

# Enhanced Kernel Uncorrelated Discriminant Nearest Feature Line Analysis for Radar Target Recognition

Chunyu Wan, Xuelian Yu, Yun Zhou and Xuegang Wang

*School of Electronic Engineering, University of Electronic Science and Technology of China, York, China*

**Keywords:** Radar Target Recognition, Feature Extraction, Nearest Feature Line, Uncorrelated Constraint, Kernel Technique.

**Abstract:** In this paper, a new subspace learning algorithm, called enhanced kernel uncorrelated discriminant nearest feature line analysis (EKUDNFLA), is presented. The aim of EKUDNFLA is to seek a feature subspace in which the within-class feature line (FL) distances are minimized and the between-class FL distances are maximized simultaneously. At the same time, an uncorrelated constraint is imposed to get statistically uncorrelated features, which contain minimum redundancy and ensure independence, and thus it is highly desirable in many practical applications. Optimizing an objective function in a kernel feature space, nonlinear features are extracted. In addition, a weighting coefficient is introduced to adjust the proportion between within-class and between-class information to get an optimal effect. Experimental results on radar target recognition with measured data demonstrate the effectiveness of the proposed method.

## 1 INTRODUCTION

Automatic target recognition (ATR) (Chen *et al.*, 2005) is a research topic of high interest in modern radar technology. High resolution range profiles (HRRP) contain rather detailed structural information of a target, thus providing us with a more reliable tool for ATR. One of the key problems of radar target recognition using HRRP is how to extract robust and effective features (Yu and Liu, 2008). Over the past few years, many classical methods have been developed and applied to radar target recognition successfully, such as principal component analysis (PCA) (Turk and Pentland, 1991), linear discriminant analysis (LDA) (Belhumeur *et al.*, 1997), locality preserving projections (LPP) (He *et al.*, 2005), neighborhood preserving projections (NPP) (Pang *et al.*, 2005), neighborhood preserving embedding (NPE) (He *et al.*, 2005) and etc. Although these methods have achieved reasonably good performance for radar target recognition, they cannot perform satisfactorily when the number of training samples per class is small.

In the NN-based classification, the representational capacity and the error rate depends on how the prototypes are chosen to account for possible

variations and also how many prototypes are available. In practical applications, only a small number of training samples are available. In order to expand the representational capacity of limited training samples, Li *et al.* (1999) proposed nearest feature line (NFL), which uses linear interpolation and extrapolation between each pair of feature points to cope with various changes. The classification is done by using the minimum distance between the feature point of the query and the FL's. The classification result also provides a quantitative position number as a byproduct which can be used to indicate the relative change between the query point and the two associated training samples. Owing to the excellent generalization capacity, NFL has been successfully used to address many recognition problems. However, it only used the NFL metric in classification stage.

Over the past few years, some subspace learning algorithms based on the idea of NFL have been proposed. For instance, Zheng *et al.* (2006) proposed nearest feature line-based nonparametric discriminant analysis (NFL-NDA), Pang *et al.* (2007) put forward nearest feature line space (NFLspace). The good properties are achieved by adopting the idea of the nearest feature line to both subspace learning stage and classification stage. Lu *et al.* (2010) presented uncorrelated discriminant nearest feature

line analysis (UDNFLA), and Yan *et al.* (2011) proposed neighborhood discriminant nearest feature line analysis (NDNFLA). Among these methods, NFL-NDA and NFLspace only use the within-class information and do not consider the between-class information, which is deficient for subspace learning. UDNFLA and NDNFLA use both within-class and between-class information, but they all use linear technique to compute the feature space, which is inadequate to describe the complexity of real data structure that is usually nonlinear. In addition, all of them give the same emphasis on within-class and between-class scatter matrix, which is not optimum because the two matrices make different influence on the recognition result.

Motivated by the above observations, we propose in this paper a new nonlinear NFL-based subspace learning method, called enhanced kernel uncorrelated discriminant nearest feature line analysis (EKUDNFLA), for radar target recognition. Firstly, the data is nonlinearly mapped into an implicit high dimensional feature space, in which the data is as linearly separable as possible. Then, proposed method minimized the within-class FL distances and maximized the between-class FL distances simultaneously, as more discriminant information can be exploited. And meanwhile it imposed an uncorrelated constraint to make the extracted features statistically uncorrelated. Uncorrelated features contain minimum redundancy and ensure independence of features. They are highly desirable in practical applications. EKUDNFLA can exploit more discriminant information and is more suitable for recognition tasks. Kernel technique will be used to solve the transformation matrix in the high dimensional feature space. Finally, a weighting coefficient is introduced into the objective function to get the optimum proportion between the within-class and between-class information.

