# A CONSTRAINED FINITE TIME OPTIMAL CONTROLLER FOR THE DIVING AND STEERING PROBLEM OF AN AUTONOMOUS UNDERWATER VEHICLE

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Keywords: AUV, Constrained finite time, Optimal control.

Abstract: In this paper a Constrained Finite Time Optimal Controller (CFTOC) is designed and applied to the diving and steering problem of an Autonomous Underwater Vehicle. The non–linear model of the AUV is presented and the decoupled linear models for the steering and diving motions of the vehicle are derived, based on certain modeling assumptions and simplifications, while the cruising speed of the vehicle is considered to be small and constant. The proposed control scheme has the merit to take under consideration: a) the mechanical and physical constrains of the AUV, b) uncertainties produced from modeling errors and environmental noise, c) constrains in the motors, and produce an optimal controller for the vehicle that will guarantee the stability of the closed loop system. The proposed CFTO–controller is applied to simulation studies and relevant simulation results are presented that prove the efficacy of the proposed scheme.

## 1 INTRODUCTION

In the last years there was a strong interest towards the development of Autonomous Underwater Vehicles (AUV) and Remotely Operated Vehicles (ROV). These classes of underwater vehicles are intended to provide simple, long–range, low–cost measurements of environmental data or surveillance studies. Although ROVs have been utilized in the past in many applications, in the recent years there has been a growing demand for the utilization of AUVs as these vehicles are superior to the ROVs, are completely autonomous and are not suffering from the demand of high operating costs, dedicated cables for carrying the data links between the vessel and the ROV and experienced crew to guide the vehicle (Yuh, 2000; E. An, 2001; Foresti, 2001).

The superiority of AUVs and their complete autonomy are generating more demands for modeling approaches and applied control algorithms as more accurate and fast control actions should be applied to the vehicle to improve the overall performance. In the utilized model of an AUV, when other factors are taken under consideration, including: a) parametric uncertainties such as added mass, hydrodynamic coefficients, lift and drag forces, b) highly and coupled non linearities, and c) environmental disturbances like ocean currents and wave effects, the problem of applying a most suitable control law is widely increased.

In the area of mathematical modeling for AUVs, there have been extended analytical approaches with the main variation being the level of association between the hydrodynamic phenomena and the underwater rigid body dynamics. In the general case the mathematical models contain hydrodynamic forces and moments expressed in terms of a set of hydrodynamic coefficients, therefore it is of paramount importance to *a priori* know the true values of these coefficients to control the AUV accurately. In most of the cases experimental measurements are needed to tune the hydrodynamic parameters of the models while the relative literature is providing sufficient references and methodologies for calculating these parameters in various types of underwater vehicles.

In the area of controlling AUVs, until now various classical approaches have been utilized. More specifically many control strategies have been appeared in


Nikolakopoulos G., J. Roussos N. and Alexis K. (2010).
A CONSTRAINED FINITE TIME OPTIMAL CONTROLLER FOR THE DIVING AND STEERING PROBLEM OF AN AUTONOMOUS UNDERWATER VEHICLE.
In *Proceedings of the 7th International Conference on Informatics in Control, Automation and Robotics*, pages 260-267
DOI: 10.5220/0002948202600267

the literature for the diving and steering control of an AUV such as: Optimal control (Field, 2000), Neural Networks (Kawano and Ura, 2002), Fuzzy control (Debitetto, 1995), Adaptive Sliding Mode control (Cristi et al., 1990), Proportional and Derivative (PD) control (Bjorn, 1994; Pestero, 2001), Sliding Modes Control (SMC) (Healey and Lienard, 1993; Rodrigues et al., 1996) and Linear Quadratic Gaussian (LQG) controller (Fossen, 1994b). In all these approaches, in the first stage of the controller design, decoupling can be applied to the movements of the AUV and each movement can be modeled and controlled by a different set of differential equations and different controllers.

In the current research effort the aim is to utilize a more accurate and realistic modeling approach for a torpedo like AUV, the REMUS AUV, and based on the derived decoupled model of the vehicle's motions to design a novel constrained finite time optimal control scheme for the diving and steering motions. The proposed control scheme has the advantage of taking under consideration in the design phase: a) the physical and mechanical constrains, b) the disturbances from the environmental noise, and c) the additive uncertainty on the system transfer function due to modeling errors and non–linearities. In spite of the complexity of the control design stage (off–line), the on–line controller implementation results in an exhaustive search in a multidimensional look–up table, depending on the number of the system's states and control inputs, that can be easily implemented in an on board micro–controller .

This article is structured as follows. In Section 2 the utilized modeling approach for the AUV is presented, while the design of the proposed CFTO–control scheme is presented in Section 3. The validity of the proposed scheme is provided in Section 4 by simulation results, resulting from the application of the proposed scheme to the diving and steering motions of the AUV. Finally the conclusions are drawn in Section 5.

