THE UNIFORM DECOMPOSITION
Properties and Applications

Franklin César Flores and Airton Marco Polidorio
Department of Informatics, State University of Maringá, Av. Colombo, 5790, Jd. Universitário, Maringá, PR, Brazil

Keywords: Uniform Decomposition, Morphological Segmentation, Hierarchical Segmentation.

Abstract: The uniform decomposition is an hierarchical segmentation method which main property is the preservation of a given characteristic for each subset of the partition provided by the decomposition. Its computation is incremental and, in each iteration, a candidate region is split only if it is found a valid split where each new subset respects an input criterion, for instance size or shape. This paper discusses some properties of the uniform decomposition and proposes several applications for this decomposition such as hierarchical segmentation, stereo vision pre-processing and interactive segmentation.

1 INTRODUCTION

Let $E \subset \mathbb{Z} \times \mathbb{Z}$ be a rectangular finite subset of points. A *partition* $\mathcal{X}$ of E is a family of subsets $X_i \in E$, such that $X_i \cap X_j = \varnothing$, $i \neq j$, and $\bigcup_i X_i = E$. Let $\mathcal{X} = (\mathcal{X}_i)$ be a family of partitions such that $\mathcal{X}_1 = E$ and

$$X_j \in \mathcal{X}_i \Rightarrow \exists X_k \in \mathcal{X}_{i-1} : X_j \subseteq X_k, \quad (1)$$

$\forall i > 1$. The family of partitions $\mathcal{X}$ defines an *hierarchy* (F. Meyer, 2001) of nested partitions: the union of adjacent regions in finer partitions forms a region in a coarser partition. And the contour of a region in a coarser partition will also belong to the finer partitions.

Hierarchical segmentation is the segmentation of an image in function of an hierarchy of nested partitions. Given a family of such partitions $\mathcal{X} = (\mathcal{X}_i)$, assigned to an image f, the hierarchical segmentation of f may be given by a partition $\mathcal{X} \in \mathcal{X}$. Such segmentation makes possible to represent the objects from an image in several levels of details and has been applied to solve problems such as segmentation of MR images (M. A. G. Carvalho, R. A. Lotufo and M. Couprie, 2003b), cells segmentation (M. A. G. Carvalho, R. A. Lotufo and M. Couprie, 2003a) and video coding (P. Salembier; A. Oliveras and L. Garrido, 1998).

One way to provide hierarchical segmentation is by application of the watershed from markers technique (Beucher and Meyer, 1992; Vincent and Soille, 1991). The idea is to compute nested partitions by discarding regional minima that are not relevant and using the important minima as markers to the application of the watershed operator. An hierarchy may be defined by the way the regional minima from an image are qualified and selected, usually according to structural features such as area, volume or contrast (C. Vachier and F. Meyer, 1995; C. Vachier, 1995).

The method described above is classical under the morphological hierarchical segmentation framework. However, it does not provide a way to control the properties of each subset in the output partition and, thus, nothing can be *globally* stated about any subset of the partition. It could be interesting to have an hierarchical morphological segmentation where each subset of the partition should respect a given criterion, such as size or shape. It is useful when the subsets of the partitions should stand after an area filtering or a erosion by a given structuring element (s.e.).

The *uniform decomposition* is a proposal to compute partitions with all of its subsets respecting a given criterion. Such computation is incremental and, in each iteration, a candidate region is split in two new ones only if both new regions are approved by a criterion function. In that way, the method guarantees that all regions in the decomposed image respect the input criterion. More, the decomposition is done inside a domain of application, what makes possible to decompose only a specific region of the image.

Besides the presentation of the uniform decomposition (Section 2), the goal of this paper is to point some of decomposition properties (Section 3) and to show three applications of the decomposition (Section 4). That is the way the paper, concluded in Section 5, is organized.