## 2 NFL AND UDNFLA

### 2.1 NFL

Consider a data set  $X = [x_1, \dots, x_N]$  in  $R^D$ . Suppose that  $x_m$  and  $x_n$  are two samples coming from the same class, the straight line passing through the two samples is called a feature line (FL), denoted as  $\overline{x_m x_n}$ . The membership of query point  $x_q$  is measured by the Euclidean distance between  $x_q$  and

its projection point  $x_p$  on the line  $\overline{x_m x_n}$ , which is termed as the FL distance and denoted as  $\|x_q - x_p\|$ . The less the FL distance is, the more probability that  $x_q$  belongs to the same class as  $x_m$  and  $x_n$ . The projection point (Pang, *et al.*, 2007)  $x_p$  can be computed as below:

$$x_p = x_m + \mu(x_n - x_m) \quad (1)$$

Where,  $\mu = \frac{(x_q - x_m)^T(x_n - x_m)}{(x_n - x_m)^T(x_n - x_m)}$ .

### 2.2 UDNFLA

The aim of UDNFLA is to find a projection matrix  $V$  that maps each data point  $x_i$  to a lower dimensional subspace  $R^d$  ( $d \ll D$ ) by  $y_i = V^T x_i$ . The optimal transformation matrix  $V$  is obtained by solving the following optimization problem (Lu *et al.*, 2010):

$$\begin{aligned} \min_V J(V) &= \text{tr}[V^T(A - B)V] \\ \text{s.t.} \quad &V^T S_i V = I \end{aligned} \quad (2)$$

where,  $A = (1/N_p) \sum_{i=1}^N \sum_{m,n \in P(i)} (x_i - x_m^i)(x_i - x_m^i)^T$ ,  $B = (1/N_R) \sum_{i=1}^N \sum_{m,n \in R(i)} (x_i - x_m^i)(x_i - x_m^i)^T$ ,  $P(i)$  denotes the samples sharing the same class label with  $x_i$ ,  $R(i)$  denotes the samples with different class label with  $x_i$ ,  $N_p$  and  $N_R$  are the numbers of samples in  $P(i)$  and  $R(i)$ , respectively.

The minimization of (2) can be converted to solve the following generalized eigenvalue problem

$$(A - B)v = \lambda S_i v \quad (3)$$

Let  $v_1, \dots, v_d$  be the eigenvectors of (3) corresponding to the  $d$  smallest eigenvalues, then the transformation matrix of UDNFLA is obtained by  $V = [v_1, \dots, v_d]$ .

## 3 EKUDNFLA

As mentioned in section I, since UDNFLA is linear method, it may not perform satisfactorily when the data structure is highly nonlinear. Moreover, same emphasis is laid on the within-class distances and

between-class distances, which is not optimum for improving recognition performance.

In this section, we first extend the UDNFLA to nonlinear form using kernel technique and yield kernel uncorrelated discriminant nearest feature line analysis (KUDNFLA), then it is modified with a weighting coefficient, which finally gives rise to the enhanced kernel uncorrelated discriminant nearest feature line analysis (EKUDNFLA).

### 3.1 KUDNFLA

To begin with, the data set is mapped into an implicit high-dimensional feature space  $F$  by using a nonlinear function  $\phi : x \in R^D \rightarrow \phi(x) \in F$ . Then, in the feature space  $F$ , the projection of  $\phi(x_i)$  onto the FL formed by  $\phi(x_m)$  and  $\phi(x_n)$  can be defined as:

$$\begin{aligned} x_{\phi, mn}^i &= \phi(x_m) + \mu^\phi (\phi(x_n) - \phi(x_m)) \\ &= (1 - \mu^\phi) \phi(x_m) + \mu^\phi \phi(x_n) \end{aligned} \quad (4)$$

with

$$\begin{aligned} \mu^\phi &= \frac{(\phi(x_i) - \phi(x_m))^T (\phi(x_n) - \phi(x_m))}{(\phi(x_n) - \phi(x_m))^T (\phi(x_n) - \phi(x_m))} \\ &= \frac{\langle \phi(x_i) - \phi(x_m), \phi(x_n) - \phi(x_m) \rangle}{\langle \phi(x_n) - \phi(x_m), \phi(x_n) - \phi(x_m) \rangle} \end{aligned} \quad (5)$$

where,  $\langle \cdot, \cdot \rangle$  denotes inner product. By introducing a kernel function,  $k(x_1, x_2) = \langle \phi(x_1), \phi(x_2) \rangle$ , (5) can be rewritten as:

$$\mu^\phi = \frac{k(x_i, x_n) - k(x_i, x_m) - k(x_m, x_n) + k(x_m, x_m)}{k(x_n, x_n) - k(x_n, x_m) - k(x_m, x_n) + k(x_m, x_m)} \quad (6)$$

