail Contract to Transport Gasoline in Thailand

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Keywords: Newsvendor model, Rail freight, Stochastic Model Applications in logistics management.

Abstract: A long-term contract between a railroad company and a shipper who wants to transport gasoline daily is studied. The contract specifies an upfront payment for reserving bogies and a per-container freight rate. The shipper also uses a trucking company to transport the excess demand, but its per-liter transportation cost is higher. The shipper's problem is to determine the number of bogies to reserve at the beginning of the contract duration, before daily demand is revealed. Since demand is perishable, we formulate the problem as a newsvendor model. The expected cost function is not convex. We show that the expected cost is unimodal and derive an optimal solution under certain conditions, which are not very restrictive. We also provide a sensitivity analysis with respect to the change in the contract parameter.

1 INTRODUCTION

Global demand for gasoline and diesel continue to grow over the next few decades (Organization of the Petroleum Exporting Countries (OPEC), 2013). Population growth is one of the key drivers in growing demand for transportation and motor gasoline consumption (Energy Information Administration (EIA), 2008). The supply chain for fuel starts from crude oil sources and ends at gas stations, passing through refinery factories and storages (e.g., tank farms, depots, terminals, and so on). From the distribution centers to gas stations, gasoline and diesel can be transported through road, rail, or pipeline if exists.

Pipeline is environment-friendly and always available all year round 24-7-365 with high service level at a relatively low cost. Products are delivered on time via pipelines, because the flows can be continuously monitored and controlled by a computer, and they are not affected by weather or climate conditions. Spills or product losses are minimal, compared to trucks or trains. Accidents involving trucks or trains with petroleum products are fairly common. Nevertheless, in some developing countries, e.g., Thailand, pipelines for refined products are not extensive and cover only a limited geographic area. In this article, pipeline is not considered.

In Thailand, gasoline and diesel are transported among regional depots by road or rail. Rail is better for the environment, produces less greenhouse emission gas, and the transportation cost is usually lower. On the other hand, road usually allows faster speed, better delivery consistency, and less product losses.

A rail company serves both passengers and cargo shippers/freighters. Bogies (under-carriage assemblies commonly referred to as "trucks" in US) are needed for both cargo tanks and passenger rail cars. When there are not enough bogies for both, the rail company often allocates bogies first to the passenger cars and then box cars for general commodities or tanks for liquid and gas.

To alleviate the problem of bogie availability, some petroleum companies enter a long-term contract with the railroad company. The long-term contract we study is found in practice between the State Railway of Thailand and one of the biggest petroleum companies in Thailand. The contract specifies an upfront payment for reserving bogies and a per-liter freight rate. For each day during the contract period, the number of bogies provided by the railroad company is at most that reserved at the beginning of the contract period. If daily demand is greater than total capacity of reserved bogies, then the excess demand is handled by a trucking company. A per-liter charge by the trucking company is typically higher. The shipper needs to decide how many bogies to reserve before knowing actual daily demand.

The shipper's problem is similar to the newsvendor problem, in which an order quantity must be determined prior to the start of the selling season. In

Amaruchkul K..
Newsvendor Model in Rail Contract to Transport Gasoline in Thailand.
DOI: 10.5220/0005198102370246
In *Proceedings of the International Conference on Operations Research and Enterprise Systems* (ICORES-2015), pages 237-246
ISBN: 978-989-758-075-8

both ours and the newsvendor problem, a fixed quantity is committed before random demand is materialized. Reviews of the newsvendor model can be found in e.g., (Qin et al., 2011) and (Khouja, 1999). Standard textbooks in operations research/management science also discuss a newsvendor problem; e.g., Chapter 10 in (Silver et al., 1998), Chapter 11 in (Cachon and Terwiesch, 2009) and Chapter 5 in (Nahmias, 2009). In the standard newsvendor model, the total expected cost is shown to be convex, whereas ours is not convex, because the daily transportation rail cost is based on the number of containers actually used, not the demand actually served, as in the standard newsvendor model.

A newsvendor model with standard-sized containers is studied in e.g., (Pantumsinchai and Knowles, 1991) and (Yin and Kim, 2012). Quantity discount pricing is offered to the shipper. In these papers, if a larger container is used, the per-unit rate is cheaper. Nevertheless, their transportation costs are based on the volume actually shipped, not the number of containers as in ours. Although we do not have quantity discount as in theirs, we have a fixed upfront payment and variable transportation costs. They do not have an upfront payment or a secondary transportation option.

Our contract scheme is related to returns policies or buyback contracts in the newsvendor setting. In our model, the total payment from the shipper to the rail company consists of two parts, namely the upfront payment proportional to the number of reserved bogies, and the variable payment proportional to the actual number of bogies actually used. In the newsvendor model in which a supplier and a buyer enter into a buyback contract, the buyer who places an order quantity of x pays wx to the supplier, where w is the per-unit wholesale price. After demand D materializes, the supplier buys back all unsold units from the buyer at a per-unit buyback price b. The net payment from the buyer to the supplier is

$$wx - b(x-D)^+ = (w-b)x + b\min(x,D)$$

where $(t)^+ = \max(t,0)$ denotes the positive part of a real number t. The payment under the buyback contract can be viewed as two parts, namely the "upfront" payment, $(w-b)x$, proportional to the committed order quantity and the variable payment, $b\min(x,D)$, proportional to the actual sales. The order quantity in the newsvendor model is analogous to the number of reserved bogies in ours, and the actual sales to the actual number of bogies actually used. Literature on buyback contracts in the newsvendor setting is extensive; see reviews in (Cachon, 2003) and (Lariviere, 1999). Ours differs from the buyback contract, because our variable payment is not linear on the actual volume shipped via rail but on the actual number of bogies used. For each realization of demand $D = d$, the variable payment $b\min(x,d)$ in the buyback contract is continuous piecewise linear function in d, whereas our payment is not linear, not continuous in d and has some jumps.