César Flores F. and Marco Polidorio A..
THE UNIFORM DECOMPOSITION - Properties and Applications.
DOI: 10.5220/0003870304980504
In *Proceedings of the International Conference on Computer Vision Theory and Applications* (VISAPP-2012), pages 498-504
ISBN: 978-989-8565-03-7

2 THE UNIFORM DECOMPOSITION

Let $K = [0,k]$ be a totally ordered set. Denote by $Fun[E,K]$ the set of all functions $f : E \to K$. An *image* is one of these functions (called graylevel functions). Particularly, if $K = [0,1]$, f is a binary image. An *image operator* (*operator*, for simplicity) is a mapping $\psi : Fun[E,K] \to Fun[E,K]$.

Let $N(x)$ be the set containing the *neighbourhood* of x, $x \in E$. We define a *path* from x to y, $x,y \in E$ as a sequence $P(x,y) = (p_0, p_1, ..., p_n)$ from E, where $p_0 = x$, $p_n = y$ and $\forall i \in [0, n-1], p_i \in N(p_{i+1})$.

A *connected subset* of E is a subset $X \subset E$ such that, $\forall x,y \in X$, there is a path C entirely inside X.

The *criterion function* is a function that will be applied to assess connected subsets according to a given criterion `crit` and a parameter p. Its general formula is given by

$$\mathbb{C}_{\texttt{crit}}(C,p) = \begin{cases} 1, \&\text{if } C \text{ satisfies the criterion } \texttt{crit}, \\ 0, \&\text{otherwise}, \end{cases} \tag{2}$$

where $C \subseteq E$ and p is a numerical parameter.

This paper will use two criterion functions : the *area* criterion and the *disc* one. The area criterion function outputs 1 if the area of the connected subset C is greater or equal to p. It is given by,

$$\mathbb{C}_{\texttt{area}}(C,p) = \begin{cases} 1, & \text{if } \#(C) \geq p, \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

where $\#(C) : E \to \mathbb{Z}_+$ returns the cardinality of $C \subseteq E$.

The disc criterion function outputs 1 if $C \subseteq E$ stands when erode by a disc s.e. with radius p. It is given by,

$$\mathbb{C}_{\texttt{disc}}(C,p) = \begin{cases} 1, & \text{if } \varepsilon_{B_p}(C) \neq \varnothing, \\ 0, & \text{otherwise}, \end{cases} \tag{4}$$

where ε is the morphological erosion (Serra, 1982) B_p is the disc s.e. with radius p.

Let $A,B \subset E$ be two disjoint connected subsets. Let $\mathcal{W}_{A,B}(g)$ be the *watershed from markers operator*, which computes the partition of E in function of g, applying A and B as markers. This partition is given by two subsets E_A and E_B, such that **i)** $E_A \bigcup E_B = E$, **ii)** $E_A \bigcap E_B = \varnothing$, **iii)** $A \subseteq E_A$ and **iv)** $B \subseteq E_B$.

Let $V \subseteq E$ be a *domain of validation*. Given a criterion function $\mathbb{C}_{\texttt{crit}}$ and a parameter p, the disjoint sets $A,B \subset V$ define a *valid pair of markers* if,

$$\mathbb{C}_{\texttt{crit}}(E_A \cap V, p) = \mathbb{C}_{\texttt{crit}}(E_B \cap V, p) = 1, \tag{5}$$

where $E_A, E_B \in E$ are the two subsets that denote a partition of E computed by $\mathcal{W}_{A,B}$. The *validation* concept introduced above will be applied in the decomposition, in order to check if a subset of the hierarchical segmentation may be split in function of a pair of markers. The splitting must only occur if both new subsets satisfies the decomposition criterion.

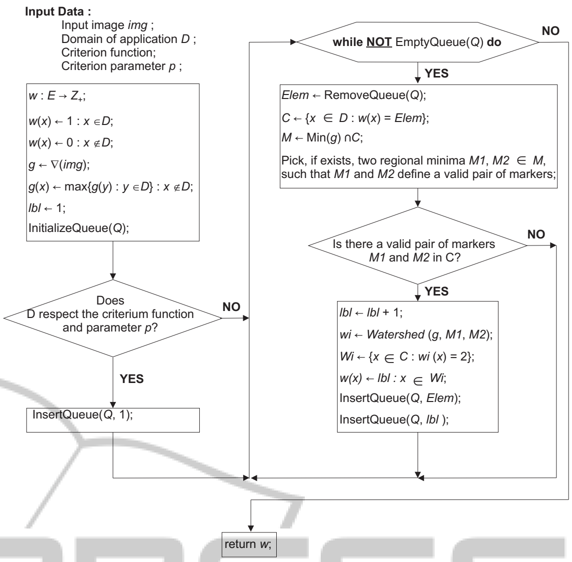

Figure 1: The uniform decomposition.

