

Flatness based Control of a 2 DOF Single Link Flexible Joint Manipulator

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Abstract: The demand for high speed robotic manipulators with little or no vibrations has been a challenging research problem. In this paper, a position control for a 2 DOF single link flexible manipulator with joint elasticity is studied. It is shown that using the flatness control approach, faster response and less oscillations and overshoots can be achieved. The flat output of the linearized system is determined as the tip of the manipulator end effector. This output and a finite order of its derivatives is defined in terms of the input and states variables of the manipulator. Using the parameters of the output in flat space, a trajectory is planned and executed to test the effectiveness of the designed control.

1 INTRODUCTION

The control of flexible robotic manipulators is required in many applications where faster response, lower energy consumption, lighter body mass, and high position accuracy at the end effector are demanded. The problem of vibration control in these systems has been a subject of research over the years (Ghorbel et al., 1989; Sira-Ramirez et al., 1992; Ider and Özgören, 2000; Ozgoli and Taghirad, 2006; Tokhi and Azad, 2008; Jiang and Higaki, 2011). Manipulators with flexible links are difficult to control due to their slow control response, high oscillations and high overshoots. The flexible joint manipulator is also known to exhibit a nonminimum phase behaviour (Tokhi and Azad, 2008). This makes trajectory tracking for the system harder to achieve. From a robot manipulator design perspective, these disadvantages are minimised by building the robot from rigid links and joints that results in high stiffness. However such stiff systems have been shown to be ineffective in terms of high power consumption and positional inaccuracy.

Many mathematical and analytical models have been proposed in the past to achieve control of these flexible systems (Dwivedy and Eberhard, 2006; Tokhi and Azad, 2008). Among these include the classical PID control, feedback linearization, fuzzy logic control, sliding mode, H_∞ control, linear quadratic control and neuro-fuzzy inference system. A comprehen-

sive survey of research in the control of flexible manipulators can be found in (Dwivedy and Eberhard, 2006; Ozgoli and Taghirad, 2006).

The concept of differential flatness proposed by Fliess, Levine, Martin and Rouchon(1995) has been applied to complex control problems(Fliess et al., 1995). This study will apply the differential flatness technique for the control of the single link flexible joint robot manipulator. The differential flatness approach through the flat output is used to design an asymptotically stable controller for suppressing vibrations of the flexible joint manipulator. Abdul-Razak (2007) and Quanser (2012) have reported the use of the linearized model of the flexible manipulator(Abdul Razak, 2007; Quanser, 2012). The linearized model is simulated with a PID controller and compared with the flatness based control.



Figure 1: Physical model of flexible joint robot manipulator.

The main contribution of this study is to show that using the differential flatness control model, faster response and less oscillations and overshoots can be achieved for the flexible manipulator. Furthermore, the problem of finding rest to rest trajectories for the nonminimum phase system is easily achieved without resorting to iterative solutions by complex numerical methods.

2 SYSTEM MODELING

The model used for the study is the standard Quanser flexible joint manipulator platform (Quanser, 2012) shown in figure(1). The nonlinear dynamic model of the flexible joint robot is formulated using Lagrange equations(Groves and Serrani, 2004). Other studies have used this model to design control for the flexible manipulator (Abdul Razak, 2007; Akyuz et al., 2011).

However, published results still suffer from oscillations and overshoots due to the flexible nature of the system. The single link flexible manipulator has a flexible joint and an arm which is oriented vertically. This introduces non-linearities in the system as a result of the potential energy due to gravity. Fig (2) shows the schematic diagram of the single link manipulator with flexible joint.

The input to the system is the voltage applied to the motor and the output is the tip angle which is a sum of the motor angle and the joint deflection. The system has two degrees of freedom which corresponds to the motor rotation angle and the rotation of the flexible joint. The coordinates of the flexible joint manipulator are reflected in fig(3).

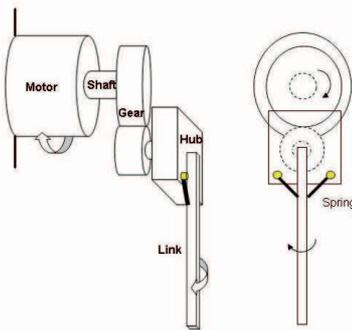


Figure 2: The schematic diagram of single link flexible manipulator with flexible joint.