Demands in the newsvendor model and ours are perishable. In our model, demand to transport gasoline must be met on daily basis. The rail company can segment customers, e.g., by freight types. Different freight types can be changed at different prices. The rail company is interested in maximizing revenue because variable costs are small, compared to the fixed sunk cost of acquiring rail cars. Rail freight is a prime candidate for perishable-asset revenue management (RM) techniques. However, papers on railway RM are quite limited, compared to those in "traditional" RM industries, e.g., airline, hotel and car rental. Railway RM papers include, e.g., (Armstrong and Meissner, 2010) and (Kraft et al., 2000).

The rest of the paper is organized as follows: Sections 1 and 2 give an introduction and a formulation of the problem. We provide an analysis and a numerical example in Section 3. Section 4 contains a short summary and a few future research directions.

2 FORMULATION

Throughout this article, let $\mathbb{Z}_+$ denote the set of nonnegative integers, and $\mathbb{R}_+$ the set of nonnegative real numbers.

Consider a shipper who needs to transport fuel (e.g., diesel and gasoline) daily using either road or rail. Let D_i be a random demand (volume in liters) for transport on day i for each $i = 1,2,\ldots,n$, where n is the length of the planning horizon. Prior to the start of the planning horizon, the shipper and the rail company establish a long-term contract: The rail company guarantees to provide up to $y \in \mathbb{Z}_+$ bogies on each day throughout the planning horizon, and the shipper pays an upfront of $\tilde{f}y$ where $\tilde{f}$ is the per-bogie upfront fee. The upfront payment is collected at the beginning of the planning horizon.

Throughout the planning horizon, the shipper also pays an additional transportation cost, which is linearly proportional to the number of tanks actually used on that day. Let κ be the capacity of the tank (in liters). For day $i = 1,2,\ldots,n$, the number of tanks actually used by the shipper is

$$Z_i = \min(y, \lceil D_i/\kappa \rceil) \tag{1}$$

$$= \begin{cases} y & \text{if } y \le \lceil D_i/\kappa \rceil \\ \lceil D_i/\kappa \rceil & \text{if } y > \lceil D_i/\kappa \rceil. \end{cases} \tag{2}$$

$$= \begin{cases} y & \text{if } D_i \geq \kappa y \\ \lceil D_i/\kappa \rceil & \text{if } D_i < \kappa y. \end{cases} \tag{3}$$

To ship total volume of D_i liters, we need $\lceil D_i/\kappa \rceil$ tanks. (We divide the total demand by the tank capacity and round up to its ceiling, because the number of tanks has to be an integer.) However, the rail company provides up to the number of reserved bogies y. One tank needs one bogie. Hence, the number of tanks actually used is the minimum of these two quantities. Detailed explanation is as follows: Suppose the demand is larger than the total capacity of reserved bogies ($D_i > \kappa y$). The number of reserved bogie is less than or equal to the number of tanks actually required to accommodate all demand ($y \leq \lceil D_i/\kappa \rceil$) All reserved bogies are used; this corresponds to the upper case in (2) and (3). On the other hand, suppose that the demand is less than the capacity of reserved bogies ($D_i < \kappa y$). We do not need to use all bogies ($y > \lceil D_i/\kappa \rceil$), and only $\lceil D_i/\kappa \rceil$ is actually needed to carry all demand D_i. This corresponds to the lower case in (2) and (3).

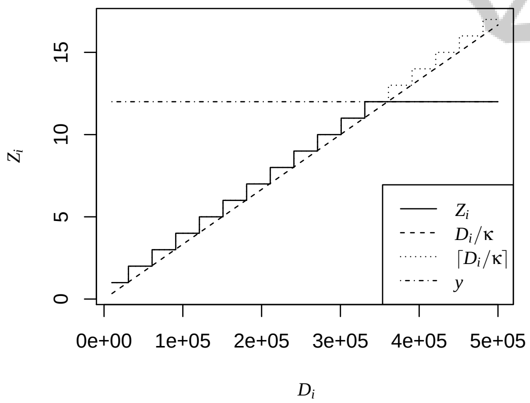

Figure 1: Number of bogies actually used $Z_i = \min(y, \lceil D_i/\kappa \rceil)$ versus demand, D_i.

Figure 1 shows the number of bogies actually used, when demand ranges from 0 up to 500000 given that capacity is $\kappa = 30000$ and the number of reserved bogies is $y = 12$ (shown in the horizontal dashed-dotted line). The number of bogies needed for all demand is $\lceil D_i/\kappa \rceil$, the stepwise nondecreasing function shown in the dotted line. (The quantity inside the ceiling D_i/κ is a linear increasing function shown in the dashed line.) The solid line in Figure 1 is the number of bogies actually used, $Z_i = \min(y, \lceil D_i/\kappa \rceil)$ as in (1). (In fact, two bogies are often fitted to each carriage train at the two ends. By "one bogie," we actually mean one pair of bogies. Furthermore, we do not consider a double-stack car/well wagon.)

The additional payment of gZ_i is transferred to the rail company, where the per-liter rail contract rate is r, and the per-tank transportation fee is $g = r\kappa$. If demand exceeds the total volume of tanks initially reserved upfront, κy, then the excess $(D_i - \kappa y)^+$ is transported by the trucking company at a per-liter rate t, where $0 < r < t$. The expected total transportation cost incurred by the shipper is

$$\begin{aligned} \psi(y) &= \tilde{f} y + E\Big\{ \sum_{i=1}^{n} \big[g \min(y, \lceil D_i/\kappa \rceil) + t(D_i - \kappa y)^+ \big] \Big\} \\ &= E\Big\{ \sum_{i=1}^{n} \big[f y + g \min(y, \lceil D_i/\kappa \rceil) + t(D_i - \kappa y)^+ \big] \Big\} \end{aligned} \tag{4}$$

where $f = \tilde{f}/n$ the upfront payment fee per day. Inside the square brackets in (4), the first term is the fixed cost of reserving y bogies, whereas the second and third terms are the random transportation costs by a train and a truck, respectively. Note that (4) is also valid at $y = 0$: If we do not use rail and use only truck, then the expected cost is $\psi(0) = t \sum_{i=1}^{n} E[D_i]$, the per-unit truck cost times the expected total demand.