## 2 AUV MODELING

The AUV under study is a torpedo like underwater vehicle and it is illustrated in Figure 1 with the relevant body–fixed and inertial coordinate systems. The modeling of the equations of motion will be derived by the utilization of a relative standard framework that has been established in (Gertler and Hagen, 1967) and revised in (Humphreys, 1976) and (Feldman, 1979) with the assumptions: 1) the vehicle is deeply submerged in a homogeneous and unbounded liquid, 2) the vehicle does not experience memory effects, 3) the simulator neglects the effects of the vehicle passing through its own wake, and 4) the vehicle does not experience underwater currents. In addition, the following assumptions for the vehicle's dynamics are also necessary: 1) the vehicle is a rigid body of constant mass, 2) the control fins do not stall regardless of angle of attack, and 3) the propulsion model treats the vehicle propeller as a source of constant thrust and torque. The vehicle equations

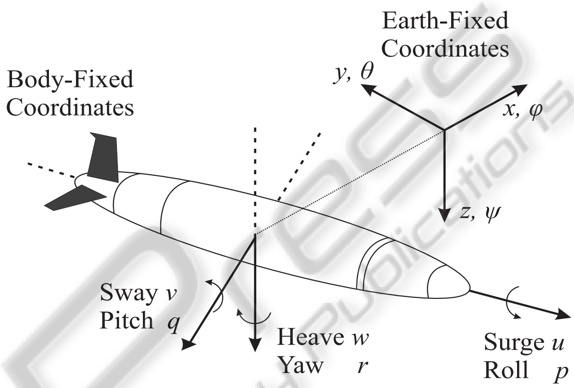


Figure 1: Body Fixed and Inertial Coordinate System of the AUV.

of motion in Figure 1, consist of the kinematics, rigid-body and mechanic terms (Fossen, 1994a) and by combining the equations for the vehicle rigid–body dynamics with the equations for the forces and the moments of the vehicle, we can conclude in the following non–linear set of equations in a general framework of six degrees of freedom plane. These equations follow the SNAME convention (SNAME, 1950) for the assignment of the body–fixed vehicle coordinate system and are presented in the above for retaining clarity.

*Eq.1 - Surge*:

$$\begin{aligned}(m-X_{\dot{u}})\dot{u}+mz_g\dot{q}-my_g\dot{r} &= X_{HS}+X_{u|u|}u|u|+(X_{\omega q}-m)\omega q\\ &+(X_{qq}+mx_g)q^2+(X_{\upsilon r}+m)\upsilon r+(X_{rr}+mx_g)r^2\\ &-my_g pq-mz_g pr+X\,prop\end{aligned} \tag{1}$$

*Eq.2 - Sway*:

$$\begin{aligned}(m-Y_{\dot{\upsilon}})\dot{\upsilon}+mz_g\dot{p}-(mx_g-Y_{\dot{r}})\dot{r} &= Y_{HS}+Y_{\upsilon|\upsilon|}\upsilon|\upsilon|+Y_{r|r|}r|r|\\ & my_g r^2+(Y_{ur}-m)ur+(Y_{\omega p}+m)\omega p+(Y_{pq}-mx_g)pq\\ &+Y_{u\upsilon}u\upsilon+my_g p^2+mz_g qr+Y_{uu\delta_r}u^2\delta_r\end{aligned} \tag{2}$$

*Eq.3 - Heave*:

$$\begin{aligned}(m-Z_{\dot{\omega}})\dot{\omega}+my_g\dot{p}-(mx_g+Z_{\dot{q}})\dot{q} &= Z_{HS}+Z_{\omega|\omega|}\omega|\omega|\\ &+Z_{q|q|}q|q|+(Z_{uq}+m)uq+(Z_{\upsilon p}-m)\upsilon p+(Z_{rp}-mx_g)rp\\ &+Z_{u\omega}u\omega+mz_g(p^2+q^2)-my_g rq+Z_{uu\delta_s}u^2\delta_s\end{aligned} \tag{3}$$

*Eq.4 - Roll*:

$$-mz_g\dot{\upsilon} + my_g\dot{\omega} + (I_{xx} - K_{\dot{p}})\dot{p} = K_{HS} + K_{p|p|}p|p| \\ -(I_{zz} - I_{yy})qr + m(uq - \upsilon p) - mz_g(\omega p - ur) + K_{prop} \tag{4}$$

*Eq.5 - Pitch*:

$$mz_g\dot{u} - (mx_g + M_{\dot{\omega}})\dot{\omega} + (I_{yy} - M_{\dot{q}})\dot{q} = M_{HS} + M_{\omega|\omega|}\omega|\omega| \\ +M_{q|q|}q|q| + (M_{uq} - mx_g)uq + (M_{\upsilon p} + mx_g)\upsilon p \\ +[M_{rp} - (I_{xx} - I_{zz})]rp + mz_g(\upsilon r - \omega q) + M_{u\omega}u\omega + M_{uu\delta_s}u^2\delta_s \tag{5}$$

*Eq.6 - Yaw*:

$$-my_g\dot{u} + (mx_g + N_{\dot{\upsilon}})\dot{\upsilon} + (I_{zz} - N_{\dot{r}})\dot{r} = N_{HS} + N_{\upsilon|\upsilon|}\upsilon|\upsilon| \\ +N_{r|r|}r|r| + (N_{ur} - mx_g)ur + (N_{\omega p} + mx_g)\omega p \\ +[N_{pq} - (I_{yy} - I_{xx})]pq - my_g(\upsilon r - \omega q) + N_{u\upsilon}u\upsilon + N_{uu\delta_r}u^2\delta_r \tag{6}$$

In these equations, the vehicle's cross products of inertia $I_{xy}$, $I_{xz}$, $I_{yz}$ were assumed to be small and neglected. Moreover, zero value coefficients have not been included in the current formulation.

