# Optimal Multidimensional Signal Processing in Wireless Sensor Networks

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Abstract: Wireless sensor networks involve a set of spatially distributed sensors and a fusion center. Three methods for finding models of the sensors and the fusion center are proposed.



## 1 INTRODUCTION

Wireless sensor networks (WSNs) have recently emerged as a promising technology for a wide range of multimedia applications (Vaseghi, 2007). A related scenario involves a set of spatially distributed sensors making local observations $\mathbf{y}_j$ correlated with a signal of interest $\mathbf{x}$. Due to some external and instrumental factors, observations are noisy. Each sensor $\mathcal{Q}_j$ transmits information about its measurements to a fusion center $\mathcal{P}$ whose primary goal is to recover the original signal within a prescribed accuracy. Fig 1 illustrates the case.

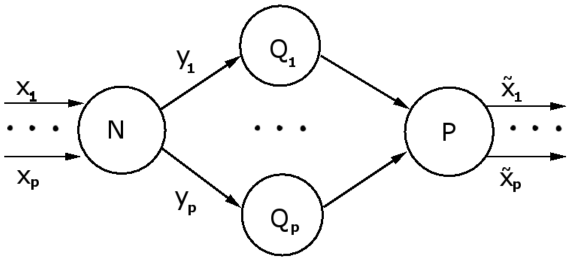


Figure 1: Block diagram of the WSN. Here, $N$ designates a noisy environment, $\widetilde{\mathbf{x}}_1, \ldots, \widetilde{\mathbf{x}}_p$ are estimations of $\mathbf{x}_1, \ldots, \mathbf{x}_p$.

It is widely recognized that efficient transmission strategies should reduce (compress) the amount of information transmitted by sensors. In this paper, the above-mentioned efficient transmission strategies are studied. We propose a novel approach based on a reduction of the multidimensional signal processing problem in WSNs to the new optimization problem.

We adopt a transform-based approach to determine the optimal transmission strategies in WSNs. More precisely, each sensor applies a suitable linear transform $\mathcal{Q}_j$ to its random observation vector $\mathbf{y}_j$ with $n_j$ components so as to reduce its dimensionality to $r_j$ components. The fusion center applies a linear transform $\mathcal{P}$ to reconstruct the random source vector of interest $\mathbf{x}$ with $m$ components. Thus, $\mathcal{Q}_j$ and $\mathcal{P}$ are given by matrices $Q_j \in \mathbb{R}^{r_j \times n_j}$ and $P \in \mathbb{R}^{m \times r}$, respectively, where $r_j \leq n_j$, $r = r_1 + \ldots + r_p$ and $r \leq m$.

Let us write $(\Omega, \Sigma, \mu)$ for a probability space. For $i = 1, \ldots, p$, let $\mathbf{x}_i \in L^2(\Omega, \mathbb{R}^{m_i})$ be a random signal with realizations $x_i = \mathbf{x}_i(\omega) \in \mathbb{R}^{m_i}$. We denote

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_p \end{bmatrix} \quad \text{and} \quad \|\mathbf{x}(\cdot)\|^2_\Omega = \int_\Omega \|\mathbf{x}(\omega)\|_2^2 d\mu(\omega) \tag{1}$$

where $\mathbf{x} \in L^2(\Omega, \mathbb{R}^m)$, $m = m_1 = \ldots + m_p$ and $\|\mathbf{x}(\omega)\|_2$ is the Euclidean norm of $\mathbf{x}(\omega) \in \mathbb{R}^m$. We also denote $\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_p \end{bmatrix}$ where $\mathbf{y}_i \in L^2(\Omega, \mathbb{R}^{n_i})$ and $n = n_1 + \ldots + n_p$.