2.1 The Algorithm

The uniform decomposition algorithm has the following input data:

- the input image $img \in Fun[E,K]$;
- the subset $D \in E$ that defines the domain of application of the decomposition;
- the criterion $\mathbb{C}_{\texttt{crit}}$, applied to the iterative splitting of regions; and
- the parameter $p \in \mathbb{Z}_+$ used when the criterion above is applied to.

The algorithm outputs an image $w \in Fun[E, \mathbb{Z}_+]$ containing the image decomposed in one or more subsets. Each subset will receive a distinct label $lbl \in \mathbb{Z}_+$ in order to be uniquely identified.

The fluxogram in Figure 1 shows how the decomposition is computed. The first block of the fluxogram initializes the output w and other auxiliary structures, such as the queue Q that will store the label of the regions being processed. The morphological gradient is computed and the minima located outside the domain of application are removed. Image w receives, in all points $x \in D$, the label 1. The remaining points are not labeled and set to 0.

The domain of application is evaluated according to the criterion function and its parameter. If it satisfies the criterion, its label is inserted in Q. Otherwise, the process stops and returns D labeled with just one label. In the following, the iterative section runs until Q is empty.

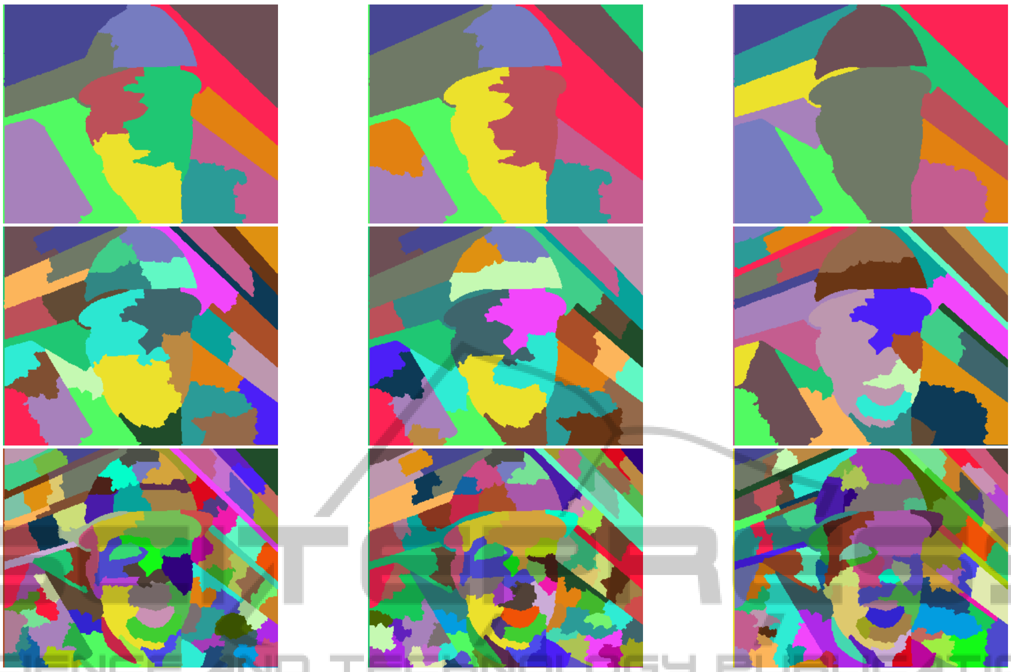

Figure 2: Hierarchical segmentation. First line : 13 regions. Second line : 34 regions. Third line : 101 regions. First column : Area criterion. Second column : Disc criterion. Third column : Classic method (extinction value threshold).

In each iteration, a label $Elem$ is removed from Q and the current subset $C \subseteq D$ labeled by $Elem$ is selected. The set M of all regional minima located inside C are selected as candidate markers to define a valid pair of markers.