## 2.1 Diving Plane Motion

For deriving the equations of the diving plane motion, we should take under consideration only the body–relative surge velocity $u$, heave velocity $\omega$, the pitch rate $q$, the earth–relative vehicle forward position $x$, the diving $z$, and the pitch angle $\theta$. Before linearizing these equations in (1-6) we will integrate the terms for the hydrostatics, the axial and crossbow drag, the added mass, the body and fin lift and finally the moments. By assuming that the other velocities ($\upsilon$, $p$, $r$) are negligible, we can result in the following linearized relationships between the body and earth fixed vehicle velocities (Prestero, 2001):

$$\begin{aligned} (m - X_{\dot{u}})\dot{u} + mz_g\dot{q} - X_u u - X_q q - X_\theta\theta &= 0 \\ (m - Z_{\dot{\omega}})\dot{\omega} - (mx_g + Z_{\dot{q}})\dot{q} - Z_\omega\omega - (mU + Z_q)q &= Z_{\delta_s}\delta s \\ mz_g\dot{u} - (mx_g + M_{\dot{\omega}})\dot{\omega} + (I_{yy} - M_{\dot{q}})\dot{q} - M_\omega\omega + & \\ (mx_gU - M_q)q - M_\theta\theta &= M_{\delta_s}\delta_s \end{aligned} \tag{7}$$

where at this point and for clarity in our presentation, the nomenclature that is ruling equations (1-7) is presented in Table 1.

If we assume that $z_g$ is small compared to the other terms, we can decouple heave and pitch from surge, which results in the following set of equations:

$$(m - Z_{\dot{\omega}})\dot{\omega} - (mx_g + Z_{\dot{q}})\dot{q} - Z_\omega\omega - (mU + Z_q)q = Z_{\delta_s}\delta s \tag{8}$$

$$-(mx_g + M_{\dot{\omega}})\dot{\omega} + (I_{yy} - M_{\dot{q}})\dot{q} - M_\omega\omega + (mx_gU - M_q)q - M_\theta\theta = M_{\delta_s}\delta_s \tag{9}$$

and based on the mentioned assumptions the vehicle's kinematic equations of motion are formulated as (Triantafyllou and Franz, 2003):

Table 1: Utilized parameters and their values in the linearized description of AUV's diving motion.

| Par. | Name | Par. | Name |
|---|---|---|---|
| $x_g$ | Center of gravity | $z_g$ | Center of gravity |
| $M_\theta$ | Hydrostatic | $m$ | AUV's mass |
| $X_u$ | Axial drag | $X_{\dot{u}}$ | Added mass |
| $X_q$ | Added mass | $I_{yy}$ | Moment of inertia |
| $Z_{\dot{q}}$ | Heave velocity | U | Steady velocity |
| $Z_\omega$ | Combined term | $Z_{\dot{\omega}}$ | Added mass |
| $Z_q$ | Combined term | $Z_{\dot{q}}$ | Added mass |
| $M_\omega$ | Combined term | $M_{\dot{\omega}}$ | Added mass |
| $M_q$ | Combined term | $M_{\dot{q}}$ | Added mass |
| $X_\theta$ | Hydrostatic | | |

$$\dot{x} = \cos(\theta)u + \sin(\theta)\omega \tag{10}$$

$$\dot{z} = -\sin(\theta)u + \cos(\theta)\omega \tag{11}$$

$$\dot{\theta} = q \tag{12}$$

For the linearization of equations in (10-12) it is assumed that the vehicle motion consists of small perturbations around a steady point. In this case, $U$ represents the steady–state forward velocity of the vehicle. If heave and pitch are linearized about zero, we have $u = U + u^{'}$, $\omega = \omega^{'}$ and $q = q^{'}$. By utilizing: a) the equations in (10–12), b) the Maclaurin expansion of the trigonometric terms, c) dropping the higher order terms, and d) with the above assumption for the decoupling of heave and pitch from surge, the following linearized kinematic equations of motion are derived:

$$\dot{z} = \omega - U\theta \tag{13}$$

$$\dot{\theta} = q \tag{14}$$

The combination of equations (8) and (9) with those in equations (13) and (14) results in the following matrix form for the description of the diving plane motion of the AUV:

$$\begin{bmatrix} m - X_{\dot{u}} & -(mx_g + Z_{\dot{q}}) & 0 & 0 \\ -(mx_g + M_{\dot{\omega}}) & I_{yy} - M_{\dot{q}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\omega} \\ \dot{q} \\ \dot{z} \\ \dot{\theta} \end{bmatrix} - \begin{bmatrix} Z_\omega & mU + Z_q & 0 & 0 \\ M_\omega & -mx_gU + M_q & 0 & M_\theta \\ 1 & 0 & 0 & -U \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega \\ q \\ z \\ \theta \end{bmatrix} = \begin{bmatrix} Z_{\delta_s} \\ M_{\delta_s} \\ 0 \\ 0 \end{bmatrix} [\delta_s] \tag{15}$$