The shipper's problem is to determine $y \in \mathbb{Z}_+$ the number of bogies to reserve upfront at the beginning of the planning horizon. The trade-offs are obvious: The shipper must commit y before daily demand is materialized. If y is larger than the number of bogies actually needed on that day Z_i, no refund in provided for unused bogies, and the shipper pays too much upfront. On the other hand, if $y < Z_i$, the excess volume needs to be transported by truck, whose per-liter freight rate is more expensive. A firm commitment is hedged against demand uncertainty; this is similar to the newsvendor problem.

Despite some similarity, our payment does differ from that in the newsvendor model. In the standard newsvendor problem, the objective function is based on the demand actually served, $\min(x, D)$, where D is a random demand and x is the order quantity. If the daily payment to the rail company were linearly proportional to daily volume actually shipped, then the standard newsvendor problem would be applied directly. Nevertheless, the daily payment to the rail company is linearly proportion to the total volume of tanks actually used, $\kappa \min(y, \lceil D_i/\kappa \rceil)$, which is greater than or equal to the total volume actually shipped, $\min(\kappa y, D_i)$. Then,

$$\begin{aligned} g \min(y, \lceil D_i/\kappa \rceil) &= r\{\kappa \min(y, \lceil D_i/\kappa \rceil)\} \\ &\geq r\kappa \min(y, D_i/\kappa) = r[\min(\kappa y, D_i)]. \end{aligned} \tag{5}$$

The left-hand-side (LHS) can be viewed as the freight rate times the total volume of tanks, whereas the right-hand-side (RHS) is the freight rate times the total volume actually carried in the tanks. The total volume of tanks (LHS), the term inside the curly brackets, is at least the total volume actually carried (RHS), the term inside the square brackets. Our scheme implies higher payment to the rail company, compared to other schemes based on the volume actually shipped. In the analysis below, we will show that, because of this payment scheme, the expected cost is not convex, whereas that in the newsvendor model is convex.

3 ANALYSIS

Assume that the vector of daily demands $D_1, D_2, \ldots, D_n$ are independent and identically distributed $\mathbb{R}_+$-valued random variables. Let F be the common distribution and D the random variable with such distribution, i.e., the random daily demand. Let $\bar{F}$ be the complementary cumulative distribution function; i.e., $\bar{F}(t) = P(D > t) = 1 - P(D \leq t) = 1 - F(t)$. Let F^{-1} denote the quantile function. Define the first-order loss function of D as $L(x) = E[(D-x)^+] = \int_x^\infty \bar{F}(u)du$. Also define the limited expected value as $\mathrm{LEV}(x) = E[\min(x, D)] = \int_0^x \bar{F}(u)du$. For well-known continuous distributions (e.g., lognormal, gamma, Weibull, beta), the formulas for the limited expected value and the loss function are readily available. Furthermore, if an expression for $L(x) = E[(D-x)^+]$ is available, then we can easily obtain a formula for $\mathrm{LEV}(x) = E[\min(x, D)]$ by using $E[\min(x, D)] = E[D] - E[(D-x)^+]$.

Define the expected daily cost, if the number of reserved bogies is y, as follows:

$$\pi(y) = fy + gE[\min(y, \lceil D/\kappa \rceil)] + tE[(D - \kappa y)^+] \tag{6}$$

$$= fy + g\sum_{i=0}^{y-1} \bar{F}(i\kappa) + t\int_{\kappa y}^{\infty} \bar{F}(u)du. \tag{7}$$

Then, the total expected cost over n day is

$$\psi(y) = n\pi(y) \tag{8}$$

since the expectation of the sum of random variables is the sum of the expected values. [Derivation of (7) is provided in Appendix.] Henceforth, we focus on the expected daily cost. Note that expression (7) provides an easy way to evaluate $\pi(y)$: The last term $E[(D-x)^+] = \int_x^\infty \bar{F}(u)du$ where $x = \kappa y$ is the first-order loss function of D.

Proposition 1 characterizes an optimal solution $y^* = \mathrm{argmin}\, \pi(y)$ when demand is deterministic; specifically, $P(D = d) = 1$. Proofs of all propositions are given in Appendix.

Proposition 1. *Suppose $f \leq (t-r)\kappa$.*

1. *If $d < (f+g)/t$, $y^* = 0$, and only truck is used.*
2. *If $(f+g)/t \leq d \leq \kappa$, $y^* = 1$, only train is used.*
3. *If $d > \kappa$, both train and truck are used, and*

$$y^* = \mathrm{argmin}\{\pi(\lfloor d/\kappa \rfloor), \pi(\lceil d/\kappa \rceil)\}.$$

Suppose $f > (t-r)\kappa$. Then, $y^* = 0$ and only truck is used.

The results in Proposition 1 make sense economically. If the fixed upfront is very large (i.e., $f > (t-r)\kappa$), then we use only truck. On the other hand, suppose that the fixed upfront is not that large. Then we have to take into account the size of demand. If demand is smaller than the cutoff $(f+g)/t$, we use only truck to avoid paying upfront. If demand is larger than the cutoff and less than the bogie capacity, then it is optimal to reserve only one bogie, and no truck is needed, because all demands can be put into one bogie. If demand is larger than the bogie capacity, then we use both truck and train.

For shorthand, denote $\Delta(y) = \pi(y+1) - \pi(y)$. If the expected daily cost were convex, then we could easily derive an optimality condition,

$$\mathrm{argmin}\, \pi(y) = \mathrm{argmin}_{y \in \mathbb{Z}_+} \{\Delta(y) \geq 0\}. \tag{9}$$

Unfortunately, $\pi(y)$ is not convex: Figure 2 shows $\pi(y)$, with constant demand $P(D = d) = 1$ where $d = 458000$, $\kappa = 30000$, $r = 0.3691$, $t = 0.49$, $f = 2000$.