The evaluation of a pair of markers is done by Equation 5. If a valid pair of markers is found, the label lbl (initialized as 1) is updated, and the watershed operator is applied to the image gradient g using $M1$ and $M2$ as markers. The resulting image w_i is labeled as 1 in the region that contains $M1$ and labeled as 2 in the region related to $M2$.

The set $W_i \subset C$ is selected by taken all points valued 2 in W_i (i.e., the subregion of C related to $M2$). The set W_i defines the points in the output image w that will be updated with the new label lbl.

Finally, label $Elem$ returns to Q and, in the following, label lbl is inserted in the queue.

A more detailed and formal presentation of this algorithm is found in (F. C. Flores and R. A. Lotufo, 2010).

3 PROPERTIES OF THE UNIFORM DECOMPOSITION

Besides the hierarchical property, the uniform decomposition also has other important characteristics:

- Given $f \in Fun[E,K]$ and a set of domains of application $D_i \subseteq E : D_i \cap D_j = \varnothing, i \neq j$. The decomposition may be applied independently to each domain D_i and different criterions $\texttt{crit}_i$ and parameter p_i may be assigned to each D_i.
- All partition subsets given by the decomposition respect the criterion and parameter applied to the domain of application. I.e., all subsets measurements are greater than p according to criterion `crit`. It provides a more balanced (or uniform) distribution of subsets.
- The most important objects in the image tend to have a more robust representation.
- The application of criteria in the decomposition computation allows the design of image operators that preserves the structures represented by the subsets of the decomposition. For instance, if the area criterion is applied to, all subsets stand to an area opening application (with parameter p), According to disc criterion, all subsets stand to erosions which s.e's are discs with diameter lower or equal to p.

4 APPLICATIONS

The support for a new version of the watershed from propagated markers (F. C. Flores and R. A. Lotufo,

Table 1: Segmentation error assessed by function $F'(I)$ of Borsotti et al.

Method ╲ Regions	13	34	101
Classic	57.3692	30.8142	23.0695
Area	57.4575	49.3614	23.1594
Disc	55.3578	49.4325	51.5475

Table 2: Segmentation error assessed by function $Q(I)$ of Borsotti et al.

Method ╲ Regions	13	34	101
Classic	581.4834	229.1602	110.9979
Area	535.1354	319.6443	103.0404
Disc	524.6157	325.5783	270.1857

2010) was the first reported application of the uniform decomposition. This Section presents and discusses other applications of the uniform decomposition, such as hierarchical segmentation, stereo vision pre-processing and interactive segmentation.

4.1 Hierarchical Segmentation

The first application presented in this paper is the use of the uniform decomposition to do hierarchical segmentation. Since it is a straightforward application of the decomposition, it is preferable to present the application along the comparison with the classical hierarchical segmentation by selection of regional minima via an extinction value threshold. It will be demonstrated here that the uniform decomposition is comparable to the classical hierarchical segmentation besides retaining the respect the decomposition criterion.

The comparison will be done with three decompositions: the uniform decomposition under the area criterion, the uniform decomposition under the disc criterion and the classical one. The three decompositions require different kind of parameters, so we decided to do the comparison according to the number of regions generated by the segmentation methods. The three methods will be applied to in order to produce the same number of regions and then the segmentation results will be assessed subjectively and quantitatively by application of two functions designed to evaluate the quality of a segmentation.

Classical decomposition is the easiest way to segment an image in *n* regions: after compute the extinction value for each regional minima, it is taken the *n*-th greatest extinction value to be the threshold value to select the *n* regional minima with the highest extinction values[1]. There is no straightforward way to segment an image in *n* regions by applying the uniform decomposition and it is a drawback of the method. So, in order to compare the uniform decomposition to the classical one using the chosen criterion, we searched for area and disc parameters that generated segmentations with the same number of regions and, then, applied this number to segment the image using the classical method.

