Propagation and Amplification of a Short Subterahertz Pulse in a Plasma Channel in Air Created by Intense Laser Radiation

A. V. Bogatskaya[1,2,3], A. M. Popov[1,2,3] and E. A. Volkova[2]
[1]*Department of Physics, Moscow State University, Moscow, 119991, Russia*
[2]*D. V. Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, 119991, Russia*
[3]*P. N. Lebedev Physical Institute, RAS, Moscow, 119991, Russia*

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Abstract: The evolution of the electron energy distribution function in the plasma channel created in air by the third harmonic of the Ti:Sa-laser pulse of femtosecond duration is studied. It is shown that such a channel can be used to amplify few-cycle electromagnetic pulses in subterahertz frequency range at the time of relaxation of the energy spectrum in air determined by the vibrational excitation of the nitrogen molecules. The coefficients of the gain as a function of time, electron concentration and frequency of the amplifying radiation are obtained. The propagation of few-cycle radio-frequency pulses through the amplifying medium is analyzed.

1 INTRODUCTION

An important feature of the plasma structure appearing in the field of an ultrashort laser pulse is its strong nonequilibrium. Such nonequilibrium can be used for a number of applications, in particular, for generation of XUV attosecond pulses (Agostini and Di Mauro, 2004, Krausz and Ivanov, 2009). The energy spectrum of photoelectrons appearing in multiphoton ionization of the gas under the conditions where the pulse duration is compareable or smaller than the average time interval between the electron - atomic collisions consists of a number of peaks corresponding to the absorption of a certain number of photons. Such an electron energy distribution function (EEDF) is characterized by the energy intervals with the inverse population. It is known, such situation can be used to amplify electromagnetic radiation in a plasma (Bunkin et al, 1972).

The possibility of using of the plasma channel created by a high intensity ultrashort pulse of a KrF excimer laser ($\hbar\Omega = 5$ eV) in xenon for the amplification of radio-frequency pulses was analyzed in the paper (Bogatskaya and Popov, 2013). In this paper time dependences of the gain factor with various frequencies ω of the amplyfied radio-frequency radiation in the xenon plasma channel were obtained. In (Bogatskaya et al, 2013) the possibilty to amplify the subtrerahertz radiation in different gases was analyzed. It was demonstrated that the xenon plasma has some advantages as the amplifying medium in comparison with other rare and molecular gases. In this paper we discuss the possibility of using of the plasma channel created in the atmospheric air as an amplifying medium for radio-frequensy radiation. The evolution of the electron energy spectrum in the relaxing plasma created by the femtosecond laser pulse is examined using the Boltzmann kinetic equation and the gain factor of electromagnetic radiation in the plasma channel is calculated as a function of time and electronic concentration in dependence of frequency in subterahertz band. It is found that for definite range of the laser frequencies there exists also a rather short time interval when such a relaxing air plasma can be also used as an amplifying medium for radio-frequency ultrashort pulses. The propagation of such pulses through the amplifiyng medium is studied in the frames of optical parabolic approximation.

It should be mentioned that mechanism of the amplification of electromagnetic radiation in the plasma channel discussed in this paper is close from physical point of view to the effect of the negative

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absolute conductivity in the gas-discharge plasma predicted by (Rokhlenko, 1978) and (Shizgal and McMahon, 1985), experimentally detected by (Warman et al, 1985), and discussed in detail in reviews (Aleksandrov and Napartovich, 1993) and (Dyatko, 2007).