Removing the nonlinear sinusoids enables the computation of the flat output for the linear system. This is computed using the technique proposed by (Levine and Nguyen, 2003). The energy equation for the system is formulated using the Lagrangian energy

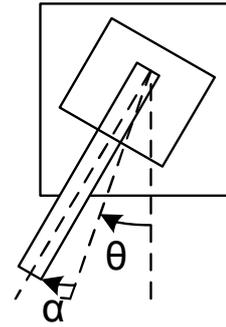


Figure 3: Co-ordinates of the flexible joint manipulator.

equation.

$$L = K - V \quad (1)$$

where

$$\begin{aligned} K &= K_h + K_l \\ V &= V_g + V_s \end{aligned} \quad (2)$$

K and V are kinetic and potential energy respectively. For a complete derivation of the dynamic model of the single link flexible joint manipulator, see (Akyuz et al., 2011). Choosing our state variables as:

$$\begin{aligned} \theta &= x_1 \\ \dot{\theta} &= x_2 \\ \alpha &= x_3 \\ \dot{\alpha} &= x_4 \end{aligned} \quad (3)$$

The linearized equations of motion about zero equilibrium point of the manipulator represented in state space are given as a fourth order system in the equation below:

$$\dot{\mathbf{x}} = \begin{bmatrix} x_2 \\ \frac{K_s}{J_h}x_3 - \frac{K_m^2 K_g^2}{R_m J_h}x_2 + \frac{K_m K_g}{R_m J_h}V \\ x_4 \\ -\frac{K_s}{J_h}x_3 + \frac{K_m^2 K_g^2}{R_m J_h}x_2 - \frac{K_m K_g}{R_m J_h}V - \frac{K_s}{J_l}x_3 + \frac{mgh}{J_l}(x_1 + x_3) \end{bmatrix} \quad (4)$$

2.1 Differential Flatness

A linear system

$$f(x, \dot{x}, u)$$

$$x \in \mathfrak{R}^n, u \in \mathfrak{R}^m, n \geq m + 1 \quad (5)$$

is said to be differentially flat if there exists a variable or set of variables $h_1 \in \mathfrak{R}^n$ called the flat output of the form:

$$h_1 = y(x, u, \dot{u}, \ddot{u}, \dots, u^{(p)}) \quad (6)$$

defined by:

$$\begin{aligned} x &= \alpha(h_1, \dot{h}_1, \ddot{h}_1, \dots, h_1^{(q)}), \\ u &= \beta(h_1, \dot{h}_1, \ddot{h}_1, \dots, h_1^{(q+1)}) \end{aligned} \quad (7)$$

p and q are finite integers, Such that the system of equations

$$\begin{aligned} \frac{d}{dt} \alpha(h_1, \dot{h}_1, \ddot{h}_1, \dots, h_1^{(q+1)}) = \\ f(\alpha(h_1, \dot{h}_1, \ddot{h}_1, \dots, h_1^{(q)}), \beta(h_1, \dot{h}_1, \ddot{h}_1, \dots, h_1^{(q+1)})) \end{aligned} \quad (8)$$

are identically satisfied (Rouchon et al., 1993).

2.2 Determination of the Flat Output

According to Levine and Nguyen (2003), a linear system of the form of equation (4) with one input can be expressed in terms of equation (9) (Levine and Nguyen, 2003).

$$A(s)x = Bu \quad (9)$$

where $x = P(s)h_1(s)$, $u = Q(s)h_1(s)$ and $A(s) = sI - A$

The variable $h_1 = (h_1, \dots, h_m)$ is the linear flat output and the matrices \mathbf{P} and \mathbf{Q} are given by equations (10) and (11) respectively

$$C^T A(s)P(s) = 0 \quad (10)$$

$$A(s)P(s) = BQ(s) \quad (11)$$

The matrix \mathbf{C} is an $n \times (n - m)$ matrix of rank $n - m$ orthogonal to \mathbf{B} such that:

$$C^T B = 0 \quad (12)$$

and

$$Q(s) = (B^T B)^{-1} B^T A(s)P(s) \quad (13)$$

Expressing equation (4) in terms of $\mathbf{A}(s)$, we have

$$A(s) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & s + \frac{K_m^2 K_g^2}{R_m J_h} & -\frac{K_s}{J_h} & 1 \\ 0 & 0 & s & -1 \\ -\frac{mgh}{J_l} & -\frac{K_m^2 K_g^2}{R_m J_h} & \frac{K_s}{J_h} + \frac{K_s}{J_l} - \frac{mgh}{J_l} & s \end{bmatrix} \quad (14)$$

$$B^T = \begin{bmatrix} 0 & \frac{K_m K_g}{R_m J_h} & 0 & -\frac{K_m K_g}{R_m J_h} \end{bmatrix} \quad (15)$$

For $C^T B = 0$ we select

$$C^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (16)$$

Noting that

$$P(s) = [P_1(s) \ P_2(s) \ P_3(s) \ P_4(s)] \quad (17)$$

Equation (10) will yield

$$\begin{aligned} P_1(s) &= \left(s^2 + \frac{K_s - mgh}{J_l} \right) h_1(s) \\ P_2(s) &= sP_1(s) \\ P_3(s) &= \left(-s^2 + \frac{mgh}{J_l} \right) h_1(s) \\ P_4(s) &= sP_3(s) \end{aligned} \quad (18)$$

From equation (18), we can express all the states of the system in terms of the flat output h_1 and its derivatives

$$\begin{aligned} x_1(t) = \theta(t) &= \ddot{h}_1(t) + \frac{K_s - mgh}{J_l} h_1(t) \\ x_2(t) = \dot{\theta}(t) &= \ddot{h}_1(t) + \frac{K_s - mgh}{J_l} \dot{h}_1(t) \\ x_3(t) = \alpha(t) &= -\ddot{h}_1(t) + \frac{mgh}{J_l} h_1(t) \\ x_4(t) = \dot{\alpha}(t) &= -\ddot{h}_1(t) + \frac{mgh}{J_l} \dot{h}_1(t) \end{aligned} \quad (19)$$

and $u(t) = v$

From equation (13), $Q(s) = (B^T B)^{-1} B^T A(s)P(s)$ yields

$$Q(s) = \beta_1 s^4 + \beta_2 s^3 + \beta_3 s^2 + \beta_4 s + \beta_5 \quad (20)$$

where:

$$\beta_1 = \frac{J_h R_m}{K_g K_m}$$

$$\beta_2 = K_g K_m$$

$$\beta_3 = \frac{K_s R_m}{2K_g K_m} + J_h R_m \left(\frac{K_s}{J_h} + \frac{K_s}{J_l} - \frac{mgh}{J_l} \right) + \frac{J_h R_m (K_s - mgh)}{2K_g K_m J_l}$$

$$\beta_4 = \frac{K_g K_m (K_s - mgh)}{J_l}$$

and

$$\beta_5 = \frac{mgh R_m J_h (K_s - mgh)}{2K_g K_m J_l^2}$$

$$-mgh \frac{K_s R_m + J_h R_m \left(\frac{K_s}{J_h} + \frac{K_s}{J_l} - \frac{mgh}{J_l} \right)}{2K_g K_m J_l}$$

Putting $\frac{K_s - mgh}{J_l} = W$ and $\frac{mgh}{J_l} = Y$ into equation (20)

Then $u(t)$ becomes

$$u(t) = \frac{J_h R_m}{K_g K_m} h_1^{(4)} + K_g K_m h_1^{(3)}$$

$$+ \left[\frac{K_s R_m + J_h R_m \left(\frac{K_s}{J_h} + W \right) + J_h R_m W}{2K_g K_m} \right] \ddot{h}_1 + K_g K_m W \dot{h}_1 + \frac{R_m J_h Y W - Y K_s R_m - R_m J_h Y \left(\frac{K_s}{J_h} + W \right)}{2K_g K_m} h_1 \quad (21)$$

Expressing the flat output h_1 in terms of the states

$$\begin{aligned} h_1(t) &= \frac{J_l}{K_s} (\theta + \alpha) \\ \dot{h}_1(t) &= \frac{J_l}{K_s} (\dot{\theta} + \dot{\alpha}) \\ \ddot{h}_1(t) &= \frac{mgh}{K_s} (\theta + \alpha) - \alpha \\ h_1^{(3)}(t) &= \frac{mgh}{K_s} (\dot{\theta} + \dot{\alpha}) - \dot{\alpha} \end{aligned} \quad (22)$$