Now, in order to minimize the within-class FL distances and maximize the between-class FL distances simultaneously, we need to solve the following minimization problem:

$$\begin{aligned} \min_V J(V) &= \frac{1}{N_p} \sum_{i=1}^N \sum_{m, n \in P(i)} \|V^T \phi(x_i) - V^T x_{\phi, mn}^i\|^2 \\ &\quad - \frac{1}{N_R} \sum_{i=1}^N \sum_{m, n \in R(i)} \|V^T \phi(x_i) - V^T x_{\phi, mn}^i\|^2 \\ &= \text{tr} [V^T (S_w^\phi - S_b^\phi) V] \end{aligned} \quad (7)$$

where  $S_w^\phi$  and  $S_b^\phi$  are the within-class and between-class FL distance scatter matrix, respectively, which are defined as:

$$S_w^\phi = \frac{1}{N_p} \sum_{i=1}^N \sum_{m, n \in P(i)} \|\phi(x_i) - x_{\phi, mn}^i\|^2 \quad (8)$$

$$= \frac{1}{N_p} \sum_{i=1}^N \sum_{m, n \in P(i)} (\phi(x_i) - x_{\phi, mn}^i) (\phi(x_i) - x_{\phi, mn}^i)^T$$

$$S_b^\phi = \frac{1}{N_R} \sum_{i=1}^N \sum_{m, n \in R(i)} \|\phi(x_i) - x_{\phi, mn}^i\|^2 \quad (9)$$

$$= \frac{1}{N_R} \sum_{i=1}^N \sum_{m, n \in R(i)} (\phi(x_i) - x_{\phi, mn}^i) (\phi(x_i) - x_{\phi, mn}^i)^T$$

Next, to make the extracted features statistically uncorrelated, the following uncorrelated constraint (Yu and Wang, 2008) is considered:

$$V^T S_i^\phi V = I \quad (10)$$

where,  $S_i^\phi = \frac{1}{N} \sum_{i=1}^N (\phi(x_i) - m^\phi) (\phi(x_i) - m^\phi)^T$  is the total scatter matrix in  $F$ , with  $m^\phi = (1/N) \sum_{i=1}^N \phi(x_i)$ .

Combing (7) and (10), the KUDNFLA can be formulated as the following constrained minimization problem:

$$\min_{V^T S_i^\phi V = I} \text{tr} [V^T (S_w^\phi - S_b^\phi) V] \quad (11)$$

Since each column of  $V$  should lie in the span of  $\phi(x_1), \phi(x_2), \dots, \phi(x_N)$ , we can write

$$V = \left[ \sum_{i=1}^N \alpha_{i,1} \phi(x_i), \dots, \sum_{i=1}^N \alpha_{i,d} \phi(x_i) \right] = \phi(X) A \quad (12)$$

where  $\alpha_{j,i}$  ( $j = 1, 2, \dots, d$ ) denotes the  $i$ -th entry of the coefficient vector  $\alpha_j$ , and  $A = [\alpha_1, \alpha_2, \dots, \alpha_d] \in R^{N \times d}$ .

Let  $Q = V^T (\phi(x_i) - x_{\phi, mn}^i)$ , and considering (12) and (4), we have

$$\begin{aligned} Q &= A^T \phi^T(X) [\phi(x_i) - (1 - \mu^\phi) \phi(x_m) - \mu^\phi \phi(x_n)] \\ &= A^T [\zeta_i - (1 - \mu^\phi) \zeta_m - \mu^\phi \zeta_n] \end{aligned} \quad (13)$$

where,

$$\begin{aligned} \zeta_i &= \phi^T(X) \phi(x_i) = [\langle \phi(x_1), \phi(x_i) \rangle, \dots, \langle \phi(x_N), \phi(x_i) \rangle]^T \\ &= [k(x_1, x_i), \dots, k(x_N, x_i)]^T \end{aligned}$$

$$\zeta_m = \phi^T(X) \phi(x_m) = [k(x_1, x_m), \dots, k(x_N, x_m)]^T$$

$$\zeta_n = \phi^T(X) \phi(x_n) = [k(x_1, x_n), \dots, k(x_N, x_n)]^T$$

Then, we can get:

$$\begin{aligned}
 V^T S_w^\phi V &= \frac{1}{N_p} \sum_{i=1}^N \sum_{m,n \in P(i)} V^T (\phi(x_i) - x_{\phi, mn}^i) \\
 &\quad \cdot (\phi(x_i) - x_{\phi, mn}^i)^T V \\
 &= \frac{1}{N_p} \sum_{i=1}^N \sum_{m,n \in P(i)} Q Q^T \\
 &= A^T K_w^{FL} A
 \end{aligned} \tag{14}$$

$$V^T S_b^\phi V = A^T K_b^{FL} A \tag{15}$$

where  $K_w^{FL}$  and  $K_b^{FL}$  are called kernel within-class and between-class FL distance scatter matrix, respectively, with

$$\begin{aligned}
 K_w^{FL} &= \frac{1}{N_p} \sum_{i=1}^N \sum_{m,n \in P(i)} \left\{ \left[ \zeta_i - (1 - \mu^\phi) \zeta_m - \mu^\phi \zeta_n \right] \right. \\
 &\quad \left. \cdot \left[ \zeta_i - (1 - \mu^\phi) \zeta_m - \mu^\phi \zeta_n \right]^T \right\}
 \end{aligned} \tag{16}$$

$$\begin{aligned}
 K_b^{FL} &= \frac{1}{N_R} \sum_{i=1}^N \sum_{m,n \in R(i)} \left\{ \left[ \zeta_i - (1 - \mu^\phi) \zeta_m - \mu^\phi \zeta_n \right] \right. \\
 &\quad \left. \cdot \left[ \zeta_i - (1 - \mu^\phi) \zeta_m - \mu^\phi \zeta_n \right]^T \right\}
 \end{aligned} \tag{17}$$

Similarly, (10) is converted to

$$A^T K_i A = I \tag{18}$$

where,

$$K_i = \frac{1}{N} \sum_{i=1}^N (\zeta_i - c)(\zeta_i - c)^T \tag{19}$$

is kernel total scatter matrix,

with  $c = (1/N) \sum_{i=1}^N \zeta_i$ .

Therefore, (11) becomes

$$\min_{A^T K_i A = I} \text{tr} \left[ A^T (K_w^{FL} - K_b^{FL}) A \right] \tag{20}$$

### 3.2 EKUDNFLA

To further enhance the discrimination power of the learned subspace, we introduce a weighting coefficient  $\beta$  into (20) and give rise to the EKUDNFLA algorithm:

$$\min_{A^T K_i A = I} \text{tr} \left\{ A^T \left[ \beta K_w^{FL} - (1 - \beta) K_b^{FL} \right] A \right\} \tag{21}$$

Where  $\beta$  is a constant between 0 and 1. Obviously, if  $\beta=0.5$ , the EKUDNFLA is reduced to KUDNFLA.

Generally speaking, the between-class FL distances in the feature space  $F$  have been enlarged

by some extent, and more emphasis should be paid on minimizing the within-class FL distances, so the value of  $\beta$  should be greater than 0.5.

Finally, the constrained minimization problem above is reduced to a generalized eigenvalue problem:

$$\left[ \beta K_w^{FL} - (1 - \beta) K_b^{FL} \right] \alpha = \lambda K_i \alpha \tag{22}$$

The matrix  $A$  is determined by eigenvectors corresponding to the eigenvectors corresponding to the  $d$  smallest nonzero eigenvalues of (22). Once  $A$  is obtained, for any point  $x \in R^D$ , it can be mapped to a  $d$ -dimensional point  $z$  by:

$$z = V^T \phi(x) = A^T \zeta \tag{23}$$

Where  $\zeta = [k(x_1, x), \dots, k(x_N, x)]^T$ .

## 4 EXPERIMENTS AND RESULTS

To evaluate the performance of the proposed algorithm, two experiments were performed on radar target recognition with measured HRRPs from three flying airplanes, including An-26, Yark-42, and Cessna Citation S/II. For each airplane, 260 profiles over a wide range of aspects are adopted, and each profile is preprocessed by energy normalization (Yu and Wang, 2008).

In the first experiment, the performance of EKUDNFLA is compared with two classical kernel methods KPCA (Scholkopf, *et al.*, 1998) and KFDA (Mika, *et al.*, 1999). The Gaussian kernel  $k(x_1, x_2) = \exp(-\|x_1 - x_2\|^2 / \sigma^2)$ , is adopted, and the parameter  $\sigma$  is empirically set as 0.2. For EKUDNFLA, the parameter  $\beta$  should be greater than 0.5 as what is said above. It is difficult to determine the optimal value of  $\beta$  analytically since it depends on the original data. But we can get a value which is optimal for our data experimentally with limited training samples. We find that a relatively good and stable result can be obtained if  $\beta$  is within the range between 0.85 and 0.98. So, the parameter  $\beta$  is set as 0.9 in the following experiments. Since we only focus on feature extraction, as for classification, the nearest neighbor classifier using Euclidean distance is employed for the sake of simplicity.