Three experiments were accomplished. In the first one, two uniform decompositions were done by applying an area parameter equals to 5000 and a disc parameter equals to 33. Both decompositions generated 13 regions. To complete the first experiment, it was applied the classical method to achieve a segmentation with 13 regions. The second experiment applied area parameter equals to 1600 and disc parameter equals to 20. The two segmentations provided 34 regions and were compared to the classical method. Finally, the third experiment compares three decompositions that generated 101 regions. The applied area and disc parameters to this experiment were, respectively, 500 and 10. Figure 2 shows all images generated to the three experiments.

Following an subjective criteria, we can see that the segmentation provided by the uniform decomposition is comparable to the provided by the classical one. In some case, the uniform decomposition presents a better representation of some objects. Besides the uniform decomposition presents similar results to the classical one, the subsets of the uniform results respect the criteria applied to generated them. For instance, in Fig. 2 (second line-left), no region has its area lower than 1600 pixels and, in Fig. 2 (second line-center), all regions stands an erosion done using a s.e. with radius equals to 10. Such structures preservation does not occur in the classical decomposition.

In order to demonstrate the similarity among both uniform decompositions and the classical one, it was done and evaluation experiment. The segmentation evaluations were done by applying two functions proposed by Borsotti et al. (M. Borsotti, P. Campadelli

[1]Let us assume, for simplification, that each regional minimum has a distinct extinction value.

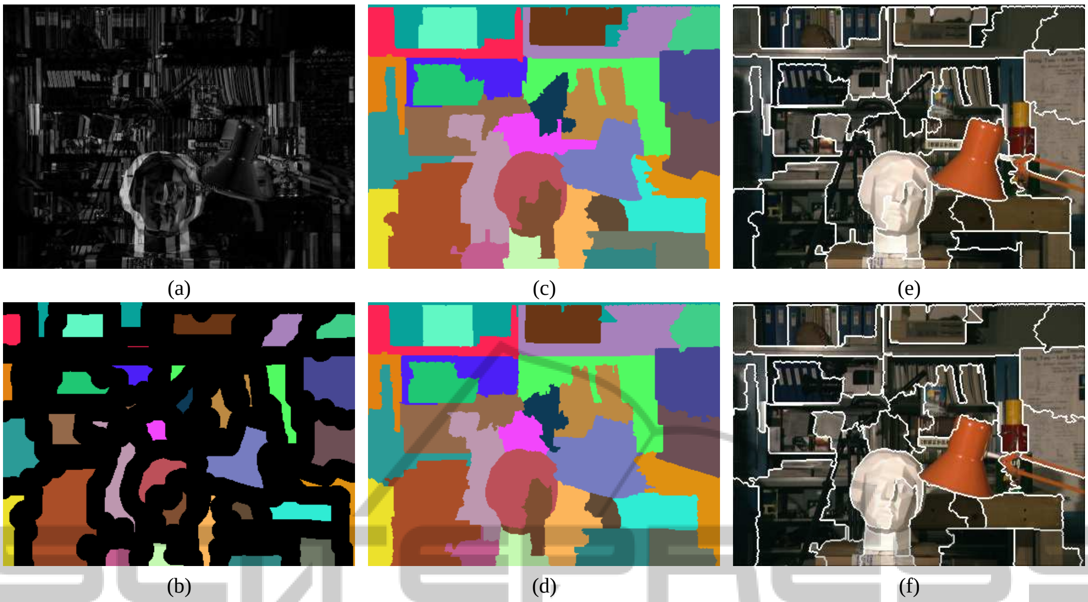

Figure 3: Support to image registration. (a) Image displacement. (b) Markers applied to segment Image Two. (c) Uniform Decomposition of Image One. (d) Segmentation of Image Two. (e) Image One (with overlaid segmentation result). (f) Image Two (with overlaid segmentation result).

and R. Schettini, 1998) to assess segmentation of color images according to heuristic criteria such as homogeneity and simplicity. When comparing segmentation results, the lower results are provided by the best segmentation according to the cited criteria.

Tables 1 and 2 show, respectively, the evaluation results given by the application of functions $F'(I)$ and $Q(I)$ proposed by Borsotti et al. Note that the values are very similar among themselves and each decomposition wins in at least one situation. However, no one victory condition was significative enough to highlight a decomposition in particular. Quantitatively, the three segmentation were equivalent. And, as a reminder, the uniform decomposition holds the criteria advantages.

