EXPLORATION OF QUANTITATIVE ASSESSMENT IN SYSTEMS ARCHITECTURE USING EVIDENTIAL REASONING

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Abstract: A key issue in successful developing complex systems is how to assess the performance of architecture model during the development process. Traditional assessment techniques are subjective and usually highlight weaknesses rather than provide quantitative and objective results. In addition, the increasing complexity of systems nowadays; has led to a move from federated systems which were closed and target unique to embedded open systems, which extends new criteria to be considered in the assessment. This paper provides an insight into how Evidential Reasoning (ER) approach as the specific Multi-Attribute Decision Making (MADM) method can be used as a type of system assessment technique. The theory and implementation details of ER with an initial case study are presented.

1 INTRODUCTION

Due to the cost of building and implementing large-scale systems, it is vital to make sure architecture designs are as accurate as possible before they are built (IWAG 2007). The assessment of an architecture model at early stage of the systems development process will allow stakeholders and engineers to have a better understanding of the strengths and weaknesses of the architecture for better decision making, to verify whether the system is right against specifications and requirements in order to meet the customer needs. Fixing defects after software code has been implemented is at least three times more costly (McConnell 2001). For embedded systems, this cost saving could be more when considering its complexity and hardware costs.

Many existing assessment techniques are subjective and merely a step-by-step guide on how to design architecture and qualitatively explain how the architecture will meet the requirements or outline possible weak points (Barbacci 2002). Although they describe themselves as architecture assessment methods, they appear to be more risk identification or weakness analysis.

A possible way to assess an system more accurately would be to quantitatively assess the quality attributes, which are defined as a high level system characteristic which cannot normally be measured directly, specific to the system (Purewal, Yang and Grigg 2009). Hence, the architecture assessment process becomes an increasing complex decision making problem in which one must simultaneously cater in a rational way for many inter-related criteria. It is also noted that the architecture assessment has to be conducted on the basis of both precise data and subjective judgments that are vague in nature.

In this paper, we will focus on how Evidential Reasoning (ER) approach primarily used in management and business studies can be implemented into the domain of systems engineering to create a method of assessment for systems architecture. The rest of the paper is organized as follows: after a belief explanation of the open systems and traditional architecture assessment methods, we will introduce Multiple Attribute Decision Making concept and describe the details of

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ER approach. This is then followed by a case study that demonstrates the application of the method and its implementation of possible architecture assessment in the future. Conclusion and future work are provided at the end of the paper.

2 OPEN SYSTEMS AND ARCHITECTURE ASSESSMENT

Open systems architecture implements sufficient open specifications for interfaces, services, and supporting formats to enable properly engineered components to be utilized across a wide range of systems with minimal changes, to interoperate with other components on local or remote systems, and to interact with users in a style that facilitates portability (SEI-CMU 2008). The idea is that of a desktop computer, where components can be obtained from many independent sources and are still compatible with the system (Henderson 2006). Common advantages include reduced cost by using commercial off-the shelf components (Zalcman 2002), better supportability which is based on the fact that a faulty module can be replaced with a new module without being necessary to be identical, thus encouraging upgrade strategies (Murdoch and Hardy 2000), increased operational flexibility and simplifying maintenance (Clements and Bergey 2005), combining affordable cost with the ability to deal with rapidly evolving technology (Borky et al 1998).

To allow these development enhancements, a number of underlying principles are used, including: modularity of system functionality, adaptive architecture (at design time), modular and layered architecture components, flexible data interconnect architecture. These characteristics extend other criteria need to be considered in the architecture assessment such as the use of standards, well defined documentation, and level of impact when upgrading or expanding the system (Purewal, Yang and Grigg 2009).

Currently, most existing assessment techniques are based on the traditional federated system, which do not support independent analysis of applications on a shared resource (Conmy and McDermid 2001). Among them, the most popular assessment methods are those created by the Software Engineering Institute (SEI), including the Quality Attribute Workshop (QAW) and the Architecture Trade-Off Analysis Method (ATAM) (Barbacci 2002). For QAW, it does assessment in a very subjective manner by exploring scenarios of an architecture model and most results are documents with written test cases and how the system will handle such cases so that the stakeholders may be interpreted into different ways. While for ATAM, it is more of a guide on how to build the system rather than precisely predict the behaviour of the system characteristic. They are more types of analysis methods rather than assessment scoring.

