COMBINATORIAL DETECTION OF ARRHYTHMIA

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Abstract: Three problems that arise from electrocardiogram (ECG) interpretation and analysis are presented, followed by algorithmic solutions based on a combinatorial model. First, the beat classification problem is discussed and possible solutions are investigated. Secondly, given the $\mathcal{RR}$-intervals, which can be determined using this combinatorial model, or any $\mathcal{QRS}$ detection algorithm, the heart rate is determined in a statistical manner from which sinus bradycardia and sinus tachycardia are inferred. Finally, a new combinatorial method for measuring heart rate variability (HRV) is presented and an algorithm for detecting atrial fibrillation is described. The developed algorithms were implemented and tests were carried out on records from the MIT-BIH arrhythmia database. The results of the tests are presented and discussed.

1 INTRODUCTION

1.1 The ECG and its Elements

A ECG is obtained by placing electrodes on the skin and measuring the direction of electrical current discharged by the heart. The current is plotted into waveforms and displayed as in Figure 1.

A *lead* provides a view of the heart's electrical activity between one positive and one negative pole (Conover, 2002; Springhouse (Editor), 2007). Most standard ECG recordings are obtained using a 12-lead device in clinical settings, and a 2-lead device in Holter (ambulatory) monitors. The different leads provide alternate views of the heart and often a combination of many leads are required to perform a diagnoses; other times, one or two are enough.

The sample in Figure 1 is from a 2-lead reading, as are all 48 records in the MIT-BIH (Massachusetts Institute of Technology - Beth Israel Hospital) arrhythmia database (Massachusets Institute of Technology, 1999) that was used for development and testing purposes. The figure shows readings from *modified limb lead II* and *precordial lead* V_1.

Each *beat* of the heart corresponds to an $\mathcal{ECG}$ *complex*, and each $\mathcal{ECG}$ complex is made up of elements, which are *waves*, *intervals* and *segments*. The presence and configuration of these elements depends on various factors, including the lead used to take the reading, the physical device, the presence of noise, the health of the heart and the age and physiology of the subject (Thaler, 2006; Stein, 2000).

Figure 2 depicts a closeup view of the $\mathcal{ECG}$ complex and its elements. Note that the figure is a freehand drawing of a 'normal' $\mathcal{ECG}$ complex, and the intervals are not accurately depicted. The two elements of particular significance for this paper are the $\mathcal{QRS}$ *complex* and the $\mathcal{P}$ *wave*.

The $\mathcal{QRS}$ complex comprises the $\mathcal{Q}$, $\mathcal{R}$ and $\mathcal{S}$ waves. Not all waves are always present. The $\mathcal{P}$ wave appears before the $\mathcal{QRS}$ complex and is smooth, rounded and upright in contour in a healthy heart. The interval between consecutive $\mathcal{ECG}$ complexes, usually measured between consecutive $\mathcal{QRS}$ complexes, determines the *heart rate* and *rhythm* of the heart.

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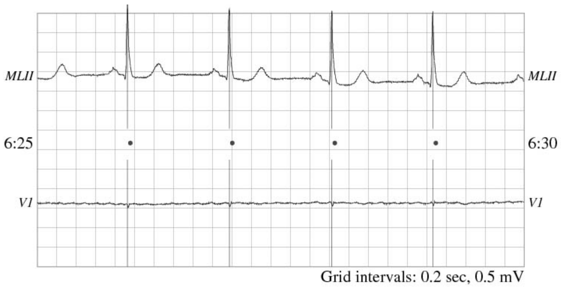


Figure 1: Section of ECG for record 101 from MIT-BIH arrhythmia database.

1.2 Arrhythmias and HRV

Any cardiac irregularity is an *arrhythmia*, not everyone of which indicates disease or requires treatment. Nevertheless, some arrhythmias can indicate serious cardiac danger and need immediate attention.