2 PHOTOIONIZATION OF AIR BY THE ULTRASHORT LASER PULSE

To analyse the properties and evolution of the plasma channel created by a high intensity femtosecond laser radiation, it is significant to take into account that the channel appears only due to the multiphoton ionization of molecules. In this case, the avalanche ionization of the gas molecules can be neglected. Moreover, for pulses with the duration of $\tau_p \sim 100$ fs, elastic collisions of electrons with molecules of the medium during the pulse can also be neglected. Indeed, the characteristic time of collisions of electrons with nitrogen or oxygen molecules in air at atmospheric pressure and room temperature ($T \approx 0.03$ eV) can be estimated as $T_c \approx 1/N\sigma v$, where $N \approx 2.5\times10^{19}$ cm^{-3} is the density of the particles, $\sigma \sim 10^{-15}$ cm^2 is the elastic collision cross section, and $v \sim 10^8$ cm/s is the velocity of electrons appearing in photoionization process. Under these conditions $T_c \sim 4\times10^{-13}$ s. This time exceeds the duration of the laser pulse. This means that the energy spectrum of photoelectrons by the end of the laser pulse is determined only by the photoionization of molecules of the gas and can be obtained from the solution of the problem of the ionization of a single atom or molecule in a strong laser field. The evolution of the spectrum caused by elastic, inelastic and electron-electron collisions, which is described by the Boltzmann kinetic equation, takes place in the postpulse regime. For this reason, under the conditions of interest, the problem of the ionization of the gas by laser radiation can be considered independently from the problem of the evolution of the spectrum of photoelectrons. The solution of the former problem is used as the initial condition for the latter problem.

For the intensity range $I \le 10^{13}$ W/cm^2 the ionization probability of O_2 molecules is a cubic function of the radiation intensity I for the third harmonic of the Ti:Sa – laser: $w_i \sim I^3$. For the N_2 molecules we have four-photon ionization in this intensity range: $w_i \sim I^4$. For the moderate fields with the laser intensity of the third harmonic of the Ti:Sa laser ~10^{11}–10^{12} W/cm^2 in accordance with the perturbation theory the probability of the three-photon ionization is significantly larger than the four-photon ionization probability. So plasma channel is formed mainly by the three-photon ionization of O_2 molecules. Also in such fields the AC Stark shift of the continuum boundary can be neglected and the position of the first peak in the spectrum of photoelectrons corresponds to the energy $\varepsilon_0 = 3\hbar\Omega - I_i$, where $I_i \approx 12.08$ eV is the ionization potential of the oxygen molecule, and Ω is the frequency the laser radiation. For the above mentioned intensity range the degree of ionization in air by the end of the laser pulse with the duration $\tau_p \sim 100$ fs can be estimated as $\alpha = N_e/N \approx 10^{-7} \div 10^{-6}$ (Delone and Krainov, 2001). Here N_e is the electron density.

3 BOLTZMANN EQUATION FOR THE EVOLUTION OF THE PHOTOELECTRON ENERGY SPECTRUM

Analyzing the evolution of the energy spectrum, we assume that the plasma channel with a given degree of ionization and strongly nonequilibrium electron energy distribution function is formed at the initial (zero) instant of time. The electron energy distribution function (EEDF) is approximated by the Gaussian

$$n(\varepsilon, t=0) = \frac{1}{\Delta\varepsilon\sqrt{\pi\varepsilon}} \exp\left(-\frac{(\varepsilon-\varepsilon_0)^2}{(\Delta\varepsilon)^2}\right). \qquad (1)$$

The width of the peak is determined by the pulse duration and for $\tau_p \sim 100$ fs can be estimated as $\Delta\varepsilon \approx 0.2$ eV. For the above mentioned intensity range above-threshold ionization peaks can be neglected.

This electron energy distribution function is normalized as

$$\int_0^\infty n(\varepsilon, t=0)\sqrt{\varepsilon}\, d\varepsilon = 1. \qquad (2)$$

The quantity $n(\varepsilon,t)\sqrt{\varepsilon}$ is the probability density of the existence of the electron with the energy ε.

The temporal evolution of the initial spectrum (1) was analyzed using the kinetic Boltzmann equation for the EEDF in the two-term approximation. We also assumed that the radio-frequency field amplifying in the plasma was weak enough and was not taken into account in the Boltzmann equation. Under above assumptions the kinetic equation was written in a form (Ginzburg and Gurevich, 1960), (Raizer, 1977):

$$\frac{\partial n(\varepsilon,t)}{\partial t}\sqrt{\varepsilon} = Q_{ee}(n) + Q^*(n) + \sum_i \frac{2m}{M_i}\frac{\partial}{\partial \varepsilon}\left(\nu_{tr}^{(i)}(\varepsilon)\varepsilon^{3/2}\left(n(\varepsilon,t) + T\frac{\partial n(\varepsilon,t)}{\partial \varepsilon}\right)\right). \qquad (3)$$