3 MCDM AND ER

Multiple Attribute Decision Making (MADM) refers to making decision in the presence of multiple, ususlly conflicting, criteria (Xu and Yang 2001). Since 1970s, many MCDM methods have been developed, such as the well-know Analytical Hierarchy Process (AHP) (Saaty, 1977, 1988, 1994) and Multiple Attribute Utility Theory (Keeney and Raiffa 1993; Jacquet-Lagreze and Siskos 1982; Belton and Stewart 2002). In those methods, as well as their extensions, such as the interval-valued assessment approach, MADA problems are modelled as different alternatives which are assessed on each criterion by either a single real number of an interval value. While in many decision situations, information will be of different types of forms, such as a subjective judgement with uncertainty, a probability distribution, or an incomplete piece of data. Thus, using a single number or an interval value to represent a judgement proves to be difficult and sometimes unacceptable.

Evidential Reasoning (ER) (Yang and Singh 1994; Yang and Sen 1994; Yang 2001; Yang and Xu 2002a,b) is one of the latest developments within MCDM literature. Based on a belief decision structure and the evidence combination rule of the Dempster-Shafer (D-S) theory (Dempster 1967; Shafer 1976), the ER approach can both model precise data and capture various types of uncertainties such as ignorance (incomplete information) and fuzziness (vague judgments). It has been applied to decision problems in engineering design, safety and risk assessment, organizational self-assessment, and supplier assessment (Yang and Xu 2002a).

3.1 Details of the ER Approach

In the ER application, assessment process can be divided into five main parts: setting up the hierarchy, weighting the basic attributes, normalizing the attribute value into one scale, evidence combination and final assessment by aggregating the attributes.

3.1.1 Assessment Hierarchy and Belief Decision Matrix

The assessment hierarchy is typically constructed with attributes at different levels. A high level attribute may represent a type of system characteristic so that it is comparable with other attributes. Such an attribute may only be evaluated through a set of detailed factors which are associated with measurable attributes or another embedded hierarchical structure. The definition of attributes and hierarchical structure is typically based on the experts' knowledge and the literature. Once the hierarchy set up, a belief decision matrix D_g can be defined as follows:

Suppose M upper level attributes a_m, $m = 1, . . . , M$, which are referred to as problem alternatives, are to be assessed based on L lower level attributes e_i, $i = 1, . . . , L$, named basic attributes.

Meanwhile, M alternatives are all assessed using the same set of N assessment grades H_n, $n = 1, . . . , N$, which are required to be mutually exclusive and collectively exhaustive for the assessment of all attributes.

If alternative a_m is assessed to a grade H_n on an attribute e_i to a belief degree of $B_{n,i}$, this assessment will be denoted by $S(e_i (a_m)) = \{(H_n , B_{n,i} (a_m)), n = 1, . . . , N\}$.

The individual assessments of the M alternatives on the L basic attributes can be then represented by the belief decision matrix $D_g = (S (e_i (a_m)))_{L\times M}$.

In above definition, the belief degree is originally designed to model a subjective assessment with uncertainty (Yang and Singh 1994; Yang and Sen 1994). The expert may not always be one hundred percent sure that the state of a factor is exactly confirmed to one of the assessment grades. Thus one or more single assessment grades may simultaneously be combined to confirm the total confidence of anything up to one hundred percent. For example, to evaluate the attribute "Coupling level" of a modular system at the development stage a_m, an expert may be 60% sure it is low and 30% sure it is medium. In the statement above, "low" and "medium" denote distinctive assessment grades, and the percentage values of 60 and 30 are referred to as the degrees of belief, which indicate the extents that the corresponding grades are assessed to. Stage a_m corresponds to an assessment alternative. Such assessment can be thus expressed as follows:

$$S(\text{Coupling}(a_m)) = \{(\text{low}, 0.6), (\text{medium}, 0.3)\} \quad (1)$$

It can be also noted that as the total degree of belief is $0.6 + 0.3 = 0.9 < 1$, the assessment in above example is described as incomplete assessment.