Assuming that the heave velocities are small compared to the other terms, and that the center of gravity is equal to the buoyancy center ($x_g = 0$), the equations in (15) are simplified to the following:

$$\begin{bmatrix} I_{yy} - M_{\dot{q}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{q} \\ \dot{z} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} -M_q & 0 & -M_\theta \\ 0 & 0 & U \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ z \\ \theta \end{bmatrix} = \begin{bmatrix} M_{\delta_s} \\ 0 \\ 0 \end{bmatrix} [\delta_s] \tag{16}$$

and finally for the state space description of the linearized system, it is derived that:

$$\begin{bmatrix} \dot{q} \\ \dot{z} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{M_q}{I_{yy}-M_{\dot{q}}} & 0 & \frac{M_\theta}{I_{yy}-M_{\dot{q}}} \\ 0 & 0 & -U \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ z \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{M_{\delta_S}}{I_{yy}-M_{\dot{q}}} \\ 0 \\ 0 \end{bmatrix} [\delta_s] \quad (17)$$

Given the state vector $x_1 = [q \;\; z \;\; \theta]' \in \Re^3$ and the input $u_{con_1} = \delta_s \in \Re$ we can write the matrix form in (17) as:

$$\dot{x_1} = A_{div}x_1 + B_{div}u_{con_1} \quad (18)$$

$$y_1 = C_{div}x_1 \quad (19)$$

with $A_{div} = \begin{bmatrix} \frac{M_q}{I_{yy}-M_{\dot{q}}} & 0 & \frac{M_\theta}{I_{yy}-M_{\dot{q}}} \\ 0 & 0 & -U \\ 1 & 0 & 0 \end{bmatrix}$, $B_{div} = \begin{bmatrix} \frac{M_{\delta_S}}{I_{yy}-M_{\dot{q}}} \\ 0 \\ 0 \end{bmatrix}$ and $C_{div} = [1\ 0\ 0]$.

## 2.2 Steering Plane Motion

As it was presented in Section 2.2, the diving subsystem controls depth and pitch errors while the steering subsystem controls heading errors. In the presented approach it is assumed that the upper–bow rudder and the lower–bow rudder, as also the upper–stern and the lower–stern rudder of the AUV, are of identical size and shape and are receiving the deflection command equally, at the same time instant, but in an opposite direction. Moreover the following assumptions are made: 1) the center of mass of the vehicle lies below the origin ($z_G$ is positive), 2) $x_G$ and $y_G$ are zero, 3) the vehicle is symmetric in its inertial properties, 4) the motions in the vertical plane are negligible ($[w_r, p, q, r, Z, \phi, \theta] = 0$), and 5) $u_r$ equals the forward speed, $U$. Based on these assumptions and on the calculation of the hydrodynamic coefficients in (Fodrea, 2002), equations (1–6) are simplified to the following linearized equations:

$$m\dot{\upsilon} = -mUr + Y_{\dot{\upsilon}}\dot{\upsilon} + Y_\upsilon\upsilon + Y_{\dot{r}}\dot{r} + Y_r r + Y_{\delta_s}\delta_r \quad (20)$$

$$I_{zz}\dot{r} = N_{\dot{\upsilon}}\dot{\upsilon} + N_\upsilon\upsilon + N_{\dot{r}}\dot{r} + N_r r + N_{\delta_s}\delta_r \quad (21)$$

$$\dot{\psi} = r \quad (22)$$

where the nomenclature that is ruling the above set of equations is presented in Table 2.

Equations (20–21) in a matrix form could be written as:

$$\begin{bmatrix} m-Y_{\dot{\upsilon}} & -Y_{\dot{r}} & 0 \\ -N_{\dot{\upsilon}} & I_{zz}-N_{\dot{r}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\upsilon} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} Y_\upsilon & Y_r - mU & 0 \\ N_\upsilon & N_r & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \upsilon \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} Y_{\delta_S} \\ N_{\delta_S} \\ 0 \end{bmatrix} [\delta_r] \quad (23)$$

For the steering subsystem there are three state variables: $r, \psi$, ad $\upsilon$. The $r$ variable is the yaw rate of turn, $\psi$ variable represents the heading angle, while $\upsilon$ represents the sway velocity. Based on the assumption made for the vehicle dynamics is that the cross coupling terms in the mass matrix are zero due to the assumed symmetry in the rudders, equation (23) can be given as:

Table 2: Utilized parameters and their values in the linearized description of AUV's Steering motion.