Equation (3) has the form of the diffusion equation in the energy space. Here, T is the gas temperature (below, we take $T \approx 0.03$ eV), m is the mass of the electron, M_i ($i=1,2$) are the masses of the nitrogen and oxygen molecules respectively, and $\nu_{tr}^{(i)} = N_i\sigma_{tr}^{(i)}(\varepsilon)\sqrt{2\varepsilon/m}$ is the partial transport frequency, where $\sigma_{tr}^{(i)}(\varepsilon)$ is the transport scattering cross section for N_2 (i=1) and O_2 (i=2) molecules, $N_1 = 0.79\times N$ and $N_2 = 0.21\times N$ are the concentrations of N_2 and O_2 molecules in the air, $Q_{ee}(n)$ is the integral of electron-electron collisions, $Q^*(n)$ is the integral of inelastic collisions. Equation (3) with initial condition (1) was solved numerically using an explicit scheme in the energy range $\varepsilon = 0-5$ eV. The elastic and necessary inelastic cross sections for N_2 and O_2 molecules were taken from (Phelps, 1985) and (Phelps and Pitchford, 1985). The total transport cross section for the electrons in air is presented at Fig.1.

Among a lot of inelastic collisions of electrons with nitrogen and oxygen molecules the excitation of vibrational levels of $N_2(X^1\Sigma^+)$ is of most importance. These cross sections are high enough in the energy range ~ 2–4 eV and contribute significantly to the temporal evolution of the EEDF discussed below.

The obtained from Eq. (3) EEDF makes it possible to calculate the temporal dependence of the optical properties of the plasma channel created by laser pulse. For example, the expression for the

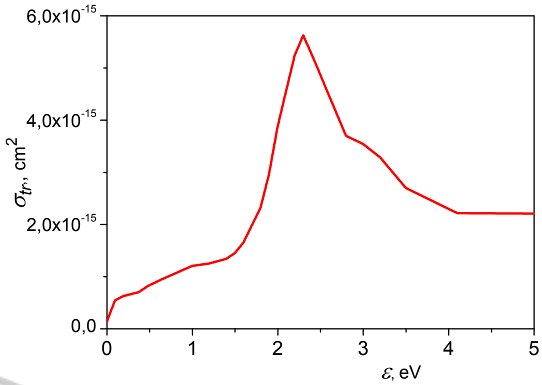

Figure 1: Transport cross section for the electrons in air.

complex conductivity $\sigma(\omega) = \sigma'(\omega) + i\sigma''(\omega)$ at the frequency ω can be written in the form (Ginzburg and Gurevich, 1960; Bunkin et al, 1972):

$$\sigma(\omega) = \frac{2}{3}\frac{e^2 N_e}{m}\int_0^\infty \frac{\varepsilon^{3/2}(\nu_{tr}(\varepsilon) + i\omega)}{\omega^2 + \nu_{tr}^2(\varepsilon)}\left(-\frac{\partial n(\varepsilon,t)}{\partial \varepsilon}\right)d\varepsilon \qquad (4)$$

The real part of this expression describes the dissipation of the energy of the electromagnetic wave in the plasma. So the absorption coefficient at the frequency ω can be represented in the form:

$$\mu_\omega = \frac{4\pi\sigma'}{c} = \frac{8\pi}{3}\frac{e^2 N_e}{mc}\int_0^\infty \frac{\varepsilon^{3/2}\nu_{tr}(\varepsilon)}{\omega^2 + \nu_{tr}^2(\varepsilon)}\left(-\frac{\partial n(\varepsilon,t)}{\partial \varepsilon}\right)d\varepsilon. \qquad (5)$$