3.1.2 Weighting

Weighting refers to assigning relative weights to the L basic attributes e_i by $W = (w_1 , . . . , w_L)$, which are supposed to be known and satisfy the conditions $0 \le w_i \le 1$ and $\sum_{i=1}^{L} w_i = 1$.

It plays an important role in assessment. In determining the weights of different assessment attributes, different methods can be implemented, such as simple direct rating by an expert, or more elaborate methods based on the pair-wise comparison technique (Saaty 1988; Yang, Deng and Xu 2001).

3.1.3 Normalization

Normalization refers to transforming the various types of information at basic attributes into the belief degree space which can be recognized by the algorithm.

For qualitative assessment, the transformation is to transform the various sets of evaluation standards to a unified set so that all attributes can be assessed in a consistent and compatible manner. For example, the set of grade $H = \{H_n , n = 1, ... , 7\}$ can be transformed to the set of grade $K = \{K_n , n = 1, ... , 5\}$ as follows:

$$\begin{aligned} K_1 &= \{(H_1 , 1.0)\}, \\ K_2 &= \{(H_2 , 0.5),(H_3 , 0.5)\}, \\ K_3 &= \{(H_4 , 1.0)\}, \\ K_4 &= \{(H_5 , 0.5),(H_6 , 0.5)\}, \\ K_5 &= \{(H_7 , 1.0)\}, \end{aligned} \quad (2)$$

For quantitative assessment, suppose any precisely known attribute value y_i , it must lie between two adjacent assessment grades: $Y_{n,i} \le y_i \le Y_{n+1,i}$, where $n \in \{1, ... , N-1\}$. It is obvious that we can use these two assessment grades to characterize the attribute value y_i . Let $B_{n,i}$ and $B_{n+1,i}$ characterize the belief degrees to which y_i is assessed to the grades $Y_{n,i}$ and $Y_{n+1,i}$ respectively, calculated by

$$B_{n,i} = (Y_{n+1,i} - y_i) / (Y_{n+1,i} - Y_{n,i}) \quad (3)$$

$$B_{n+1,i} = (y_i - Y_{n,i}) / (Y_{n+1,i} - Y_{n,i}) \quad (4)$$

Further detailed of transformation technique can be referred in paper (Yang 2001).

3.1.4 Evidence Combination

In the evidence combination step, the ER algorithm first transforms the original belief degrees, which have been obtained in the scaling step, into basic probability masses by combining the relative weights and the belief degrees using the following equations:

$$p_{n,i} = p_i(H_n) = w_i B_{n,i}(a_m),\ n = 1, \ldots, N,\ i = 1, \ldots, L \quad (5)$$

$$p_{H,i} = p_i(H) = 1 - \sum_{n=1}^{N} p_{n,i} = 1 - w_i \sum_{n=1}^{N} B_{n,i}(a_m),\ i = 1, \ldots, L \quad (6)$$

$$\bar{p}_{H,i} = \bar{p}_i(H) = 1 - w_i,\ i = 1, \ldots, L \quad (7)$$

$$\hat{p}_{H,i} = \hat{p}_i(H) = w_i\left(1 - \sum_{n=1}^{N} B_{n,i}(a_m)\right),\ i = 1, \ldots, L \quad (8)$$

with $p_{H,i} = \bar{p}_{H,i} + \hat{p}_{H,i}$ and $\sum_{i=1}^{L} w_i = 1$

Here we can see the ignorance and uncertainty in the individual assessment is broke down into two parts, $\bar{p}_{H,i}$ and $\hat{p}_{H,i}$, so that can be treated differently, where $\bar{p}_{H,i}$ is the unassigned probability mass caused by the relative importance of the attribute e_i, $\hat{p}_{H,i}$ is the unassigned probability mass caused by the incompleteness of the assessment on e_i. The $p_{H,i}$ is the total of them, named the probability mass assigned to the whole set H.