A normal heart rate is between 60 and 100 *beats per minute* (bpm). Arrhythmia is caused by a disturbance either of the heart rate, the regularity of the beat, the site of origin or the conduction through the heart, (Thaler, 2006).

Normally, the heart rate stays within the above mentioned range, but is not entirely steady, due to inspiration (expiration), which causes it to speed up (slow down). The measuring of the "steadiness" of the heart rate is *heart rate variability* (HRV).

The elements of the ECG, together with the HRV, provide information as to what type of arrhythmia the patient has. Some arrhythmias, though they may have a different cause, manifest themselves in similar ways on the ECG. The added study of the HRV can give more indications and allow a more accurate diagnosis.

1.3 Paper Organization

The paper is organized as follows. Next, in Section 2, the terms and parameters are defined. In Section 3 the three problems are presented, suggested solutions are discussed, and comments are made on the implementation and the experiments carried out. Each problem, algorithmic solution and test results occupy their own subsection, (3.1 to 3.3). Finally, Section 4 contains further works and conclusions.

2 DEFINITIONS

A *signal* s is a k-tuple $(t, p_{MLII}, p_{V1}, \ldots)$, where t is the time in seconds, and p_E is the electrical potential at lead E in millivolts. When the signal read is from a single lead, it is represented as a pair, a 2-tuple, (t, p).

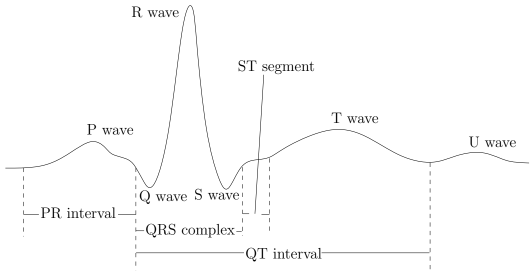


Figure 2: A 'normal' $\mathcal{ECG}$ complex and its elements.

A *sequence of signals* $s_1, s_2, \ldots, s_n$ is a sequence of pairs, (t_1, p_1), (t_2, p_2), …, (t_n, p_n).

A *beat* b is a pair (t, c), where t denotes the time in seconds of occurrence of the beat, and c is a character that describes the type of beat. The values that c can take are described in Section 3.1.

A *sequence of beats* $\mathcal{B} = b_1, \ldots, b_n$ is a sequence of pairs, (t_1, c_1), …, (t_n, c_n). The $\mathcal{RR}$*-interval*, rr, is the time between two consecutive beats:

$$rr_i = t_i - t_{i-1} \tag{1}$$

The *rate of change* g is the difference between two consecutive $\mathcal{RR}$-intervals i.e., $g_i = rr_i - rr_{i-1}$. The *cumulative rate of change* for k beats is the sum of the absolute rates of change of consecutive pairs i.e.,

$$|g_i| + |g_{i+1}| + \ldots + |g_{i+k-1}| = \sum_{j=0}^{k-1} |g_{i+j}|$$

Let $\hat{\Sigma}$ and Σ be the following alphabets:

$$\hat{\Sigma} = \{-, 0, +\}$$

$$\Sigma = \{C^-, C^0, C^+\}$$

and the set of non-trivial pairs:

$$\mathcal{A} = \{(+,0), (-,0)\}$$

(trivial pairs are $\{(0,0), (+,+), (-,-)\}$).

Let d be a symbol from the alphabet Σ, which denotes a decrease, no significant difference, or an increase in the rate of change. Small differences, discriminated by ν, are considered negligible, thus ν is a threshold.

The value of d at position i, is determined by Equation 2:

$$d_i = \begin{cases} C^0, & \text{if } |rr_i - rr_{i-1}| \leq \nu \\ C^-, & \text{if } rr_i - rr_{i-1} < -\nu \\ C^+, & \text{if } rr_i - rr_{i-1} > \nu \end{cases} \tag{2}$$

The set of all strings over Σ is denoted by Σ^*.

Two consecutive rate changes have *direction equivalence*, when there's a smooth transition from one rate change to the other, without a significant change of direction.