The electron energy distribution function typically decreases with the energy, i.e., $\partial n/\partial \varepsilon < 0$ and, consequently, the integral in Eq. (5) is positive and, hence, $\mu_\omega > 0$. However, in the process of the photoionization of atoms by short pulses, energy ranges with the positive derivative, $\partial n/\partial \varepsilon > 0$, appear to exist for the initial instant of time. Such energy intervals make a negative contribution to the integral in Eq. (5) and reduce the absorption coefficient. In (Bunkin et al, 1972) it was demonstrated that the integral in Eq. (5) can become even negative in the low-frequency range $\omega < \nu_{tr}$ in gases with the pronounced Ramsauer effect for the EEDF with energy interval with positive derivative, $\partial n/\partial \varepsilon > 0$. In the paper (Bogatskaya et al, 2013) it was found that for the plasma with the EEDF similar to (1) the amplification of the electromagnetic radiation with $\omega < \nu_{tr}$ will be possible, if the condition

$$\frac{d}{d\varepsilon}\varepsilon/\sigma_{tr}(\varepsilon) < 0 \qquad (6)$$

will be fulfilled. Typically, the condition $\omega < \nu_{tr}$ is satisfied for the subterahertz frequency range $\omega \le 10^{12}$ s^{-1}.

In the paper (Bogatskaya and Popov, 2013) it was demonstrated that the Ramsauer minimum presence in the transport cross section of xenon and as a consequence the rapidly increasing range of the $\sigma_{tr}(\varepsilon)$ can be responsible for the appearance of the amplification of electromagnetic radiation in the plasma created by multiphoton ionization by short laser pulse. Both N_2 and O_2 molecules do not characterized by the Ramsauer minimum. Nevertheless, the transport cross section for electron scattering on nitrogen molecule is characterized by

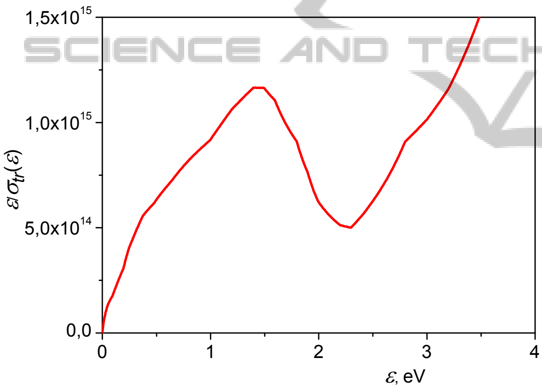

Figure 2: The value $\varepsilon/\sigma_{tr}(\varepsilon)$ for electrons in air.

large positive value of the derivative $d\sigma_{tr}/d\varepsilon$ in the energy range of ~1.5–2.3 eV. As a result, the condition (6) is satisfied in this range (see Fig. 2). It means that it is also possible to obtain the negative values of the absorption coefficient.

Results of the numerical calculations for the EEDF evolution in time are presented at Fig. 3 and Fig. 4 for two different energy positions of the initial photoelectron peak. As can be seen, for the initial energy of photoelectrons $\varepsilon_0 = 1.8$ eV (this energy value is very close to the ionization of oxygen molecules by the third harmonic of the Ti:Sa laser) the electron energy distribution function is characterized by a pronounced maximum, which is gradually shifted toward lower energies. While the average electron energy is more then ~1.5 eV (see the dependence at Fig.2), it is naturally to expect the positive value of the gain factor. It should be

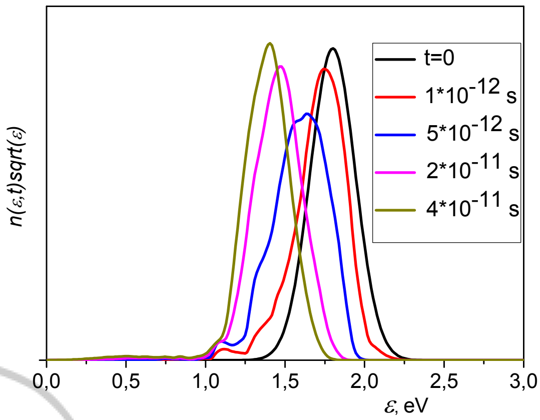

Figure 3: The EEDF in air at various times after the creation of the plasma channel. Initial peak of photoelectrons is characterized by $\varepsilon_0 = 1.8$ eV and electron concentration $N_e = 10^{13}$ cm^{-3}.