Next, based on the D-S theory, the basic probability masses on the L basic attributes are aggregated into the combined probability assignments by using the following analytical formulae:

$$\{H_n\}: p_n = k\left[\prod_{i=1}^{L}(p_{n,i} + \bar{p}_{H,i} + \hat{p}_{H,i}) - \prod_{i=1}^{L}(\bar{p}_{H,i} + \hat{p}_{H,i})\right],\ n = 1, \ldots, N \quad (9)$$

$$\{H\}: p_H = k\left[\prod_{i=1}^{L}(\bar{p}_{H,i} + \hat{p}_{H,i}) - \prod_{i=1}^{L}\bar{p}_{H,i}\right] \quad (10)$$

$$\{H\}: p_H = k\left[\prod_{i=1}^{L}\bar{p}_{H,i}\right] \quad (11)$$

where

$$k = \left[\sum_{n=1}^{N}\prod_{i=1}^{L}(p_{n,i} + \bar{p}_{H,i} + \hat{p}_{H,i}) - (N-1)\prod_{i=1}^{L}(\bar{p}_{H,i} + \hat{p}_{H,i})\right]^{-1} \quad (12)$$

Finally, the combined probability assignments are normalized into overall belief degrees by using the following equations:

$$\{H_n\}: B_n = p_n / (1 - p_n),\ n = 1, \ldots, N \quad (13)$$

$$\{H\}: B_H = p_H / (1 - p_H) \quad (14)$$

where B_n and B_H represent the overall belief degrees of the combined assessments assigned to the assessment grades H_n and H, respectively. This combined assessment can be denoted by
$S(y(a_m)) = \{(H_n, B_n(a_m)), n = 1, \ldots, N\}$

The above formulae (5)-(14) together constitute a complete ER analytical algorithm which is considered as the theory foundation of the ER approach in the business application.

3.1.5 Final Assessment with Utility

In the final assessment step, expected utility values are calculated for ranking alternatives, as distributed descriptions are not sufficient to show the difference between two assessments.

Suppose H_1 is the least preferred grade having the lowest utility and H_N the most preferred grade having the highest utility. Then the maximum, minimum and average expected utilities on alternative y are given by:

$$u_{max}(y) = \sum_{n=1}^{N-1} B_n u(H_n) + (B_N + B_H)u(H_N) \quad (15)$$

$$u_{min}(y) = (B_N + B_H)u(H_1) + \sum_{n=2}^{N} B_n u(H_n) \quad (16)$$

$$u_{avg}(y) = (u_{max}(y) + u_{min}(y)) / 2 \quad (17)$$

If all original assessments are complete, then $B_H = 0$ and $u(y) = u_{max}(y) = u_{min}(y) = u_{avg}(y)$.

We say $u(y_{n+1})$ is preferred to $u(y_n)$ if and only if $u(y_{n+1}) > u(y_n)$.

4 INITIAL CASE STUDY OF ER

The ER algorithm has been developed into a software program named IDS Multi-criteria Assessor, or IDS for short. In this section, the software will be used to carry out the assessment in our case study to provide an insight of how the approach can be implemented in order to assess the system architecture.

As the first part, an assessment hierarchy with the weight of each attribute is formed as shown in table 1 below:

Table 1: Assessment attributes hierarchy and the weights associated.

Level 1	Level 2	Level 3
1. Openness (45%)	1.1 Modularity (60%)	1.1.1 Coupling (35%)
		1.1.2 Interface Complexity (65%)
	1.2 Interoperability (40%)	1.2.1 Standard Interfaces (70%)
		1.2.2 Stable Interfaces (30%)
2. Functionality (55%)	2.1 Operability (70%)	2.1.1 Initialisation (15%)
		2.1.2 Partitioning (20%)
		2.1.3 Scheduling (45%)
		2.1.4 Timing (20%)
	2.2 Maintainability (30%)	2.2.1 Complexity (75%)
		2.2.2 Documentation (25%)

Although the attributes selecting is based on the literature review of the systems engineering and the field work conducted within Systems Engineering Innovation Centre (SEIC), the hierarchy and weights of each attribute are to some extent assumed for simplicity.

Table 2 presents the belief degrees of each measurable factor in two different development stages. The values assigned here is also by assumption. But to be noted, in the real application, these values assignment can be calculated through restrict definition and appropriate methods. For example, the level of attribute "Complexity" can be calculated by the number of classes, number of interfaces, number of operations, number of connectors etc.

Table 2: Assessment data for measurable factors.