A consecutive sequence of direction equivalent rate changes is defined as $\mathcal{C}[x,k]$ of C^x, where $x \in \hat{\Sigma}$, $k \in \mathbb{N}^+$ and for some $C^x \in \Sigma$.

Formally, C^x *is equivalent to* C^y,

$$C^x \sim C^y \tag{3}$$

if $(x,y) \in \mathcal{A}$. Furthermore, the two sequences are equivalent

$$C^{x_1}C^{x_2}...C^{x_k} \approx C^{y_1}C^{y_2}...C^{y_k} \tag{4}$$

if $C^{x_i} \sim C^{y_i}, \forall i$.

For example, $C^+ \sim C^0$, but $C^- \nsim C^+$ (*not equivalent to*) and the sequence $C^0C^+C^+C^0 \approx \mathcal{C}[+,4]$, but $C^-C^+C^+C^0C^- \not\approx \mathcal{C}[-,5]$. Also note, $\mathcal{C}[+,k] \approx C^{x_1}C^{x_2}...C^{x_k}$, where $x_i \in \{+,0\}$, for $i \in [1..k]$, and similarly for $\mathcal{C}[-,k]$.

A *window* is defined to be ω consecutive symbols of Σ i.e., for $i,j \in N$ and $0 \le i \le j$, $\omega = j-i+1$ denotes the length of the string $C^{x_i}, C^{x_{i+1}}, ..., C^{x_j}$.

A *change of direction* occurs when, given a sequence $C^{x_1}C^{x_2}...C^{x_k}$ all elements of which are direction equivalent i.e., $\mathcal{C}[x,k] \approx C^{x_1}C^{x_2}...C^{x_k}$, the $k+1$'th element in the sequence *breaks* this equivalence i.e., $C^{x_1}C^{x_2}...C^{x_k}C^{x_{k+1}} \not\approx \mathcal{C}[x,k]$.

The constant ξ is defined (ϕ is defined), to be the maximum (minimum) allowable change of direction within a window of length ω. For example, for a window of size ω, and the sequence $\mathcal{C}[-,i_1]$ $\mathcal{C}[+,i_2]$ $\ldots\mathcal{C}[-,i_m]$, the inequality $m \le \xi$ ($m \ge \phi$) must hold. Finally, ψ is the maximum cumulative rate of change within a window of length ω i.e., $\sum_{j=0}^{w} |g_{i+j}| \le \psi$.

3 PROBLEMS & ALGORITHMS

The problems in this section are an attempt to formalize ECG interpretation and analysis in a combinatorial way. In each subsection, the problem is given with its practical setting, followed by a formal definition. Next, an algorithmic solution is suggested and each subsection ends with discussion on the implementation and experimental results of the suggested solution.

First, the *beat classification problem* identifies the types of beats in an ECG, then the *heart rate problem* identifies the rhythm of the heart and finally a new method for HRV is developed and explained in the *heart rate variability problem.*

All the implementations were done on a Windows machine with an Intel Celeron M processor of 1.60GHz and 896MB RAM and the tests were run through Cygwin, a Linux-like environment for Windows. The code was written in C++ using the STL.

Table 1: Beat classes and types.

Type	Description	Class
· or N	Normal	N
A	Atrial premature	S
S	Supraventricular premature	S
V	Premature ventricular contraction	V
F	Fusion of ventricular and normal	F
E	Ventricular escape	V
/	Paced beat	Q
Q	Unclassifiable beat	Q

3.1 Beat Classification Problem

The first problem is the classification of beats. The (ANSI/AAMI, 1998b) standard recommends 5 general *heartbeat classes*:

- Class N: normal and bundle branch block beats.
- Class S: supraventricular ectopic beats (SVEB).
- Class V: ventricular ectopic beats (VEB).
- Class F: fusion of a VEB and a normal beat.
- Class Q: paced and non-classifiable beats.