Let us define a sensor model $\mathcal{Q}_i$ by the relation

$$[\mathcal{Q}_i(\mathbf{y}_i)](\omega) = Q_i[\mathbf{y}_i(\omega)] \tag{2}$$

where $\mathcal{Q}_i : L^2(\Omega, \mathbb{R}^{n_i}) \to L^2(\Omega, \mathbb{R}^{r_i})$ and $Q_i$ is a matrix, $Q_i \in \mathbb{R}^{r_i \times n_i}$. For

$$r = r_1 + \ldots + r_p, \tag{3}$$

a fusion center model, $\mathcal{P} : L^2(\Omega, \mathbb{R}^r) \to L^2(\Omega, \mathbb{R}^m)$, is defined similarly to (2), by matrix $P \in \mathbb{R}^{m \times r}$.

**Problem 1.** *For $j = 1, \ldots, p$, let $\mathbf{x}_j$ and $\mathbf{y}_j$ be reference signals and observed data, respectively. Find*



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*models of the sensors, $\mathcal{Q}_1,\ldots,\mathcal{Q}_p$, and a model of the fusion center, $\mathcal{P}$, that provide*

$$\min_{\mathcal{P},\mathcal{Q}_1,\ldots,\mathcal{Q}_p}\left\|\mathbf{x}-\mathcal{P}\begin{bmatrix}\mathcal{Q}_1(\mathbf{y}_1)\\ \vdots\\ \mathcal{Q}_p(\mathbf{y}_p)\end{bmatrix}\right\|_\Omega^2. \quad (4)$$

# 2 MAIN RESULTS

## 2.1 First Method: WSN Equipped with Orthogonal Data Convertors

Let us extend the original problem (4) to the problem equipped with additional data converters, $\mathcal{G}_1,\ldots,\mathcal{G}_p$, such that they transform observations $\mathbf{y}_1,\ldots,\mathbf{y}_p$ to vectors with the special property given by Definition 1 below. This property allows us to determine solution in a quite simple way.

For $\mathbf{x}$ and $\mathbf{y}$ presented by

$$\mathbf{x}=[\mathbf{x}^{(1)},\ldots,\mathbf{x}^{(m)}]^T \quad\text{and}\quad \mathbf{y}=[\mathbf{y}^{(1)},\ldots,\mathbf{y}^{(n)}]^T$$

with $\mathbf{x}^{(\ell)}\in L^2(\Omega,\mathbb{R})$ and $\mathbf{y}^{(q)}\in L^2(\Omega,\mathbb{R})$ where $\ell=1,\ldots,m$ and $q=1,\ldots,n$, respectively, we write

$$E[\mathbf{x}\mathbf{y}^T]=E_{xy}=\left\{\left\langle\mathbf{x}^{(\ell)},\mathbf{y}^{(q)}\right\rangle\right\}_{\ell,q=1}^{m,n}\in\mathbb{R}^{m\times n}$$

and $\left\langle\mathbf{x}^{(\ell)},\mathbf{y}^{(q)}\right\rangle=\int_\Omega \mathbf{x}^{(\ell)}(\omega)\mathbf{y}^{(q)}(\omega)\,d\mu(\omega)$.

**Definition 1.** *For $i=1,\ldots,p$, let*

$$\mathbf{u}_i=\mathcal{G}_i(\mathbf{y}_i)$$

*where $\mathcal{G}_i: L^2(\Omega,\mathbb{R}^{n_i})\to L^2(\Omega,\mathbb{R}^{n_i})$. The data converters $\mathcal{G}_1,\ldots,\mathcal{G}_p$ are called pairwise orthogonal if*

$$E_{u_iu_j}=\mathbb{O}\quad \text{when } i\neq j, \quad (5)$$

*where $\mathbb{O}$ is the zero matrix.*

The determination of the pairwise orthogonal data converters $\mathcal{G}_1,\ldots,\mathcal{G}_p$ is given in Lemma 1 below.

Let us now extend problem (4) by including data converters $\mathcal{G}_1,\ldots,\mathcal{G}_p$.