| Par. | Name |
|---|---|
| $Y_{\dot{\upsilon}}$ | Added mass is sway |
| $Y_{\dot{r}}$ | Added mass in yaw |
| $Y_\upsilon$ | Sway force induced by side slip |
| $Y_r$ | Sway force induced by yaw |
| $N_{\dot{\upsilon}}$ | Added mass in sway |
| $N_{\dot{r}}$ | Added mass in yaw |
| $N_\upsilon$ | Sway moment from side slip |
| $N_r$ | Sway moment from yaw |
| $Y_{\delta_S}$ | Linearized rudder action force |
| $N_{\delta_S}$ | Linearized rudder action force |

$$\begin{bmatrix} m-Y_{\dot{\upsilon}} & 0 & 0 \\ 0 & I_{zz}-N_{\dot{r}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\upsilon} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} Y_\upsilon & Y_r - mU & 0 \\ N_\upsilon & N_r & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \upsilon \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} Y_{\delta_S} \\ N_{\delta_S} \\ 0 \end{bmatrix} [\delta_r] \quad (24)$$

where $\delta_r$ is the control signal applied to both rudders. The state space description of the linearized system will be:

$$\begin{bmatrix} \dot{\upsilon} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{Y_\upsilon}{m-Y_{\dot{\upsilon}}} & \frac{Y_r-mU}{m-Y_{\dot{\upsilon}}} & 0 \\ \frac{N_\upsilon}{I_{zz}-N_{\dot{r}}} & \frac{N_r}{I_{zz}-N_{\dot{r}}} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \upsilon \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} \frac{Y_{\delta_S}}{m-Y_{\dot{\upsilon}}} \\ \frac{N_{\delta_S}}{I_{zz}-N_{\dot{r}}} \\ 0 \end{bmatrix} [\delta_r] \quad (25)$$

Given the state vector $x_2 = [\upsilon \;\; r \;\; \psi]' \in \Re^3$ and the input $u_{con_2} = \delta_r \in \Re$, the matrix form in Eq. (25) can be written as:

$$\dot{x_2} = A_{steer}x_2 + B_{steer}u_{con_2} \quad (26)$$

$$y_2 = C_{steer}x_2 \quad (27)$$

where

$A_{steer} = \begin{bmatrix} \frac{Y_\upsilon}{m-Y_{\dot{\upsilon}}} & \frac{Y_r-mU}{m-Y_{\dot{\upsilon}}} & 0 \\ \frac{N_\upsilon}{I_{zz}-N_{\dot{r}}} & \frac{N_r}{I_{zz}-N_{\dot{r}}} & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $B_{steer} = \begin{bmatrix} \frac{Y_{\delta_S}}{m-Y_{\dot{\upsilon}}} \\ \frac{N_{\delta_S}}{I_{zz}-N_{\dot{r}}} \\ 0 \end{bmatrix}$ and $C_{steer} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$.

# 3 CONSTRAINED FINITE TIME OPTIMAL CONTROLLER SYNTHESIS

In the proposed control strategy, the aim is to design a CFTOC–scheme for the decoupled diving and steering motion of the AUV as it is presented in Figure 2. At this point it should be mentioned that the speed

$u \in \Re$ of the AUV was considered to be constant and no control action has been considered for this motion while the control actions are applied to the linear model of the AUV after the ZOH. Prior to the design of the control algorithm, it is necessary to model and take under consideration in the controller's synthesis, the mechanical constrains of the utilized AUV, the disturbances that are introduced from the onboard sensors, the additive uncertainties due to modeling errors and the non–linearities.

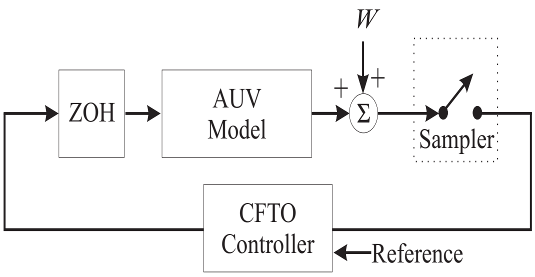


Figure 2: The Proposed Control Scheme for the AUV's Depth and Steering Motions

The derived linearized decoupled models in equations (18-19) and (26-27) are valid only for small values of pitch, yaw and fin angles around the linearization points. Moreover, the control actions $u_{con_1}$, $u_{con_2}$ should also be bounded due to physical constraints applied to the motors. We consider the state vector $\mathcal{X} = [x_1,\ x_2]^T \in \Re^6$ and the control vector $\mathcal{U} = [u_{con_1},\ u_{con_1}]^T \in \Re^2$.

Let the matrix $H_i$ be a zeroed $2 \times 8$ matrix except for its $i$–th column, which is equal to $[1, -1]^T$ and with $i \in [1, 2, \cdots, 16]$, i.e.:

$$H_i = \begin{bmatrix} 0 & \ldots & 0 & 1 & 0 & \ldots & 0 \\ 0 & \ldots & 0 & -1 & 0 & \ldots & 0 \end{bmatrix} \quad (28)$$

The bounds can be cast in a more compact form as:

$$\begin{bmatrix} H_1 \\ \vdots \\ H_{12} \\ -- \\ H_{13} \\ \vdots \\ H_{16} \end{bmatrix}_{16\times 8} \cdot \begin{bmatrix} \mathcal{X} \\ -- \\ \mathcal{U} \end{bmatrix}_{8\times 1} \leq \begin{bmatrix} \mathcal{X}_1^{\max} \\ \mathcal{X}_1^{\min} \\ \vdots \\ \mathcal{X}_6^{\max} \\ \mathcal{X}_6^{\min} \\ -- \\ \mathcal{U}_1^{\max} \\ \mathcal{U}_1^{\min} \\ \mathcal{U}_2^{\max} \\ \mathcal{U}_2^{\min} \end{bmatrix}_{16\times 1} \quad (29)$$

where the notation $\mathcal{X}_i$ corresponds to the $i$th element of the vector $\mathcal{X}$.