Level 3 Factors	Development Stages (alternatives)	
	Stage 1	Stage 2
1.1.1 Coupling	{43,1.0}	{57,1.0}
1.1.2 Interface Complexity	{82,1.0}	{49,1.0}
1.2.1 Standard Interfaces	{65,1.0}	{63,1.0}
1.2.2 Stable Interfaces	{47,1.0}	{44,1.0}
2.1.1 Initialisation	{(A,0.8), (G,0.2)}	{(A,0.4), (G,0.6)}
2.1.2 Partitioning	{(P,0.5), (A,0.5)}	{(P,0.2), (A,0.6), (G,0.1)}
2.1.3 Scheduling	{(A,0.2), (G,0.7)}	{G,1.0}
2.1.4 Timing	{A,1.0}	{(P,0.2), (A,0.7), (G,0.1)}
2.2.1 Complexity	{78,1.0}	{62.1.0}
2.2.2 Documentation	{92,1.0}	{72,1.0}

(The assessment grades for qualitative attributes are defined as W – worst, P – poor, A – average, G – good and B – best. The quantitative attributes are assessed from 0 to 100, where 100 represent the best situation. The number behind the assessment grade or value represents the belief degree.)

We can see both qualitative and quantitative attributes in the model. Assessment with uncertainty between different grades and incomplete assessment on individual attribute are demonstrated.

Figure 1 and Table 3 below shows the results of the overall assessment implemented by the IDS. It can be seen the overall score for development stage 1 (0.6418) is better than the stage 2 (0.5891), which can be concluded that the modification work in development stage 2 is not successful comparing with the original achievement (stage 1). Further examine the assessment of each high level system characteristic, only the scoring of operability is improved in development stage 2. We can thus make a possible conclusion according to the results of the assessment that other system characteristics are sacrificed in the improving of system operability in development stage 2. It is not a recommended modification from the view of overall.

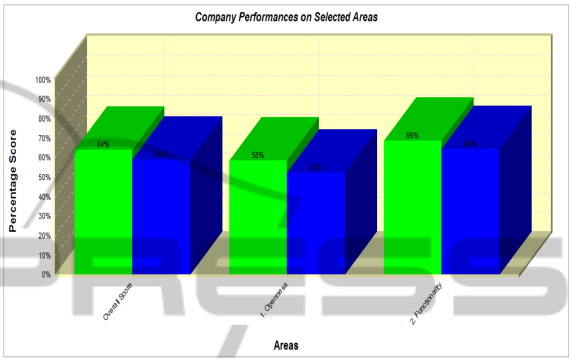

Figure 1: Overall score of the assessment.

Table 3: Utility results of the final assessment.

High level attributes	Development stages (alternatives)	
	Stage 1	Stage 2
Overall score	0.6418	0.5891
1. Openness	0.5840	0.5218
1.1 Modularity	0.6250	0.5223
1.2 Interoperability	0.5510	0.5296
2. Functionality	0.6879	0.6447
2.1 Operability	0.5220	0.5926
2.2 Maintainability	0.8433	0.6793

5 CONCLUSIONS AND FUTURE WORK

The initial investigation in this paper has shown that ER approach appears to provide a method to quantitatively assess the current system architectures, with the ability to deal with various types of uncertainties.

In recent years, the original ER approach has been further developed to support the solution of MADM problems with interval grades assessment (Xu, Yang and Wang 2006; Wang, Yang, Xu and Chin 2006), type of fuzziness (Yang, Wang , Xu and Chin 2006) and even both of them (Guo, Yang Chin, Wang and Liu 2009).

In Interval ER (IER) approach, the unknown portion of performance represented by the probability mass assigned to the whole set H in the original ER is narrowed on the subsets of adjacent grades so that taking the advantage with ability of handling the interval judgement and reducing the uncertainty in the final assessment.

In fuzzy ER and fuzzy IER approaches, triangular and trapezoidal fuzzy sets are incorporated into the ER and IER to simulate the overlap of adjacent assessment grades to support the solution of more sensitivity analysis in complex MADM problems. However, due to additional uncertainties caused by the fuzzy sets, the uncertainties of the final assessment will be enlarged apparently in comparison with the non-fuzzy results.

The next stage of our research is to investigate how the ER approach and its extensions can be modified in any way in order to be implemented into the actual application of architecture assessment.

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