**Problem 2.** *For $i=1,\ldots,p$, find models of the sensors, $\mathcal{Q}_1,\ldots,\mathcal{Q}_p$, and a model of the fusion center, $\mathcal{P}$, that provide*

$$\min_{\mathcal{P},\mathcal{Q}_1,\ldots,\mathcal{Q}_p}\left\|\mathbf{x}-\mathcal{P}\begin{bmatrix}\mathcal{Q}_1\mathcal{G}_1(\mathbf{y}_1)\\ \vdots\\ \mathcal{Q}_p\mathcal{G}_p(\mathbf{y}_p)\end{bmatrix}\right\|_\Omega^2. \quad (6)$$

Let us denote by $M^\dagger$ the Moor-Penrose pseudo-inverse of a matrix $M$.

First, we give the models of orthogonal data converters $\mathcal{G}_1,\ldots,\mathcal{G}_p$ that satisfy (5) as follows.

**Lemma 1.** *Let $\mathbf{u}_i=\mathcal{G}_i(\mathbf{y}_i)$ for $i=1,\ldots,p$ and let $\mathcal{G}_1,\ldots,\mathcal{G}_p$ be such that*

$$\mathcal{G}_1(\mathbf{y}_1)=\mathbf{y}_1 \quad\text{and}\quad \mathcal{G}_i(\mathbf{y}_i)=\mathbf{y}_i-\sum_{k=1}^{i-1}\mathcal{Z}_{ik}(\mathbf{u}_k) \quad (7)$$

*for $i=2,\ldots,p$ with $\mathcal{Z}_{ik}: L^2(\Omega,\mathbb{R}^{m_i})\to L^2(\Omega,\mathbb{R}^{m_i})$ defined by*

$$Z_{ik}=E_{y_iu_k}E_{u_ku_k}^\dagger+M_{ik}(I-E_{u_ku_k}E_{u_ku_k}^\dagger) \quad (8)$$

*with $M_{ik}\in\mathbb{R}^{n\times n}$ arbitrary. Then the $\mathcal{G}_1,\ldots,\mathcal{G}_p$ are pairwise orthogonal data converters.*

Next, to find a solution of Problem 2, we write $\mathcal{P}=[\mathcal{P}_1\ \ldots\ \mathcal{P}_p]$ where, for $j=1,\ldots,p$, $\mathcal{P}_j$ is defined by matrix $P_j\in\mathbb{R}^{m\times r_j}$. Then

$$\left\|\mathbf{x}-[\mathcal{P}_1\ \ldots\ \mathcal{P}_p]\begin{bmatrix}\mathcal{Q}_1\mathcal{G}_1(\mathbf{y}_1)\\ \vdots\\ \mathcal{Q}_p\mathcal{G}_p(\mathbf{y}_p)\end{bmatrix}\right\|_\Omega^2 = \left\|\mathbf{x}-[\mathcal{F}_1,\ldots,\mathcal{F}_p](\mathbf{u})\right\|_\Omega^2, \quad (9)$$

where

$$\mathcal{F}_i=\mathcal{P}_i\mathcal{Q}_i \quad\text{and}\quad \mathbf{u}=[\mathbf{u}_1^T,\ldots,\mathbf{u}_p^T]^T. \quad (10)$$

Thus, problem (6) is reduced to the equivalent problem of finding $\mathcal{F}_1,\ldots,\mathcal{F}_p$ that solve

$$\min_{\mathcal{F}_1,\ldots,\mathcal{F}_p}\left\|\mathbf{x}-[\mathcal{F}_1,\ldots,\mathcal{F}_p](\mathbf{u})\right\|_\Omega^2 \quad (11)$$

subject to

$$\operatorname{rank}\mathcal{F}_1\le r_1,\quad \ldots,\quad \operatorname{rank}\mathcal{F}_p\le r_p. \quad (12)$$