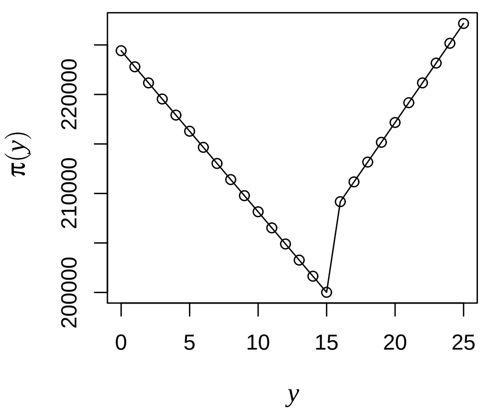

Figure 2: Expected daily cost $\pi(y)$ is not convex on $\mathbb{Z}_+$.

There are two kinks at $\lfloor d/\kappa \rfloor = 15$ and $\lceil d/\kappa \rceil = 16$.

Henceforth, we consider the problem with random demand. In Proposition 2 we identify a condition under which the optimality condition (9) holds, and provide a closed-form optimal expression for the number of reserved bogies, which minimizes the expected daily cost. Let ζ denote a density function of F.

Proposition 2. *Assume that*

$$\frac{\kappa\zeta(\kappa y)}{F(\kappa(y+1)) - F(\kappa y)} \leq \frac{t}{r} \tag{10}$$

for all $y \in \mathbb{Z}_+$.

1. *The expected cost* $\pi(y)$ *is unimodal and attains its minimum at* $y^* = \mathrm{argmin}_{y \in \mathbb{Z}_+}\{\Delta(y) \geq 0\}$ *where*

$$\Delta(y) = f + r\kappa\bar{F}(\kappa y) - t\int_{\kappa y}^{\kappa(y+1)} \bar{F}(u)du. \tag{11}$$

Furthermore, if $f \geq \kappa(t-r)$, *then* $\pi(y)$ *attains its minimum at* $y^* = 0$.

2. *The optimal number of reserved bogies increases if one or more of the following conditions hold: (i) the fixed upfront payment* f *decreases; (ii) the rail freight rate* r *decreases; (iii) the truck freight rate* t *increases.*

If the fixed upfront payment is sufficiently large, e.g., at least $\kappa(t-r)$, then the shipper should not use rail service; all daily demand should be accommodated by truck. Part 2 also gives a sensitivity analysis with respect to a change in a contract parameter. The results make economically sense.

To find an optimal solution using (11) in Proposition 2, we need to be able to evaluate the definite integral $\int_{\kappa y}^{\kappa(y+1)} \bar{F}(u)du$. This can be written as the difference of the two loss functions

$$\begin{aligned}\int_{\kappa y}^{\kappa(y+1)} \bar{F}(u)du &= \int_{\kappa y}^{\infty} \bar{F}(u)du - \int_{\kappa(y+1)}^{\infty} \bar{F}(u)du \\ &= L(\kappa y) - L(\kappa(y+1)).\end{aligned}$$

Alternatively, it can be computed as the difference of the two limited expected values

$$\begin{aligned}\int_{\kappa y}^{\kappa(y+1)} \bar{F}(u)du &= \int_{0}^{\kappa(y+1)} \bar{F}(u)du - \int_{0}^{\kappa y} \bar{F}(u)du \\ &= \mathrm{LEV}(\kappa(y+1)) - \mathrm{LEV}(\kappa y).\end{aligned}$$

As previously mentioned, a closed-form expression for either loss function or limited expected value can be found in standard probability and statistics textbooks. A numerical example, which describes how to find an optimal solution, is given in at the end of this section.

Although the expected cost $\pi(y)$ is not convex, it is unimodal, and an optimal solution can be found analytically, when condition (10) holds. We argue that this condition is not very restrictive. Recall that the cumulative distribution function is the area under the density curve: $F(t) = \int_0^t \zeta(u)du$. Then,

$$F(\kappa(y+1)) - F(\kappa y) \approx \kappa\zeta(\kappa y). \tag{12}$$

In (12), the LHS is the area under the density curve from $t_1 = \kappa y$ to $t_2 = \kappa(y+1)$, whereas the RHS is the area of the rectangle with height equal to density at t_1, $\zeta(\kappa y)$, and width equal to the length of interval $[t_1, t_2]$, κ. With this approximation, condition (10) becomes

$$1 \approx \frac{\kappa\zeta(\kappa y)}{F(\kappa(y+1)) - F(\kappa y)} \leq \frac{t}{r}.$$

In our model, the truck rate is strictly greater than the rail rate, $t > r$; so condition (10) usually holds.

We now turn our attention to a heuristic solution. We approximate the expected daily cost by removing the ceiling function in (6):

$$\begin{aligned}\nu(y) &= fy + gE[\min(y, D/\kappa)] + tE[(D-\kappa y)^+] \\ &= fy + rE[\min(\kappa y, D)] + tE[(D-\kappa y)^+].\end{aligned}$$

The approximated cost $\nu(y)$ differs from the exact cost $\pi(y)$ only on the second term: In $\nu(y)$, the cost charged by the rail company is based on the total volume actually shipped, whereas in $\pi(y)$ it is based on the total volume of tanks actually used; see (5). Clearly, the approximated cost is a lower bound on the exact cost, by the definition of the ceiling function. The approximated cost $\nu(y)$ is continuously differentiable, and a one-line formula for a minimizer is derived in Proposition 3.