Due to factors such as noise, accuracy of measurements and round–off errors, the onboard measurements are not ideal. These inaccuracies in the measurements, in the presented approach are considered as additive disturbances to the system models. Moreover we will consider uncertainty in the state space matrices, due to the existence of modelling simplifications and errors. If we consider a sampling time $T_s \in \Re^+$, the combined discrete time version of equations (18-19) and (26-27) with the effects of the additive noise, can be cast as piecewise affine (PWA) system:

$$\dot{\mathcal{X}} = \begin{bmatrix} A_{div}^{*,i} & 0_{3\times 3} \\ 0_{3\times 3} & A_{steer}^{*,i} \end{bmatrix} \mathcal{X} + \begin{bmatrix} B_{div}^{*,i} & B_{steer}^{*,i} \end{bmatrix} \mathcal{U} + W \quad (30)$$

with the constraints in (29). The notation $(\cdot)^*$ represents the discrete time value of the corresponding matrix, while $W \in \Re^8$ is an additive and of a zero mean white noise, bounded by the set $\mathcal{W} \subseteq \Re^6$. Moreover $j \in \mathcal{S}$, with $\mathcal{S} \triangleq \{1, 2, \cdots, s\}$, is a finite set of indexes and $s \in \mathbb{Z}^+$ denotes the number of affine sub–systems in (30). For polytopic uncertainty, $\Omega$ is the polytope defined as:

$$\Omega = Co\{[A_{div}^{*,1}\, A_{steer}^{*,1}\, B_{div}^{*,1}\, B_{steer}^{*,1}], \cdots, [A_{div}^{*,s}\, A_{steer}^{*,s}\, B_{div}^{*,s}\, B_{steer}^{*,s}]\}, \quad (31)$$

where $Co$ denotes the convex hull and $[A_{div}^{*,i}\ A_{steer}^{*,i}\ B_{div}^{*,i}\ B_{steer}^{*,i}]$ are vertices of the convex hull. Any $[A_{div}^{*}\ A_{steer}^{*}\ B_{div}^{*}\ B_{steer}^{*}]$ within the convex set $\Omega$ is a linear combination of the vertices:

$$[A_{div}^{*}\, A_{steer}^{*}\, B_{div}^{*}\, B_{steer}^{*}] = \sum_{j=1}^{L} a_j [A_{div}^{*}\, A_{steer}^{*}\, B_{div}^{*}\, B_{steer}^{*}] \quad (32)$$

with $\sum_{j=1}^{L} a_j = 1,\ 0 \leq a_j \leq 1$.

The CFTOC-design problem consists of computing the optimum control vector sequence $\tilde{\mathcal{U}} = [\mathcal{U}_k\ \mathcal{U}_{k+1}\ \ldots\ \mathcal{U}_{k+N-1}]^T$, where $N$ corresponds to the prediction horizon that minimizes the following cost function:

$$\begin{aligned} J_N(\mathcal{X}_k, \mathcal{X}^r) \quad &= \quad [\mathcal{X}_{k+N} - \mathcal{X}^r)]^T \tilde{P} [\mathcal{X}_{k+N} - \mathcal{X}^r] + \\ & \sum_{m=0}^{N-1} [\mathcal{U}_{k+m}]^T R [\mathcal{U}_{k+m}] + [\mathcal{X}_{k+m} - \mathcal{X}^r]^T Q [\mathcal{X}_{k+m} - \mathcal{X}^r] \end{aligned}$$

where $\tilde{P} \in \Re^{6\times 6}$ with $\tilde{P} = \tilde{P}^T \geq 0$, $R \in \Re^{2\times 2}$ with $R = R^T > 0$ and $Q \in \Re^{6\times 6}$ with $Q = Q^T \geq 0$, are full column rank weighting matrices penalizing the corresponding optimization variables i.e predicted states, control effort and the desired final state, respectively, while $\mathcal{X}^r$ is the reference set–point.

The solution to the CFTOC problem (Borelli et al., 2003; Grieder et al., 2004; Kvasnica et al., 2004) is a continuous control action of the form:

$$\mathcal{U}_k \quad = \quad F_l \mathcal{X}_k + G_l \text{ if } \mathcal{X}_k \in \tilde{R}_l \quad (33)$$

where $\tilde{R}_l$, $l \in \{1, \ldots l^{\max}\}$ corresponds to a convex polyhedron $(\tilde{R}_l \in \Re^6)$ calculated by the algorithm, and $F_l \in \Re^{2\times 6}$, $G_l \in \Re^{2,1}$. The $l^{\max}$–number of polyhedra is similarly specified by the algorithm. In general the higher the number of the PWA–systems along with the large dimension of the state vector and the number of constraints, the more complicated the solution is.