To find a solution of problem (11)–(12) we write

$$\left\|\mathbf{x}-[\mathcal{F}_1,\ldots,\mathcal{F}_p](\mathbf{u})\right\|_\Omega^2 = \|E_{xx}^{1/2}\|^2-\|E_{xu}(E_{uu}^{1/2})^\dagger\|^2+\|E_{xu}(E_{uu}^{1/2})^\dagger-FE_{uu}^{1/2}\|^2.$$

Here, the only term that depends on $F_1,\ldots,F_p$ is

$$\|E_{xu}(E_{uu}^{1/2})^\dagger-[F_1,\ldots,F_p]E_{uu}^{1/2}\|^2 = \|A-[F_1,\ldots,F_p]C\|^2 \quad (13)$$

where $A=E_{xu}(E_{uu}^{1/2})^\dagger$ and $C=E_{uu}^{1/2}$. Due to the property (5), matrix $E_{uu}$ is block-diagonal,

$$E_{uu}=\begin{bmatrix}E_{u_1u_1} & \mathbb{O} & \ldots & \mathbb{O}\\ \mathbb{O} & E_{u_2u_2} & \ldots & \mathbb{O}\\ \ldots & \ldots & \ldots & \ldots\\ \mathbb{O} & \mathbb{O} & \ldots & E_{u_pu_p}.\end{bmatrix}$$

Therefore, matrix $C$ is also is block-diagonal,

$$C=\begin{bmatrix}C_{11} & \mathbb{O} & \ldots & \mathbb{O}\\ \mathbb{O} & C_{22} & \ldots & \mathbb{O}\\ \ldots & \ldots & \ldots & \ldots\\ \mathbb{O} & \mathbb{O} & \ldots & C_{pp}.\end{bmatrix}$$

If we write $A = [A_1 \dots A_p]$ where, for $j = 1, \dots, p$, $A_j \in \mathbb{R}^{m \times n_j}$, then it follows from (13) that

$$\|A - [F_1, \dots, F_p]C\|^2 = \sum_{j=1}^{p} \|A_j - F_j C_{jj}\|^2.$$

Thus, problem (11)–(12) is reduced to $p$ individual problems of finding $F_j$, for $j = 1, \dots, p$, that solves

$$\min_{F_j} \|A_j - F_j C_{jj}\|^2 \quad \text{with rank } F_j \le r_j. \tag{14}$$

The solution has been given in (Torokhti and Friedland, 2009) as follows.

#### 2.1.1 Best Rank-constrained Matrix Approximation

Let $\mathbb{C}^{m\times n}$ be a set of $m \times n$ complex valued matrices, and denote by $\mathcal{R}(m,n,r) \subseteq \mathbb{C}^{m\times n}$ the variety of all $m \times n$ matrices of rank $r$ at most. Fix $A = [a_{ij}]_{i,j=1}^{m,n} \in \mathbb{C}^{m\times n}$. Then $A^* \in \mathbb{C}^{n\times m}$ is the conjugate transpose of $A$. Let the singular value decomposition (SVD) of $A$ be given by

$$A = U_A \Sigma_A V_A^*, \tag{15}$$

where $U_A \in \mathbb{C}^{m\times m}$, $V_A \in \mathbb{C}^{n\times n}$ are unitary matrices, $\Sigma_A := \text{diag}(\sigma_1(A), \dots, \sigma_{\min(m,n)}(A)) \in \mathbb{C}^{m\times n}$ is a generalized diagonal matrix, with the singular values $\sigma_1(A) \ge \sigma_2(A) \ge \dots \ge 0$ on the main diagonal.