There are 15 different *heartbeat types* annotated in the 48 records in the MIT-BIH arrhythmia database (Goldberger et al., 2000; Massachusets Institute of Technology, 1999). Table 1 depicts some of the most commonly found types and their equivalent classes.

Problem 1 (**Beat Classification).** *Given an ECG as a sequence of signals* $s_1, \ldots, s_n = (t_1,p_1), \ldots, (t_n,p_n)$ *and a sequence of times of occurrence of* $\mathcal{QRS}$ *complexes* $t'_1, \ldots, t'_m$*, where* $t'_i \in t_1 \ldots t_n, \forall\ 1 \le i \le n$*, determine the* class *of the beats i.e., output the list of beats,* $\mathcal{B} = b_1, b_2 \ldots, b_m = (t'_1,c_1),(t'_2,c_2)\ldots,(t'_m,c_m)$*, where* $c_j \in \{N,S,V,F,Q\}, \forall\ 1 \le j \le m$.

A number of methods to solve the beat classification problem have been proposed. Most rely on signal processing, such as (Afonso et al., 1997) and use varying computer science methodologies, for instance neural networks (Hu et al., 1997). In (Chazal et al., 2004), the authors achieved good accuracy results on the MIT-BIH arrhythmia database. Recently, in (Iliopoulos and Michalakopoulos, 2009) a new model for ECG interpretation was introduced. This model can be used for beat classification; an algorithmic outline follows.

STEP 1

Identify the $\mathcal{P}$ waves and $\mathcal{QRS}$ complexes. A detailed description of identifying the $\mathcal{QRS}$ complexes is in (Iliopoulos and Michalakopoulos, 2009).

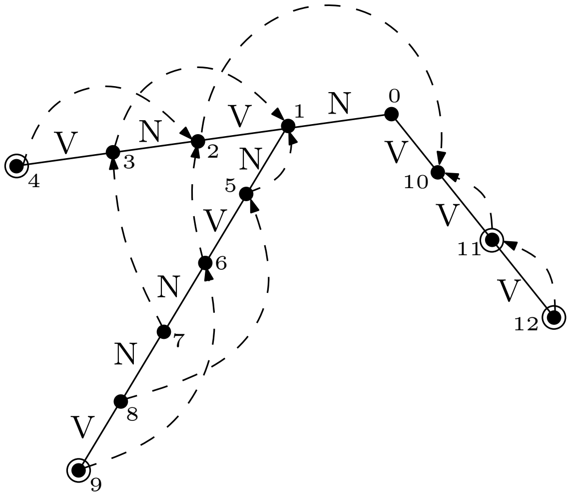


Figure 3: Aho-Corasick automaton for keyword list $\{NVNV, NNVNNV, VV, VVV\}$. Trivial failure links are not shown.

The algorithm relies on searching for the pattern $\mathcal{C}[++,2]\mathcal{C}[--,2]$ in the ECG.

The $\mathcal{P}$ wave can be identified by searching for the more complex pattern $\mathcal{C}[0,k_1]\mathcal{C}[+,k_2]\mathcal{C}[-,k_3]$, where $k_1,k_2,k_3 \in \mathbb{N}^+$ are determined during the algorithms learning period.

STEP 2

If both the $\mathcal{RR}$ interval and the $\mathcal{P}$ wave are regular, classify the beat as normal (N).

STEP 3

If the beat is not normal, then determine the beat type from $\{S,V,F,Q\}$ by examining the $\mathcal{P}$ wave, $\mathcal{RR}$ interval and the $\mathcal{QRS}$ complex and its duration.

Regardless of the method used, the output is a string of characters, $C = c_1c_2\ldots c_m$, where $c_i \in \{N,S,V,F,Q\}$. Two types of *ventricular arrhythmia*, namely *ventricular bigeminy* and *ventricular trigeminy* can be identified by a simple pattern search. For $k \in \mathbb{N}^+$ and $k \geq 2$, $(NV)^k$ indicates ventricular bigeminy, while $(NNV)^k$ ventricular trigeminy.