Proposition 3. *If* $f < \kappa(t-r)$, *then the approximated cost* $\nu(y)$ *is convex and attains its minimum at*

$$y_a = \frac{1}{\kappa}F^{-1}\left(1 - \frac{f/\kappa}{t-r}\right). \tag{13}$$

We refer to $1 - f/[\kappa(t-r)]$ as the *critical ratio*. Recall that in the newsvendor model with overage c_o and underage c_u, the optimal order quantity is $F^{-1}(c_u/(c_u+c_o))$ where F is the demand distribution, and $c_u/(c_u+c_o)$ is the so-called critical ratio. In analogous, the overage is f/κ, the per-liter cost of reserving too much and some bogies are not used. The underage occurs if we do not reserve enough bogies and need to use both road and rail; so the per-liter underage is $t - r - f/\kappa$. The underage can be viewed as the incremental per-liter cost if truck is used. Specifically, it is $t - (r + f/\kappa)$, the per-liter cost by truck t minus the per-liter cost by rail, which is the sum of the per-liter freight rate and the per-liter upfront $(r + f/\kappa)$.

Our heuristic solution is to reserve

$$y_a^* = \begin{cases} \lceil y_a \rceil & \text{if } \pi(\lceil y_a \rceil) \geq \pi(\lfloor y_a \rfloor) \\ \lfloor y_a \rfloor & \text{otherwise} \end{cases}$$

Our heuristic solution requires evaluation of the expected daily cost function $\pi(y)$ and compares the expected values at the floor and ceiling.

In practice, the per-liter rates by train and truck, r and t, may depend on daily crude oil or gas price;

they may be random when the contract is signed, and their values are realized daily. Then, the expected total cost (4) becomes

$$E\Big\{\sum_{i=1}^{n}\big[fy+\kappa R_i \min(y,\lceil D_i/\kappa\rceil)+T_i(D_i-\kappa y)^+\big]\Big\}$$

where R_i and T_i are per-liter rates by train and truck on day i, respectively. We further assume that the per-liter rates are independently and identically distributed and that they are independent of demands. Our formulation and analysis remain valid, if we now interpret $r = E[R_1]$ and $t = E[T_1]$ as the expected per-liter rates by train and truck, respectively.

Suppose that the expected per-liter rates are exogenously specified. Assume that the shipper makes decision according to (13). If the rail company wants to allocate exactly $y^u \in \mathbb{Z}_+$ bogies to the shipper, then the daily upfront payment should be

$$f = \kappa(t-r)[1-F(y^u\kappa)]. \tag{14}$$

We obtain (14) simply by solving (13) for the fixed upfront f.

NUMERICAL EXAMPLE

Consider one of the biggest petroleum companies in Thailand, who wants to transport hi-speed diesel from Saraburee province in the central region to Phare province in the northern region. Two modes of transportation are available: 1) road, service offered by a 3PL trucking company, and 2) rail from Baan Pok Pak station in Saraburee to Denchai station in Phare, service offered by State Railway of Thailand. The distance is about 500 kilometers. Either truck or train can make a round trip within a single day.

The 310-day demands in year 2013 are collected, and its histogram is shown in Figure 3. We fit a log-normal distribution with parameters $\mu = 12.756$ and $\sigma = 0.488$. (These parameters are obtained using a `fitdistr` command in R.) The corresponding density is superimposed over the histogram in Figure 3.

The daily demand has mean

$$E[D] = \exp(\mu+\sigma^2/2) = 390456.6 \text{ liters}$$

and standard deviation

$$\sqrt{\mathrm{var}(D)} = \sqrt{\exp(2\mu+\sigma^2)(\exp(\sigma^2)-1)} = 202470.4 \text{ liters}.$$

The coefficient of variation is 0.52; in words, the standard deviation is about half of the mean. The capacity of a tank is $\kappa = 33000$ liters. The rail per-liter rate is $r = 0.3169$ Thai Baht (THB), the per-tank cost is $g = r\kappa = 10457.7$, and the truck per-liter rate is $t = 0.49$ THB.

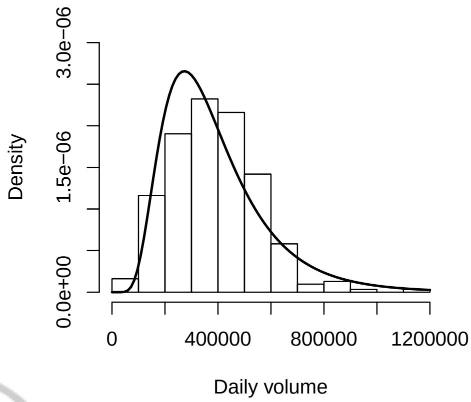

Figure 3: Histogram of daily demand.

Assume that one year has $n = 310$ days. It follows from Proposition 2 Part 1 that if the fixed daily upfront is greater than $\kappa(t-r) = 5712.3$ THB (or equivalently the fixed upfront is $\tilde{f} = nf = 1770813$ THB per bogie per year), then the shipper would not use rail service at all.

Suppose that the daily upfront is $f = 1000$ THB. For the given daily demand distribution, we choose the maximum number of reserved bogies to be $y^m = 60$; this y^m is sufficient to accommodate daily demand roughly 99.99 percent of the time. We can verify that condition (10) holds for each $y = 1,2,\ldots,y^m$; see Figure 4.

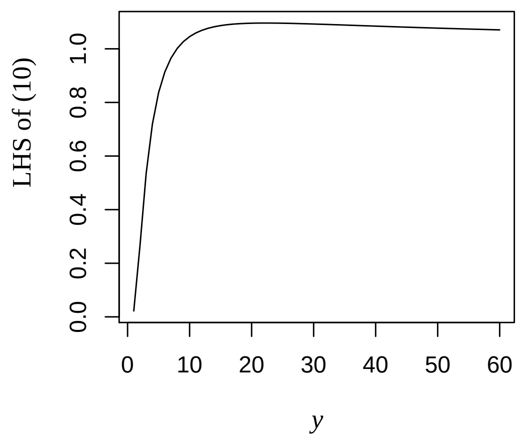

Figure 4: Sufficient condition is verified: LHS values of (10) are at most $t/r = 1.54622$.