Let $U_A = [u_1\ u_2\ \dots u_m]$ and $V_A = [v_1\ v_2\ \dots v_n]$ be the representations of $U$ and $V$ in terms of their $m$ and $n$ columns, respectively. Let

$$P_{A,L} := \sum_{i=1}^{\text{rank}\,A} u_i u_i^* \in \mathbb{C}^{m\times m} \text{and } P_{A,R} := \sum_{i=1}^{\text{rank}\,A} v_i v_i^* \in \mathbb{C}^{n\times n} \tag{16}$$

be the orthogonal projections on the range of $A$ and $A^*$, correspondingly. Define a truncated SVD, $\{A\}_r$, of matrix $A$ by

$$\{A\}_r := \sum_{i=1}^{r} \sigma_i(A) u_i v_i^* = U_{Ar} \Sigma_{Ar} V_{Ar}^* \in \mathbb{C}^{m\times n} \tag{17}$$

for $r = 1, \dots, \text{rank}\,A$, where

$$U_{Ar} = [u_1\ u_2\ \dots u_r],\ \Sigma_{Ar} = \text{diag}(\sigma_1(A), \dots, \sigma_r(A)) \quad \text{and} \quad V_{Ar} = [v_1\ v_2\ \dots v_r]. \tag{18}$$

For $r > \text{rank}\,A$, we write $\{A\}_r := A$ (or $\{A\}_r = \{A\}_{\text{rank}\,A}$). For $1 \le r < \text{rank}\,A$, the matrix $\{A\}_r$ is uniquely defined if and only if $\sigma_r(A) > \sigma_{r+1}(A)$.

Recall that $A^\dagger := V_A \Sigma_A^\dagger U_A^* \in \mathbb{C}^{n\times m}$ is the Moore-Penrose generalized inverse of $A$, where $\Sigma_A^\dagger := \text{diag}\left(\frac{1}{\sigma_1(A)}, \dots, \frac{1}{\sigma_{\text{rank}\,A}(A)}, 0, \dots, 0\right) \in \mathbb{C}^{n\times m}$.

Henceforth $\|\cdot\|$ designates the Frobenius norm.

Theorem 1 below provides a solution to the problem of finding a matrix $F$ that solves

$$\min_{F \in \mathcal{R}(p,q,r)} \|A - BFC\|. \tag{19}$$

**Theorem 1.** (Friedland and Torokhti, 2007) *Let $A \in \mathbb{C}^{m\times n}$, $B \in \mathbb{C}^{m\times p}$ and $C \in \mathbb{C}^{q\times n}$ be given matrices. Let*

$$L_B = (I_p - P_{B,R})S \quad \text{and} \quad L_C = T(I_q - P_{C,L}) \tag{20}$$

*where $S \in \mathbb{C}^{p\times p}$ and $T \in \mathbb{C}^{q\times q}$ are any matrices, and $I_p$ is the $p \times p$ identity matrix. Then the matrix*

$$F = (I_p + L_B)B^\dagger \left\{ P_{B,L} A P_{C,R} \right\}_r C^\dagger (I_q + L_C) \tag{21}$$

*is a minimizing matrix for the minimal problem (19). Any minimizing $F$ has the above form.*

#### 2.1.2 Determination of Models of Sensors and Fusion Center that Satisfy (6)

It follows from (19), (21), that a solution of the problem in (14) is a particular case of Theorem 1.

Indeed if, in (19)–(21), we write $A_j$, $F_j$, $C_{jj}$ and $r_j$ instead of $A$, $F$, $C$ and $r$, respectively, and set $n = n_j$, $p = m$, $q = n_j$ and $B = I$ then (14) coincides with (19). Its solution follows from (21) in the form

$$F_j = \left\{ A_j P_{C_{jj},R} \right\}_{r_j} C_{jj}^\dagger (I_{n_j} + L_{C_{jj}}), \tag{22}$$

where similarly to $L_C$ in (20), $L_{C_{jj}} = T_j(I_{n_j} - P_{C_{jj},L})$ with $T_j$ to be any $n_j \times n_j$ matrix. The solution of problem (11)–(12) is given by (22) as well.