4.2 Pre-processing for Stereo Vision

The second application presented in this paper is the matching of a pair of stereo images, in order to preprocess them before the displacement estimation for each object in the image registration framework. Basically, the proposed pre-processing method aims to segment the objects contained in one image and use them as markers to segment the same objects in the other image. It is not added any semantics to the preprocessing method: the segmentation of the objects is given by the uniform decomposition of Image One, using as criterion the disc with radius p. The subsets are then erode separately by a disc s.e. with radius no greater than $p/2$. This eroded subsets will be the markers applied to segment Image Two via watershed from markers.

This matching framework proposed here was designed under the assumption that the objects contained in Image One are also contained in Image Two, considering a reasonable displacement. Given an object represented by a subset in the partition of Image One, it is expected that the erosion of the subset, by a s.e. which radius is chosen in function of the image displacement, provides a marker located on the same object in Image Two.

In the following, it is presented the result of the pre-processing described in the previous paragraphs. It is applied to a pair of images extracted from the dataset available at http://vision.middlebury.edu/stereo/. Figure 3 (a) shows the displacement of two Images One and Two (this displacement was computed by the symmetrical difference between the graylevel version of both images). Figure 3 (c) shows the uniform decomposition of Image One, using as criterion a disc with radius 20. All regions are eroded separately by a disc with radius 10 and used as a set of markers (Fig. 3 (b)) to segment Image Two (Fig. 3 (d)). This segmentation is quite good.

4.3 Multiscale Segmentation by Interactive Splitting of Markers

In this application, the uniform decomposition is used

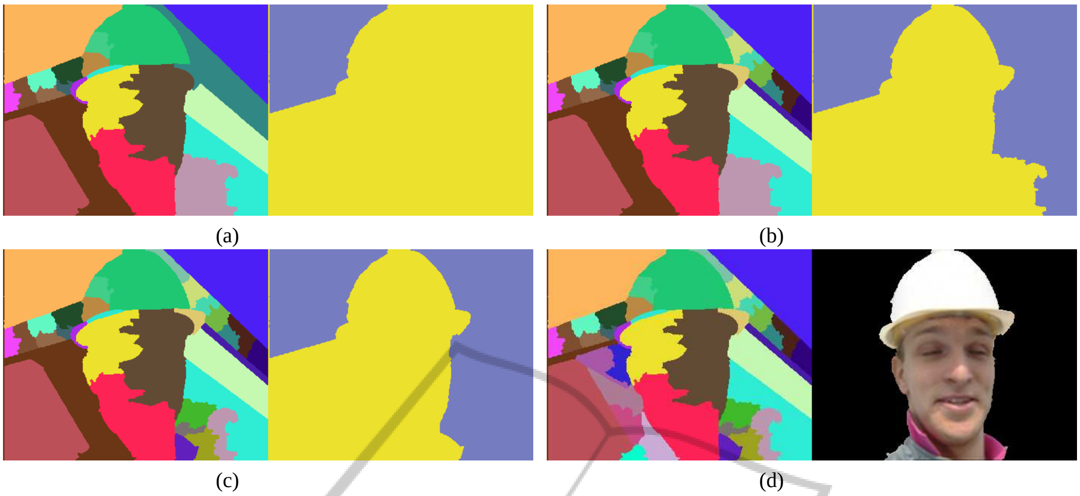

Figure 4: Interactive segmentation. Decomposition and switching of (a) upper-left region. (b) upper-right region. (c) down-right region. (d) down-left region (composed with the original image).

to achieve interactive segmentation of static images. The idea in this approach is not to add or to remove markers to segment objects via watershed from markers application. The regions of the image are segmented by the local hierarchy control in each region.

The user points the mouse cursor to a given region and selects with the mouse buttons the desired operations. The activation of such operations depends on the current mode the interface is switched. There are two interface modes: in the *decomposition mode*, the user segments hierarchically the region pointed by the mouse cursor using the mouse buttons to control the hierarchy. Left button lowers the hierarchy in a region while right button raises the hierarchy. In the *edition mode*, the user may choose two operations: to split a region in two ones using the same decomposition or to define if the region is a foreground or a background one. Except in the decomposition mode where the user goes up and down in the hierarchy of a given region, there is no merging of regions in this segmentation framework. Once a decomposition operation is done, it is not possible to join the split regions in the further steps.