A new states space model in terms of the flat output variables called the Brunovskys model can now be described for the original manipulator system as:

$$\begin{aligned} \dot{h}_1 &= h_2 \\ \dot{h}_2 &= h_3 \\ \dot{h}_3 &= h_4 \\ \dot{h}_4 &= \frac{mghK_s}{J_l J_h} h_1 - \frac{(K_g K_m)^2 (K_s - mgh)}{J_l J_h R_m} h_2 \\ &\quad - \left[\frac{K_s}{J_h} + \frac{K_s}{J_l} - \frac{mgh}{J_l} \right] h_3 - \frac{(K_g K_m)^2}{J_h R_m} h_4 + \frac{K_g K_m}{J_h R_m} u(t) \end{aligned} \quad (23)$$

3 CONTROLLER DESIGN

Designing the controller in flat output space is easy since the manipulator is represented by a chain of integrators. The flatness property decouples dynamics of the position, velocity, acceleration and jerk. Their trajectories can easily be generated without differentiation. It should be noted that only the tip position was used for feedback. Figure (4) illustrates the block diagram of the flatness based control.

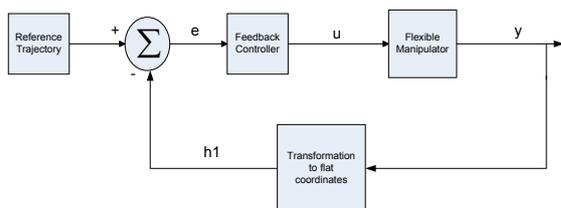


Figure 4: Block diagram of flatness based control for the flexible joint robot manipulator.

The reference trajectory is generated from the end effector tip position which is the flat output of the flexible manipulator. The state variables are transformed to flat output coordinates. The aim of the control is to track the position of the end effector as precisely as possible. Based on the linear system of equation (23), a controller will now be designed using the flat variables. For the 4th order system:

$$\dot{h}_4(t) = \hat{h}_{4d} - K_1(h_1(t) - \hat{h}_{1d}(t)) - K_2(h_2(t) - \hat{h}_{2d}(t)) - K_3(h_3(t) - \hat{h}_{3d}(t)) - K_4(h_4(t) - \hat{h}_{4d}(t)) \quad (24)$$

This can be written in the form

$$h_4(t) = \hat{h}_{4d} - K_1 e - K_2 \dot{e} - K_3 \ddot{e} - K_4 \dddot{e} \quad (25)$$

where

$$e = h_1 - \hat{h}_{1d}, \dot{e} = h_2 - \hat{h}_{2d}, \ddot{e} = h_3 - \hat{h}_{3d}, \dddot{e} = h_4 - \hat{h}_{4d}$$

$K_i, i = 1, 2, 3, 4$ are the controller gains.

The expression in the complex field is

$$s^4 + K_4 s^3 + K_3 s^2 + K_2 s + K_1 = 0 \quad (26)$$

The K parameters have to be chosen to minimise the system error. PID is used to tune the gains and drive the system error to a minimum. Figure (5) shows the simulation environment for the linear model in Simulink.

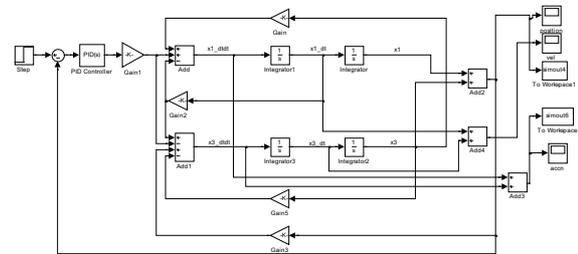


Figure 5: Simulink diagram of the linear model.

4 SIMULATION AND RESULTS

It is required to generate smooth point to point end effector tip movements. For position control, motion that has a velocity of zero at the start of motion and at the end is desired. The motion should also accelerate and decelerate smoothly. To check for the controllability of the modelled flexible manipulator, time response analysis was carried out. Results show that the system is stable and controllable. Figure (6) shows the response of the tip position $(\theta + \alpha)$ to a step input.