Since (11)–(12) is equivalent to (6), it remains to show that models of sensors, $\mathcal{Q}_1, \dots, \mathcal{Q}_p$, and a model of the fusion center, $\mathcal{P}$, that satisfy (6), follow from (22). To this end, we recall that by (10),

$$\mathcal{F}_j = \mathcal{P}_j \mathcal{Q}_j$$

where $\mathcal{F}_j$, $\mathcal{P}_j$ and $\mathcal{Q}_j$ are defined by matrices $F_j \in \mathbb{R}^{m\times n_j}$, $P_j \in \mathbb{R}^{m\times r_j}$ and $Q_j \in \mathbb{R}^{r_j\times n_j}$, respectively, where $F_j = P_j Q_j$. The matrices $P_j$ and $Q_j$ are determined as follows. Let us write the SVD of $F_j$ in (22) as

$$F_j = U_{F_j} \Sigma_{F_j} V_{F_j}^T \tag{23}$$

where matrices

$$U_{F_j} = [u_{j1}, \dots, u_{jm}] \in \mathbb{R}^{m\times m},$$
$$\Sigma_{F_j} = \text{diag}(\sigma_1(F_j), \dots, \sigma_{\min(m,n_j)}(F_j)) \in \mathbb{R}^{m\times n_j}$$
$$\text{and} \quad V_{F_j} = [v_{j1}, \dots, v_{jn}] \in \mathbb{R}^{n_j\times n_j}$$

are similar to matrices $U_A$, $\Sigma_A$ and $V_A$ for the SVD of matrix $A$ in (15), respectively. In particular, $\sigma_{j1}, \dots, \sigma_{j\min(m,n_j)}$ are the associated singular values. Let

$$U_{F_j r_j} = [u_{j1}, \dots, u_{jr_j}] \in \mathbb{R}^{m\times r_j}, \tag{24}$$

$$\Sigma_{F_j r_j} = \text{diag}(\sigma_1(F_j), \dots, \sigma_{r_j}(F_j)) \in \mathbb{R}^{r_j\times r_j} \tag{25}$$

$$\text{and} \quad V_{F_j r_j} = [v_{j1}, \dots, v_{jr_j}] \in \mathbb{R}^{n\times r_j}. \tag{26}$$

Then $F_j$ in (22) can be written in form $F_j = P_j Q_j$ where, for $j = 1, \dots, p$,

$$P_j = U_{F_j r_j} \Sigma_{F_j r_j}, \quad Q_j = V^T_{F_j r_j}, \tag{27}$$

or

$$P_j = U_{F_j r_j} \in \mathbb{R}^{m \times r_j}, \quad Q_j = \Sigma_{F_j r_j} V^T_{F_j r_j}. \tag{28}$$

Thus, we have proved the following.

**Theorem 2.** *The models of the sensors and the fusion center that satisfy (6) are given by matrices* $Q_1, \dots, Q_p$ *and* $P = [P_1, \dots, P_p]$*, respectively, determined by (27) or (28).*

## 2.2 Second Method: Direct Solution of WSN Problem (4)

Here, we consider a way to determine models of the sensors, $Q_1, \dots, Q_p$, and the fusion center, $\mathcal{P}$, for the case when the orthogonal data converters, $\mathcal{G}_1, \dots, \mathcal{G}_p$ (see (6), Definition 1 and Lemma 1), are not used, i.e. when $Q_1, \dots, Q_p$ and $\mathcal{P}$ should satisfy (4).