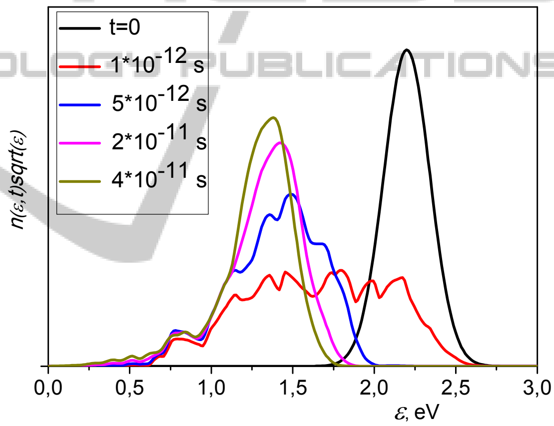

Figure 4: The EEDF in air at various times after the creation of the plasma channel. Initial peak of photoelectrons is characterized by $\varepsilon_0 = 2.2$ eV and electron concentration $N_e = 10^{13}$ cm^{-3}.

emphasized that for larger energy of the initial photoelectrons ($\varepsilon_0 = 2.2$ eV) the temporal evolution of the EEDF is quite different from that was discussed above (see Fig.4). Due to significant value of the cross section for the vibrational excitation of N_2 molecules by electrons with energies above ~2.0 eV the characteristic time of relaxation of the EEDF for $\varepsilon_0 = 2.2$ eV decreases dramatically and photoelectrons are found to be distributed over the energy range of $1.0-2.2$ eV even for the t=1 ps. Later the Gaussian-type EEDF is formed again, but as the average energy of photoelectrons for these instants of time is less 1.5 eV, and the positive value of the gain factor can not be achieved.

The electron energy distribution functions obtained in the numerical calculations were used to calculate the gain factor of electromagnetic radiation ($k_\omega = -\mu_\omega$) in the air plasma for different values of the initial peak position and the frequency of the amplified radiation $\omega = 5\times10^{11}$ s^{-1}. These data are presented at Fig.5. The data presented clearly demonstrate that the amplification of the radiation is possible if the energy of photoelectrons is less than ~2.25 eV. On the other hand, the energy of initial photoelectron peak should not be less than 1.5 eV. The maximum value of the gain factor can be obtained for the initial photoelectron peak position $\varepsilon_0 = 1.8$ eV. Such energy of photoelectrons appears to exist for the three-photon ionization by the laser radiation with $\hbar\Omega \approx 4.63$ eV which is very close to the third harmonics of the Ti:Sa laser. Even for such value of ε_0 the gain factor is found to be positive during approximately 25 ps. It means that the plasma channel in air can be used for amplification of only extremely short few-cycled radio-frequency pulses. For example, for $\omega = 5\times10^{11}$ s^{-1} it is possible to amplify the pulses of two or three cycle duration. For higher frequencies of amplified radiation the gain factor drops dramatically as the condition $\omega < \nu_{tr}$ is not satisfied already.

4 PROPAGATION OF THE RADIO-FREQUENCY PULSES IN THE PLASMA CHANNEL

As it is known, propagation of the electromagnetic radiation in the medium is described by the wave equation:

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial j}{\partial t}. \qquad (7)$$

Here E is the electric field strength, $j = \sigma E$ is the density of the electric current in the plasma and σ is the conductivity determined by expression (4). We assume that the radio-frequency pulse intensity is weak enough and do not contribute to the temporal evolution of the EEDF in the plasma channel.

We use optical parabolic approximation to find the solution of Eq. 7 (Akhmanov and Nikitin, 1997). According to this approximation for the pulse propagation along z-direction E should be represented as

$$E(\vec{r},t) = E_0(\rho,z,t)\cdot\exp(i(kz-\omega t)). \qquad (8)$$

Here E_0 is the envelope of the radio-frequency pulse, and k is the wave number. As the electronic density in the plasma channel is low enough, the permittivity at the frequency $\omega = 5\cdot10^{11}$ s^{-1} is close to unity and it is possible to assume that the radio-frequency pulse propagates in the channel also with the speed of light. Then $k = \omega/c$. After some approximations one can obtain the folowing equation for the E_0:

$$ik\left(\frac{\partial E_0}{\partial z} + \frac{1}{c}\frac{\partial E_0}{\partial t}\right) = -\frac{1}{2}\nabla_\perp^2 E_0 + \frac{i}{2}k_\omega(t - z/c)\,kE_0. \qquad (9)$$