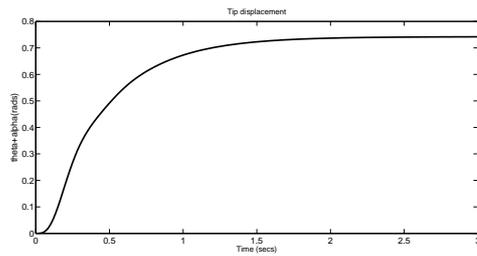


Figure 6: Step response of end effector position ($\theta + \alpha$) based on linear model.

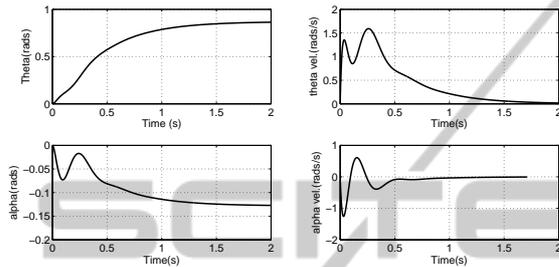


Figure 7: Step responses of motor angle (θ), joint displacement (α) and their velocities.

The step input introduces an instantaneous rotation on the motor shaft and results in a joint deflection. The system has a rise time of 0.84s, a settling time of 1.6s and steady state of 0.74rad with no overshoots. This response is satisfactory given that the flexible manipulator model has been linearized. A further check on the motor angle response and joint deflection and their velocities gives an insight of a stable system. The plots of figure (7) show the time response on motor angle θ and joint deflection α of the linear flexible manipulator model.

In order to check the steady state error performance of the proposed control, a closed loop feedback control was carried out as shown in figure (4). Comparisons were made from simulation results obtained for two different controlled platforms of the flexible manipulator using the MATLAB/Simulink environment. Position control is carried out on the linearized model and then compared with the flatness based model.

The results in figure (8) shows that both PID control and the flatness based control, achieved zero steady state error. The flatness based control has a percentage overshoot of less than 2% while the PID control has 9%. This is caused by the instantaneous effect of the step input on the motor. This effect can be seen by the large overshoots in the velocity and jerk. The reference trajectory however quickly settles to steady state with a settling time of 1.8s for the linear PID control. When compared to the flatness based control, a faster settling time of 0.3s is observed. This means that the flatness based control is more toler-

ant to oscillations and vibrations much more than the classical PID control. An observation of nonminimum phase behaviour was made in the linear flat dynamics. The plots in figure (8) show that the Flatness based controller was able to resolve this problem.

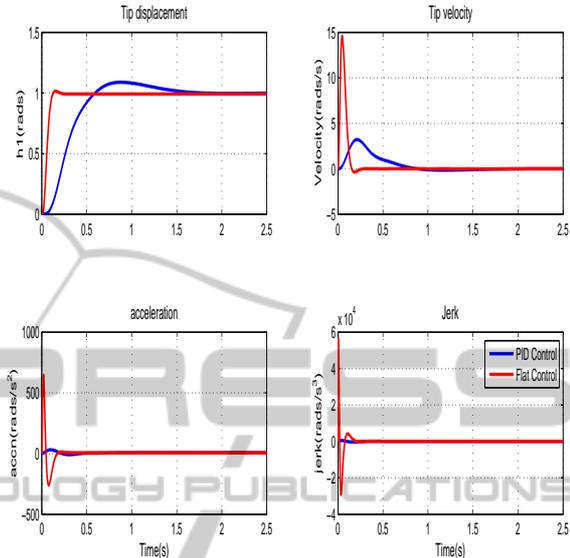


Figure 8: Comparison of system response based on PID and flatness based control.

5 CONCLUSIONS

This paper has presented a control for a single link flexible joint robot manipulator. The flat output for a linearized model of the manipulator was derived. The model was analysed and control designed based on differential flatness. The PID control on the linear model was compared with control of the flatness based model. Results show a satisfactory performance on the dynamics and control of both platforms. The flatness based control however shows faster response to instantaneous motor displacement with little vibrations and less overshoots.

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