In this case, similar to (9) and (10), we have

$$\left\| \mathbf{x} - [\mathcal{P}_1 \ \dots \ \mathcal{P}_p] \begin{bmatrix} \mathcal{Q}_1(\mathbf{y}_1) \\ \vdots \\ \mathcal{Q}_p(\mathbf{y}_p) \end{bmatrix} \right\|_\Omega^2 = \|E_{xx}^{1/2}\|^2 - \|E_{xy}(E_{yy}^{1/2})^\dagger\|^2 + \|E_{xy}(E_{yy}^{1/2})^\dagger - F E_{yy}^{1/2}\|^2,$$

where, as before, for $j = 1, \dots, p$, $F_j = P_j Q_j$. Here, the only term that depends on $F_1, \dots, F_p$ is

$$\|E_{xy}(E_{yy}^{1/2})^\dagger - [F_1, \dots, F_p] E_{uu}^{1/2}\|^2 = \|A - [F_1, \dots, F_p] C\|^2$$

where $A = E_{xy}(E_{yy}^{1/2})^\dagger$ and $C = E_{yy}^{1/2}$. Thus, problem (4) is reduced to finding $F_j$, for $j = 1, \dots, p$, that solve

$$\min_{F_1, \dots, F_p} \|A - [F_1, \dots, F_p] C\|^2 \tag{29}$$

subject to

$$\operatorname{rank} F_1 \le r_1, \quad \dots, \quad \operatorname{rank} F_p \le r_p. \tag{30}$$

A difference from (13) is that in (29), matrix $C$ is not block-diagonal. In this general case, a solution to problem (29)–(30), $F_1, \dots, F_p$, follows from the extension of Theorem 1. This result will be provided at the conference. Then, for $j = 1, \dots, p$, each matrix $F_j$ that satisfies (29)–(30) is presented in the form (27) or (28).

Thus, in this case, the models of the sensors and the fusion center that satisfy (4) are given by matrices $Q_1, \dots, Q_p$ and $P = [P_1, \dots, P_p]$, respectively, determined by( 27) or (28) provided that $F_1, \dots, F_p$ solve (29)–(30).

## 2.3 Third Method: Approximate Solution of WSN Problem (4)

Here, we consider a method which represents a compromise between the first and second methods. In (29), matrices $A = [A_1, \dots, A_p]$ and $C$ can be represented in the form

$$A = \widetilde{A}_1 + \ldots + \widetilde{A}_p \quad \text{and} \quad C = [C_1^T, \dots, C_p^T]^T, \tag{31}$$

respectively, where $\widetilde{A}_1 = [A_1, \mathbb{O}, \dots, \mathbb{O}]$, ..., $\widetilde{A}_p = [\mathbb{O}, \dots, \mathbb{O}, A_p]$ and, for $j = 1, \dots, p$, $C_j \in \mathbb{R}^{n_j \times n}$ is a block of $C$. Then

$$\|A - [F_1, \dots, F_p] C\|^2 \le \sum_{j=1}^{p} \|\widetilde{A}_j - \sum_{j=1}^{p} F_j C_j\|^2. \tag{32}$$

The latter motivates finding models of the sensors, $Q_1, \dots, Q_p$, and the fusion center, $P = [P_1, \dots, P_p]$, in the form $F_1 = P_1 Q_1, \dots, F_p = P_p Q_p$, where $F_1, \dots, F_p$ are determined from $p$ individual problems of finding $F_j$, for $j = 1, \dots, p$, that solves

$$\min_{F_j} \|\widetilde{A}_j - F_j C_j\|^2 \quad \text{with } \operatorname{rank} F_j \le r_j. \tag{33}$$

A direct comparison with (14) shows that the problem in (33) is different from that in (14). This is because, for $j = 1, \dots, p$, matrices $A_j$, $C_{jj}$ and $\widetilde{A}_j$, $C_j$ are different. Nevertheless, formally, the problems in (14) and (33) are similar. Therefore, the solution of (33)) is given in the form (22) where the notation should be changed in accordance with that in (31)–(33).

As a result, the following theorem is true.

**Theorem 3.** *The models of the sensors and the fusion center of the WSN that approximate the optimal models are given by matrices* $Q_1, \dots, Q_p$ *and* $P = [P_1, \dots, P_p]$ *determined by (27) or (28), where* $A_j$ *and* $C_{jj}$ *must be replaced with* $\widetilde{A}_j$ *and* $C_j$*, respectively.*

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