For each airplane, 26, 18 and 13 of all profiles are used for training, respectively, and the remainder for test. Table I tabulates the recognition rates

attained by each method with different number of training samples per target (NTSPT).

As can be seen from Table I, the proposed EKUDNFLA outperforms KPCA and KFDA with gains of 9.58% and 2.29% when the NTSPT is only 13, and 13.63% and 2.06% when the NTSPT is 18, and 10.69% and 0.01% when the NTSPT is increased to 26. It indicates that the FL distance can better characterize the geometrical structure of samples than the conventional euclidean distance, especially when the number of training samples per class is small.

Table 1: Recognition rate (%) obtained by each method.

Method	Recognition rate (%)		
	26	18	13
KPCA	77.06	77.41	76.51
KFDA	87.74	88.98	83.80
EKUDNFLA	87.75	91.04	86.09

In the second experiment, we fix the NTSPT as 18. Since the merit of EKUDNFLA stems from two factors: kernel technique and weighting coefficient, we also evaluated the performance when only one factor is applied. Hence, we derived the kernel uncorrelated discriminant nearest feature line analysis (KUDNFLA) and enhanced uncorrelated discriminant nearest feature line analysis (EUDNFLA), respectively. We have also compared EKUDNFLA with PCA, LDA, and two other NFL-based methods NFLspace and UDNFLA as well. Figure 1 shows the recognition rates versus feature dimensions of all those methods mentioned above except LDA, since the extracted feature dimension is only 2 for our experiment. The top recognition rates along with the corresponding dimensions obtained by each method are listed in Table II. The following observations can be made from the experimental results:

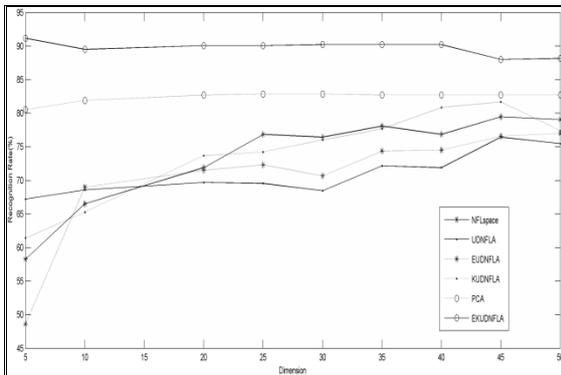


Figure 1: Recognition rates versus feature dimensions of each method.

1) The proposed EKUDNFLA is overall superior to all the other NFL metric subspace learning methods involved in our experiments in terms of recognition rate.

2) Compared with the linear NFL metric method UDNFLA, KUDNFLA attains higher recognition rate at almost each dimension. This is mainly because KUDNFLA is a nonlinear method and the kernel technique is helpful for improving its discriminative power.

3) EKUDNFLA significantly outperforms KUDNFLA in terms of recognition rate. It can be attributed to the introduction of the weighting coefficient, which gives more emphasis on minimizing the within-class FL distances than maximizing the between-class FL distance, so as to use the within-class and between-class information more effectively.

4) As we can see, the proposed EKUDNFLA also outperforms conventional linear method PCA and LDA with gains of 8.46% and 12.27%.

Table 2: Top recognition rate (%) and corresponding dimension of each method.

Method	NFL space	UDNFLA	EUDNFLA	KUDNFLA	EKUDNFLA	LDA	PCA
Recognition rate (%)	79.48	76.45	76.99	81.68	91.18	78.91	82.72
Dimension	45	45	50	45	5	2	45

## 5 CONCLUSIONS

We have proposed in this paper a new subspace learning method, called enhanced kernel uncorrelated discriminant nearest feature line analysis (EKUDNFLA), for radar target recognition. The method achieves good discrimination ability by minimizing the within-class FL distances and maximizing the between-class FL distances, simultaneously. Furthermore, an uncorrelated constraint is imposed to make the extracted features statistically uncorrelated. Mapping the input data to some high-dimensional feature space using the kernel technique, nonlinear features are extracted. In addition, weighting coefficient is introduced to adjust the proportion between within-class and between-class matrix. Experimental results on radar target recognition with measured data show that EKUDNFLA is overall superior to other NFL-based methods in terms of recognition accuracy. And compared with other conventional feature extraction methods, like PCA, LDA, KPCA and KFDA, EKUDNFLA also shows competitive performance.

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