This complexity increases significantly with the value of the prediction horizon $N$, and the number $l^{\max}$ of the convex polyhedra (regions) grows up (usually) exponentially. Certain techniques have been devised to merge the various regions into larger ones without significantly compromising the validity of the solution. For the on–line implementation of the controller the number $l^{\max}$ of the regions is not only a measure of the controller's complexity but also affects its implementation typically precomputed in a look–up table.

## 4 SIMULATION STUDIES

The proposed CFTO–control scheme has been applied in simulation studies on the model of the REMUS AUV. Based on experimental results in (Prestero, 2000) the parameters presented in Tables I and II were tuned and the continuous time state space matrices for the diving and steering motion of the AUV are:

$$A_{div} = \begin{bmatrix} -0.82 & 0 & -0.69 \\ 0 & 0 & -1.54 \\ 1 & 0 & 0 \end{bmatrix},\ B_{div} = \begin{bmatrix} -4.16 \\ 0 \\ 0 \end{bmatrix} \quad (34)$$

$$A_{steer} = \begin{bmatrix} -1.01 & -0.68 & 0 \\ -0.54 & -0.82 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ B_{steer} = \begin{bmatrix} 0.22 \\ -1.19 \\ 0 \end{bmatrix} \quad (35)$$

The constraints on the control inputs and the outputs have been arbitrary set to: $u_{con_{1_{min}}} = -10 \leq u_{con_1}(t) \leq 10 = u_{con_{1_{max}}}$, $u_{con_{2_{min}}} = -60 \leq u_{con_2}(t) \leq 60 = u_{con_{2_{max}}}$ and $y_{1_{min}} = -50^0 \leq y_1(t) \leq 50^0 = y_{1_{max}}$, $y_{2_{min}} = -70^0 \leq y_2(t) \leq 50^0 = y_{1_{max}}$ respectively. The constrains for the states have been set as:

$$\begin{aligned} -30^0 \leq & \ x_1 & \leq 30^0 \\ -20^0 \leq & \ x_2 & \leq 20^0 \\ -360 \leq & \ x_3 & \leq -360 \\ -50^0 \leq & \ x_4 & \leq 50^0 \\ -40^0 \leq & \ x_5 & \leq 40^0 \\ -360 \leq & \ x_3 & \leq -360 \end{aligned}$$

For the state space matrices $A_{div}$, $B_{div}$, $A_{steer}$, $B_{steer}$, we assume that there is an additive corrupting uncertainty of 1%, while the additive disturbances have been set to $0.01 \cdot I_{6\times 1}$.

The selection that has been made on the penalizing matrices for the CFTOC cost was $P = 10^3 \cdot I^{6\times 6}$, $R = I^{2\times 2}$ and $Q = 10^3 \cdot I^{6\times 6}$. The output set–point was selected for the diving motion as $Y_{1_{ref}} = 5^0$ and $Y_{2_{ref}} = 5^0$ for the steering motion. The initial augmented state vector was $X_{init} = O_{6\times 1}$. For the controllers formulation the 2− vector norm case was tested and the discretization has been made with a sampling period of $T_s = 1sec$.

The resulting controllers' partitions for the diving and steering motion are presented in Figures 3 and 4, while the responses of the diving and steering motions are displayed in Figures 5 and 6 respectively, while the controllers' response for the cases of the diving and steering motion are presented in Figures 7 and 8 respectively. Finally in Figure 9 the 3–D combined movement of the AUV (including displacement with a constant speed (U=1.54m/sec) is displayed.

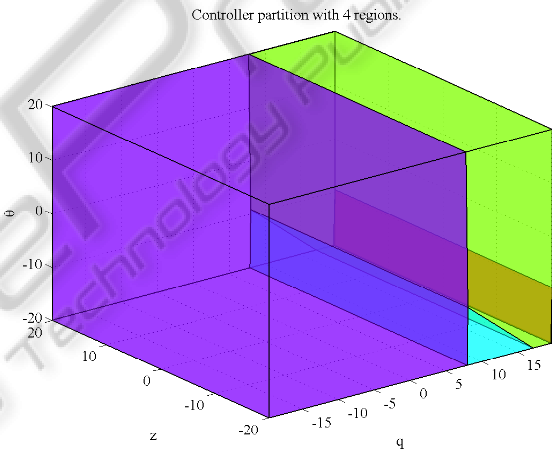


Figure 3: CFTO–Controller Partitioning for the Diving Motion.

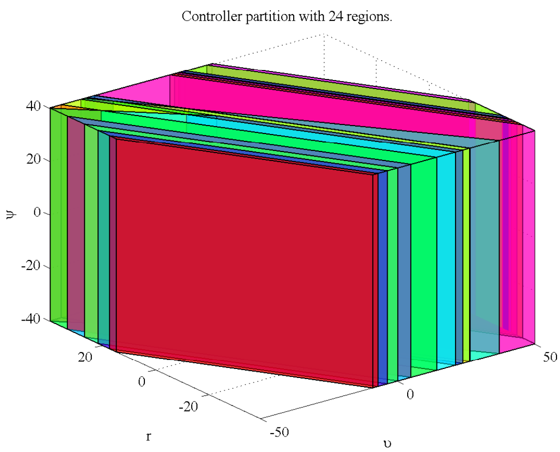


Figure 4: CFTO–Controller Partitioning for the Steering Motion.