Then, we can use (11) to find an optimal solution. The difference $\Delta(y)$ is evaluated for each $y = 1,2,\ldots,y^m$; see last column in Table 1.

The first y at which the difference is nonnegative is $y^* = \mathrm{argmin}\{\Delta(y) \geq 0\} = 16$. The optimal solution given in Proposition 2 requires the shipper to reserve $y^* = 16$ bogies.

At the optimal solution, on each day the expected

Table 1: Expected daily total cost $\pi(y)$, rail cost $gE[\min(y,\lceil D/\kappa\rceil)]$, truck cost $tE[(D-\kappa y)^+]$, upfront fy, and $\Delta(y)$.

y	upfront	cost.truck	cost.rail	$\pi(y)$	$\Delta(y)$
1	1000	175153.72	11457.70	186611.42	-4711.08
2	2000	158984.94	22915.39	181900.33	-4683.55
3	3000	142847.23	34369.55	177216.78	-4556.35
4	4000	126886.69	45773.75	172660.44	-4281.27
5	5000	111398.11	56981.05	168379.17	-3874.89
6	6000	96736.04	67768.24	164504.28	-3386.85
7	7000	83204.85	77912.57	161117.42	-2868.02
8	8000	71000.09	87249.31	158249.40	-2357.12
9	9000	60201.56	95690.71	155892.28	-1878.85
10	10000	50794.65	103218.78	154013.43	-1446.41
11	11000	42699.89	109867.13	152567.02	-1064.89
12	12000	35800.17	115701.96	151502.13	-734.20
13	13000	29961.64	120806.28	150767.93	-451.25
14	14000	25047.97	125268.71	150316.67	-211.43
15	15000	20929.22	129176.03	150105.24	-9.58
16	16000	17486.66	132609.00	150095.66	159.44
17	17000	14614.86	135640.24	150255.10	300.46
18	18000	12222.10	138333.46	150555.56	417.82
19	19000	10229.71	140743.67	150973.38	515.31
20	20000	8570.95	142917.74	151488.69	596.23
21	21000	7189.66	144895.26	152084.92	663.35
22	22000	6038.81	146709.45	152748.26	719.02
23	23000	5079.23	148388.06	153467.29	765.22
24	24000	4278.37	149954.13	154232.51	803.57
25	25000	3609.24	151426.84	155036.08	835.43
26	26000	3049.47	152822.04	155871.51	861.93
27	27000	2580.58	154152.87	156733.45	883.99
28	28000	2187.26	155430.18	157617.44	902.38
29	29000	1856.85	156662.98	158519.83	917.73
30	30000	1578.88	157858.68	159437.56	930.56
31	31000	1344.66	159023.46	160368.12	941.29
32	32000	1147.00	160162.41	161309.41	950.29
33	33000	979.94	161279.76	162259.71	957.85
34	34000	838.53	162379.03	163217.55	964.20
35	35000	718.62	163463.12	164181.75	969.54
36	36000	616.81	164534.48	165151.29	974.05
37	37000	530.21	165595.13	166125.34	977.86
38	38000	456.45	166646.75	167103.20	981.08
39	39000	393.53	167690.75	168084.28	983.81
40	40000	339.77	168728.32	169068.09	986.12
41	41000	293.77	169760.44	170054.21	988.09
42	42000	254.35	170787.94	171042.29	989.76
43	43000	220.53	171811.53	172032.06	991.19
44	44000	191.45	172831.79	173023.24	992.40
45	45000	166.44	173849.21	174015.64	993.44
46	46000	144.88	174864.21	175009.09	994.33
47	47000	126.27	175877.15	176003.42	995.10
48	48000	110.19	176888.32	176998.51	995.75
49	49000	96.28	177897.99	177994.26	996.31
50	50000	84.23	178906.35	178990.58	996.80
51	51000	73.77	179913.60	179987.37	997.21
52	52000	64.69	180919.90	180984.59	997.57
53	53000	56.79	181925.37	181982.16	997.88
54	54000	49.91	182930.14	182980.04	998.15
55	55000	43.91	183934.29	183978.20	998.39
56	56000	38.68	184937.91	184976.58	998.59
57	57000	34.10	185941.07	185975.17	998.76
58	58000	30.10	186943.83	186973.93	998.92
59	59000	26.60	187946.25	187972.85	999.05
60	60000	23.52	188948.37	188971.90	

number of tanks actually used is $E[\min(y^*,\lceil D/\kappa\rceil)] = 11.15$, and the expected total volume of tanks is $\kappa E[\min(y^*,\lceil D/\kappa\rceil)] = 367950$, whereas the expected total volume actually shipped via rail is $E[\min(\kappa y^*, D)] = 354769.5$, and the expected volume shipped via road is $E[(D-\kappa y^*)^+] = 35687$.

Alternatively, the optimal solution can be found via enumeration. In addition to $\Delta(y)$, Table 1 shows the expected total daily cost $\pi(y)$ along with its components, namely the expected rail cost, expected truck cost, and the upfront payment.

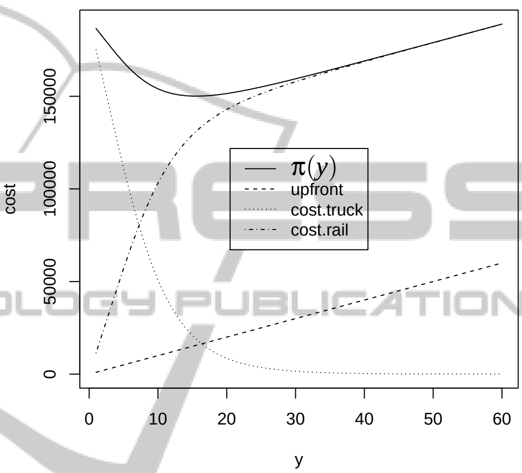

Figure 5: Expected total cost as function of number of bogies.