A potentially more dangerous arrhythmia, *ventricular tachycardia* manifests as a run of 3 or more consecutive *premature ventricular contractions* (PVC), (Thaler, 2006). Thus, ventricular tachycardia is represented by the pattern $V^\ell, \ell \geq 3$.

Another interesting pattern are *ventricular couplets*, VV. To identify these patterns online, an Aho-Corasick automaton (Aho and Corasick, 1975), is built for the dictionary $\{NVNV, NNVNNV, VV, VVV\}$ as shown in Figure 3.

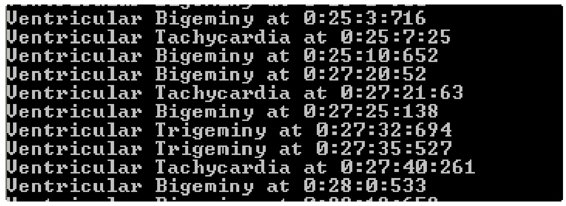


Figure 4: Output of program for Problem 1, on record 106.

The above patterns can also be determined offline in constant ($O(1)$) time, if the list of beats is indexed using e.g. a suffix array or suffix tree (Crochemore et al., 2007; Gusfield, 1997). For a prerecorded ECG and a list of beats of size $|m|$, this indexing can be done in $O(m)$ time, given that the alphabet $\{N,S,V,F,Q\}$ is of constant size.

3.1.1 Experimental Results

The Aho-Corasick automaton of figure 3 was implemented, and tests were carried out on the MIT-BIH arrhythmia database. A short sample of the output for record 106 can be seen in Figure 4.

Record 106 was chosen because it manifests many occurrences of the above mentioned arrhythmias. The program simply identifies the type of arrhythmia, and outputs the information to the screen and to a text file, for later processing.

3.2 Heart Rate Problem

The second problem identifies arrhythmias that manifest as irregular *heart rate*. The cells of the heart discharge electrical potential at different rates. The dominant cells in a healthy heart are located in the sinoatrial (SA) node. The term *heart rate* refers to the *sinus rhythm* i.e., the rate that the SA node fires. The ECG encapsulates information about the rates of other parts of the heart, such as the *atrial rate* and the *ventricular rate*.

The *heart rate problem* is concerned with calculating the sinus rhythm. A normal sinus rhythm is between 60 and 100 bpm. Anything faster is termed *tachycardia*, and anything slower *bradycardia*. According to (ANSI/AAMI, 1998a), *sinus tachycardia* should be identified when the heart rate is faster than 110 bpm and *sinus bradycardia* for rates slower than 50 bpm, for 15 consecutive seconds.

It should be noted that these arrhythmias do not always indicate irregularities. For example, a long-distance runner will in rest have a heart rate below 60 bpm, and the same subject will have a heart rate of well above 100 bpm during rigorous exercise. The algorithm presented below does not thus claim to identify dangerous conditions, but simply indicates when the rhythm has fallen or risen to a level which should be checked by a specialist.

Problem 2 (Heart Rate). *Given a sequence of beats,* $\mathcal{B} = b_1, b_2 \ldots, b_n = (t_1, c_1), (t_2, c_2) \ldots, (t_n, c_n)$*, calculate the heart rate and detect* sinus tachycardia *and* sinus bradycardia*.*

A solution to this problem is depicted in Algorithm 1. The series of beats are processed sequentially. Each step can be visualized as a shift of a window of size ω by one beat to the right.

The heart rate is determined by first summing the $\mathcal{RR}$ intervals within the window. This is only done for *valid* windows. A valid window is one which contains only normal ($c_i = N$) beats.

The average $\mathcal{RR}$ interval of the window is calculated on Line 16. This average is then converted into beats per minute i.e., heart rate, on Line 17. If the heart rate is below 50 bpm, the algorithm reports sinus bradycardia, if it's above 110 bpm, sinus tachycardia is reported.