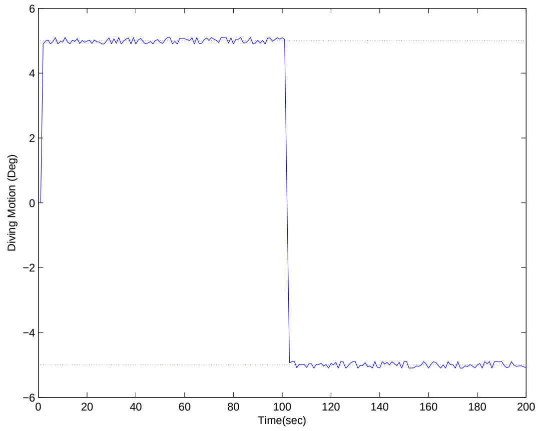


Figure 5: Diving Motion Time Response.

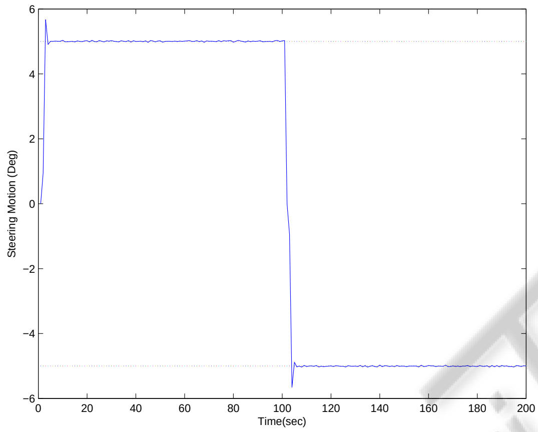


Figure 6: Steering Motion Time Response.

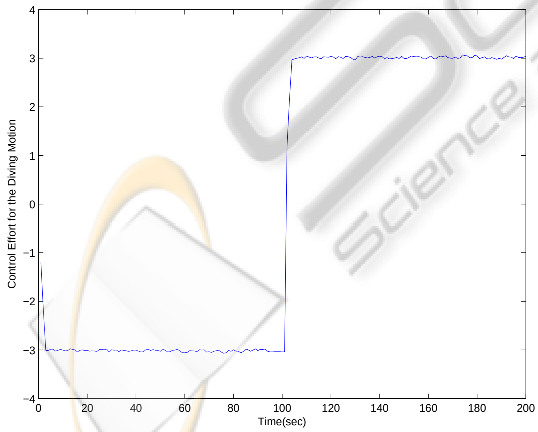


Figure 7: Controller Effort for the Diving Motion.

## 5 CONCLUSIONS

In this paper a constrained finite time optimal controller for the diving and steering motion of an AUV has been presented. The utilized proposed scheme

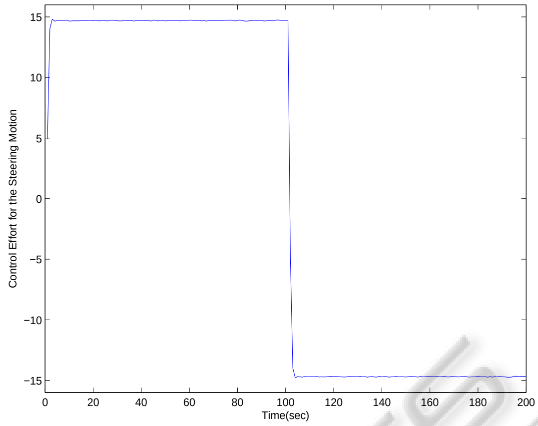


Figure 8: Controller Effort for the Steering Motion.

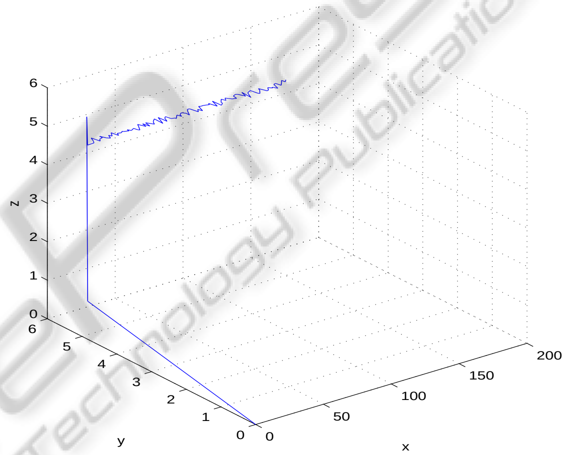


Figure 9: 3–D Combined movement of the AUV.

was developed based on the decoupled equations of motion for the diving and steering of the AUV. The derived Constrained Finite Time Optimal control had the ability to take under consideration: a) the constrains on the inputs, outputs, and the states of the model, b) the corrupting disturbances due to the existence of the noise in the measurements, and c) the uncertainty in the modeling procedures. Simulation results have been presented that prove the validity of the proposed scheme.

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