These costs are plotted in Figure 5. The lowest point of the expected total cost occurs at $y^* = 16$.

If one did not take into account any cost parameters and ignored demand uncertainty, then one could estimate the number of reserved bogies by dividing the daily mean demand by the tank capacity and rounding up, $\lceil E[D]/\kappa\rceil = \lceil 11.84\rceil = 12$, and the expected volume shipped via road would be $E[(D-12\kappa)^+] = 73061.58$, an increase of 105% from that using the optimal solution. Note that the optimal solution y^* is larger than the mean demand divided by capacity: The number of reserved bogies would be less than the optimal solution by 25%. The expected additional cost if one did not use an optimal solution would be $\pi(12) - \pi(y^*) = 151502 - 150096 = 1406$ THB per day, or equivalently $(310)(1406) = 435860$ THB per year.

To find a heuristic solution, we compute the critical ratio $1 - f/[\kappa(t-r)] = 0.8259$; the corresponding quantile divided by the capacity is $y_a = 16.59$. Since $\pi(16) = 150096 < 150255 = \pi(17)$, from our heuristic the number of reserved bogies $y_a^* = 16$. At this particular problem instance, the heuristic solution is exactly equal to the optimal solution. To evaluate performance of our heuristic approach, more numerical examples can be conducted in the future.

Suppose that the rail company can allocate at most $y^u = 7$ bogies on this leg (origin-destination pair). Further assume that the shipper makes decision according to (13). Then, the rail company should offer a daily upfront payment of at least (14), which is equal to 4553.78 THB, or equivalently $\tilde{f} = 1411669$ THB per bogie per year.

Throughout our numerical example, the LEV function is used extensively to evaluate $\Delta(y)$ and $\pi(y)$. For the lognormal distribution with parameters μ and σ, the LEV function is

$$E[\min(D,x)] = \exp(\mu+\sigma^2/2)\Phi\Big(\frac{\log(x)-\mu-\sigma^2}{\sigma}\Big) + x\Big[\Phi\Big(\frac{\log(x)-\mu}{\sigma}\Big)\Big]$$

where Φ is the cumulative distribution function of the standard normal distribution. All calculations in our numerical example are done in R. (For instance, R command for the cumulative distribution function of lognormal is `plnorm` and that of normal is `pnorm`.)

4 CONCLUSION

In summary, we formulate a newsvendor model in which the shipper determines how many bogies to be reserved before random daily demand is materialized. If the number of reserved bogies is very large, we may end up paying a large upfront fee for unused bogies. On the other hand, if we reserve not enough bogies, we incur a large transportation cost via truck. We want to determine an optimal number of bogies to reserve in order to minimize the expected cost. In the analysis, we show that under certain condition, the expected cost is unimodal and a closed-form optimal solution is derived. An easy-to-implement heuristic solution is also proposed. Some sensitivity analysis is provided: The number of reserved bogies decreases, if the upfront payment increases, or the rail freight rate increases, or the truck freight rate decreases. In the numerical example, we show that the optimal solution and the heuristic solution coincide. Furthermore, if we did not take into account of demand uncertainty as in our model, then the number of reserved bogies could differ from the optimal solution by 25% and would result in a significant increase in transportation cost.

Some future research directions are identified below. Recall that, in the analysis, we assume demands are independent and identically distributed. Note that the expected total cost (4) can be written as

$$\psi(y) = E[\sum_{i=1}^{n}\psi_i(y)] = \sum_{i=1}^{n}E[\psi_i(y)] \qquad (15)$$

where $\psi_i(y)$ is the random cost on day i

$$\psi_i(y) = fy + g\min(y,\lceil D_i/\kappa\rceil) + t(D_i-\kappa y)^+.$$

Equation (15) always holds (whether or not demands are independent) since the expectation of the sum of random variables is equal to the sum of the expectation. If demands are not independent but identically distributed, then all of our results remain unchanged, because $\psi(y) = nE[\psi_1(y)]$ as in (8), and we can focus on the expected daily cost, $E[\psi_1(y)] = \pi(y)$ in (6). However, if demands are not identically distributed, then we cannot minimize each term $E[\psi_i(y)]$ separately: We need to minimize the total expected cost $\sum_{i=1}^{n}[\psi_i(y)]$. Cases when demands are nonstationary (i.e., neither independent nor identically distributed) would be an interesting research extension.

Recall that our model considers a contract problem on a single leg in the railroad network. One extension would be to study a contract to transport gasoline over the entire railroad network. Furthermore, for each leg, there may be multiple types of containers, depending on their capacities. The upfront fee depends on the container type, and the per-container charge depends on the container capacity and the distance between an origin and a destination.

Another interesting extension is a railroad's contract design problem. Our model allows the shipper to choose the number of bogies to be reserved. In the contract design problem, we would allow the railroad company to choose the parameters of the contract, e.g., the upfront payment and the per-container freight charge. The contract design problem can be modeled using a game-theoretical framework. We hope to pursue these or related problems in the future.

ACKNOWLEDGEMENTS

The problem was materialized after some discussions with Mr. Apichat Gunthathong, our part-time master student who has been working at the petroleum company (in the numerical example) for 12 years. His independent project, a part of requirement for a master degree in logistics management at the school, was related to our model.