The first term in the right part in Eq. (9) stands for the diffraction divergence of the electromagnetic field and the second one represents the absorption (amplification) process. Actually, the amplification duration τ corresponds to the amplification distance of about $c\cdot\tau \sim 1$ cm. So the laser pulse creates the air plasma channel characterized by amplifying «trail» (see Fig.6). If we launch the laser pulse and the few-cycled radio-frequency pulse just one after another simultaneously, the last one will continually locate in the amplifying zone of the laser pulse.

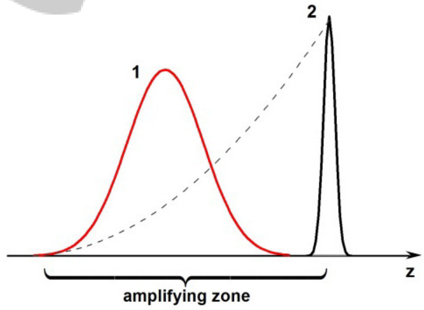

Figure 6: Spatial structure of radio (1) and laser (2) pulses for a given instant of time.

To obtain the amplification of the few-cycle radio-frequency pulse in the plasma channel, the second term in the right side of Eq. (9) should be dominant in comparison with the diffraction divergence. That is possible under the condition:

$$k_\omega > (\lambda/R)/(2\pi R). \qquad (10)$$

Here R is the plasma channel radius (about 1 cm), $\lambda = 2\pi c/\omega = 0.36$ cm for frequency $\omega = 5\cdot10^{11}$ s^{-1}. So the estimation (10) for the gain factor gives $k_\omega > 0.05$ s^{-1}. If one neglects the diffraction of the electromagnetic pulse the solution of the Eq. (9) for

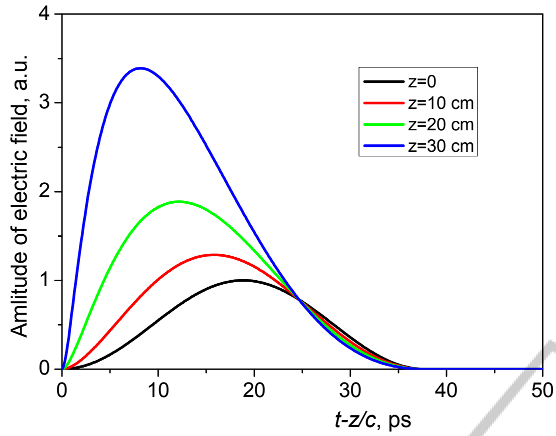

Figure 7: Time dependence of the electric field strength in the amplifying pulse for different propagation lengths.

weak fields can be found analytically. Introducing new variables $\zeta = z$, $\tau = t - z/c$, one obtains from (9):

$$\frac{\partial E_0(\zeta,\tau)}{\partial z} = \frac{1}{2} k_\omega(\tau) E_0(\zeta,\tau). \tag{11}$$

From (11) one obtains:

$$E_0(z,t) = \Phi(t - z/c)\exp\left(\frac{1}{2} k_\omega (t - z/c) z\right). \tag{12}$$

Here Φ is the initial envelope of the radio-frequency pulse. We assume that it has the Gaussian form with spatial size of 3λ. Fig. 7 shows that the sugnificant increase of the radio-frequency pulse amplitude can be obtained during its propagation despite the short time of amplification. It is worth noting, that the diffraction length of the radio-frequency pulse can be found: $l_D = kR^2 \approx 20$ cm. This length determines the applicability limit of the solution (12).

5 CONCLUSIONS

In this paper it has been shown that a plasma channel created in the atmospheric air by the third harmonic of the Ti:Sa laser can be used for amplification of few-cycle electromagnetic pulses in subterahertz frequency range. Despite the short time duration of the positive gain factor there is an opportunity to reach significant amplification by the simultaneous launching of the laser and few-cycle radio-frequency pulses with approximately the same propagation velocity.

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