Two images are provided in the method interface: one of them shows the current hierarchical segmentation; all tasks are done according to the partition shown in this image. The second image just show which subsets of the partition are classified as foreground (yellow) and which ones are classified as background (blue).

Figures 4 shows some samples of such images, taken from a sequence of operations over the *Foreman* input image. Each sample shows a significative instant where part of the foreman is segmented. Figure 4 (a)) shows the decomposition and switches of a region in the upper-left location in the image, after a global initial decomposition. Figures 4 (b-d) show the decomposition and switches of other regions in the image. Figure 4 (d) shows the composition of final result with the input image.

5 CONCLUSIONS

The uniform decomposition is a hierarchical segmentation method which incremental process guarantees that all regions in the decomposed image respect an input criterion, such as area or diameter of a contained disc. In each iteration, it is investigated is a candidate region may be split in two new ones, and this splitting is done according to the input criterion. It provides a more balanced distribution of subsets in the output image partition.

Several applications supported by the uniform decomposition were proposed and the results are quite satisfactory. It was also done a experimental comparison with the classical hierarchical segmentation provided by the threshold of the regional minima extinction values. The comparison was done subjectively and quantitatively and the results provided by the uniform and classical decomposition were comparable. More, the uniform decomposition still gives the advantage of retain the structure criterion for each subset of the partition.

Future works include the design of new criterion functions, the proposal of methods to find valid pair of markers and the proposal of new applications of the uniform decomposition.

ACKNOWLEDGEMENTS

Authors would like to thank Fundação Araucária for financial support.

REFERENCES

Beucher, S. and Meyer, F. (1992). *Mathematical Morphology in Image Processing*, chapter 12. The Morphological Approach to Segmentation: The Watershed Transformation, pages 433–481. Marcel Dekker.

C. Vachier (1995). *Extraction de caractéristiques, segmentation d'image et morphologie mathématique.* PhD thesis, Ecole des Mines de Paris.

C. Vachier and F. Meyer (1995). Extinction Value: A New Measurement of Persistence. In *Proceedings of 1995 IEEE Workshop on Nonlinear Signal and Image Processing*, volume I, pages 254–257. IEEE.

F. C. Flores and R. A. Lotufo (2010). A New Hierarchical Decomposition Applied to Object Segmentation in Image Sequences: The Uniform Decomposition. In *IEEE Proceedings of SIBGRAPI'2010*, pages 156–163, Gramado, Brazil.

F. Meyer (2001). Hierarchies of Partitions and Morphological Segmentation. In *Proceedings of the Third International Conference on Scale-Space and Morphology in Computer Vision*, volume 2106, pages 161–182.

M. Borsotti, P. Campadelli and R. Schettini (1998). Quantitative Evaluation of Color Image Segmentation Results. *Pattern Recognition Letters*, 19(1998):741–747.

M.A.G. Carvalho, R. A. Lotufo and M. Couprie (2003a). Segmentation of Images of Yeast Cells by Scale-Space Analysis. In *IEEE Proceedings of SIBGRAPI'2003*, pages 376–380, Sao Carlos, Brazil.

M.A.G. Carvalho, R. A. Lotufo and M. Couprie (2003b). Spatiotemporal Segmentation of MR Image Sequence Based on Hierarchical Analysis. In *Proceedings of Seventh International Symposium on Signal Processing and Its Applications - ISSPA 2003*, pages 677–680, Paris, France.

P. Salembier; A. Oliveras and L. Garrido (1998). Antiextensive connected operators for image and sequence processing. *IEEE Transactions on Image Processing*.

Serra, J. (1982). *Image Analysis and Mathematical Morphology*. Academic Press.

Vincent, L. and Soille, P. (1991). Watersheds in Digital Spaces: An Efficient Algorithm Based on Immersion Simulations. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 13(6):583–598.