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APPENDIX

Derivation of (7)

For shorthand, denote $M = \lceil D/\kappa \rceil$. Since D is a non-negative random variable with distribution F, M is a $\mathbb{Z}_+$-valued random variable. The cumulative distribution function of M is as follows: For any $t \in \mathbb{R}_+$

$$\begin{aligned} P(M \le t) &= P(\lceil D/\kappa \rceil \le \lfloor t \rfloor) \\ &= P(D/\kappa \le \lfloor t \rfloor) \\ &= F(\lfloor t \rfloor \kappa). \end{aligned}$$

We use the tail-sum formula for expectation:

$$\begin{aligned} E[\min(t,M)] &= \sum_{j=0}^{\infty} P(\min(t,M) > j) \\ &= \sum_{j=0}^{\lfloor t \rfloor - 1} P(M > j) \\ &= \sum_{j=0}^{\lfloor t \rfloor - 1} \bar{F}(j\kappa). \end{aligned}$$

The finite sum in the second term in (7) is obtained. The last term is the first-order loss function of D. □

Proof of Proposition 1

Proof. For shorthand, let

$$\begin{aligned} \tilde{\pi}(y) &= [f - (t-r)\kappa]y + td \\ \hat{\pi}(y) &= fy + g\lceil d/\kappa \rceil. \end{aligned}$$

Given that demand is constant and equal to d, the daily cost (6) becomes

$$\begin{aligned} \pi(y) &= fy + g\min(y, \lceil d/\kappa \rceil) + t(d - \kappa y)^+ \\ &= \begin{cases} fy + gy + t(d-\kappa y) & \text{if} y = 0,1,2,\ldots,\lfloor d/\kappa \rfloor \\ fy + g\lceil d/\kappa \rceil & \text{if } y = \lceil d/\kappa \rceil, \ldots \end{cases} \\ &= \begin{cases} \tilde{\pi}(y) & \text{if } y = 0,1,2,\ldots,\lfloor d/\kappa \rfloor \\ \hat{\pi}(y) & \text{if } y = \lceil d/\kappa \rceil, \lceil d/\kappa \rceil + 1, \ldots \end{cases} \end{aligned}$$

In particular, if $d < \kappa$,

$$\begin{aligned} \pi(0) &= td \\ \pi(y) &= fy + g \quad \text{if } y = 1,2,\ldots \end{aligned}$$

Since both $\tilde{\pi}(y)$ and $\hat{\pi}(y)$ are linear, $\pi(y)$ is a piecewise linear function. Note that $\hat{\pi}(y)$ is strictly increasing since $f > 0$. Let $y^* = \operatorname{argmin} \pi(y)$.

Suppose $f > (t-r)\kappa$. Both $\tilde{\pi}$ and $\hat{\pi}$ are increasing functions, so $y^* = 0$. In words, if the fixed upfront is sufficiently large, we use only truck and do not reserve any bogies ($y^* = 0$).

Suppose $f \le (t-r)\kappa$. That is, $(f+g)/t \le \kappa$ since $g = r\kappa$.

1. If $d < (f+g)/t$, then in this case $d < \kappa$ and $\pi(0) = td \le (f+g) = \pi(1)$, so $y^* = 0$, and we use only truck.
2. If $(f+g)/t \le d < \kappa$, then $\pi(0) > \pi(1)$, so $y^* = 1$; i.e., we reserve one bogie. Since demand does not exceed capacity, one bogie is enough to accommodate entire demand; we do not need any truck.
3. If $d \ge \kappa$, then $\pi(y)$ is linearly decreasing up to $\lfloor d/\kappa \rfloor$ and increasing from $\lceil d/\kappa \rceil$. Specifically, the coefficient of y in $\tilde{\pi}(y)$ is negative when $f \le (t-r)\kappa$, so $\tilde{\pi}(y)$ is linearly decreasing and

minimized at $\lfloor d/\kappa \rfloor$. Since $\hat{\pi}(y)$ is increasing, it is minimized at $\lceil d/\kappa \rceil$. Thus, the global minimizer is $\text{argmax}\{\hat{\pi}(\lceil d/\kappa \rceil), \tilde{\pi}(\lfloor d/\kappa \rfloor)\}$.

$\square$

Proof of Proposition 2

Proof. From (7) we have that

$$\Delta(y) = f + g\bar{F}(\kappa y) - t\int_{\kappa y}^{\kappa(y+1)} \bar{F}(u)du,$$

and

$$\Delta'(y) = -\kappa g \zeta(\kappa y) + \kappa t[F(\kappa(y+1)) - F(\kappa y)].$$

Recall $g = r\kappa$. If (10) holds, then $\Delta'(y) \geq 0$, or equivalently $\Delta(y)$ is increasing; thus, the expected cost function $\pi(y)$ is unimodal. Let $y^* = \text{argmin}_{y \in \mathbb{Z}_+} \{\Delta(y) \geq 0\}$. Then, $\pi(y)$ switches from decreasing to increasing at y^*; hence, $\pi(y)$ attains its minimum at y^*.

Finally, note that

$$\Delta(0) = f + \kappa r - t\int_0^{\kappa} \bar{F}(u)du \geq f - \kappa(t-r)$$

since $0 \leq \bar{F}(u) \leq 1$. If $f \geq \kappa(t-r)$, $\Delta(0) \geq 0$; $\pi(y)$ is increasing on $\mathbb{Z}_+$ and is, thus, minimized at $y^* = 0$. Part 2 follows from (11) and the fact that $\Delta(y)$ is increasing. $\square$

Proof of Proposition 3

Proof. For convenient let $x = \kappa y$ and define

$$\xi(x) = (f/\kappa)x + rE[\min(x,D)] + tE[(D-x)^+]$$

Then, $\text{argmin}\,\nu(y) = (\text{argmin}\,\xi(x))/\kappa$ since $\nu(y) = \xi(x)$. After some simplifications, we get

$$\xi(x) = (f/\kappa)x + rE[D] + (t-r)E[(D-x)^+].$$

The first and second derivatives with respect to x are

$$\xi'(x) = f/\kappa - (t-r)\bar{F}(x)$$
$$\xi''(x) = (t-r)F'(x) = (t-r)\zeta(x) \geq 0$$

respectively. The approximated cost function $\xi(x)$ is convex. If $f < \kappa(t-r)$, then the optimality condition is that $\xi'(x) = 0$; i.e., $x = F^{-1}(1 - f/[\kappa(t-r)])$ and (13) follows immediately. $\square$