For example, if the average is found to be a beat every 0.8 secs, the heart rate is 75 bpm, whereas if the average is every 1.2 secs, the heart rate is 50 bpm and sinus bradycardia is reported.

Algorithm 1. Process Heart Rate.

1: $ave \leftarrow 0$ ▷ average $\mathcal{RR}$-interval in window of size ω
2: $hr \leftarrow 0$ ▷ heart rate in bpm
3: $win\_count \leftarrow 0$
4: $win\_total \leftarrow 0$
5: $i \leftarrow 1$
6: **while** not end of $\mathcal{B}$ **do** ▷ for each beat in $\mathcal{B}$
7: $(t_i, c_i) \leftarrow \mathcal{B}[i]$
8: **if** $c_i \neq$ 'N' **then**
9: $win\_count \leftarrow 0$, $win\_total \leftarrow 0$
10: **else** ▷ beat is normal
11: $win\_count \leftarrow win\_count + 1$
12: $win\_total \leftarrow win\_total + (t_i - t_{i-1})$
13: **if** $win\_count \geq \omega$ **then**
14: **if** $win\_count > \omega$ **then**
15: $win\_total \leftarrow win\_total - (t_{i-\omega} - t_{i-\omega-1})$
16: $ave \leftarrow win\_total / \omega$
17: calculate hr from ave
18: **if** $hr \leq 50\ bpm$ **then**
19: report SINUS BRADYCARDIA
20: **else if** $hr \geq 110\ bpm$ **then**
21: report SINUS TACHYCARDIA
22: increment i ▷ next beat

3.2.1 Experimental Results

Algorithm 1 was implemented, and tests were carried out on the MIT-BIH arrhythmia database. The PhysioNet utility *ann2arr*, see (Goldberger et al., 2000), was used to first extract the beats from the ECG. This produces a text file with the beats in $\mathcal{B} = (t_1, c_1), \ldots, (t_n, c_n)$ format, that serves as input to the program.

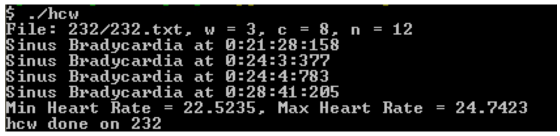


Figure 5: Output of program for Problem 2, on record 232.

The heart rate is calculated by the $\mathcal{RR}$ intervals as in Algorithm 1. The test database averages the beats over 3 consecutive $\mathcal{RR}$ intervals to calculate the heart rate, and so the window length for the purpose of calculating the normal sinus rhythm was set to $\omega = 3$. The heart rate was calculated for the beats marked with N (normal), Q (unclassifiable), L (left bundle branch block) and R (right bundle branch block).

The arrhythmias looked for are sinus bradycardia and sinus tachycardia. The only records in the test database with sinus bradycardia are records 202 and 232, whereas sinus tachycardia is not annotated in the database. Figure 5 shows the output of the program on record 232, where sinus bradycardia is correctly identified and the heart rate is shown in the last line.

Software and ambulatory ECG devices should allow user-defined input parameters, as recommended in (ANSI/AAMI, 1998a). In this implementation, the identification of sinus bradycardia was set to 50 bpm, and sinus tachycardia to 110 bpm, but it could easily be modified to allow these values to be input as parameters.

Other input parameters seen in Figure 5 are $w = 3$, which is the window length ω, $n = 12$, which is the ν threshold and $c = 8$, which is the change of direction parameter (see Section 3.3).

3.3 Heart Rate Variability Problem

Heart rate variability (HRV) measures the beat-to-beat fluctuations in heart rate. It has been associated with a number of cardiac and pathologic conditions.

A low HRV i.e., with few fluctuations, has been used in clinical practice to predict risk after *acute myocardial infarction* and is an early warning sign of *diabetic neuropathy* (Task Force of the European Society of Cardiology the North American Society of Pacing Electrophysiology, 1996). It has also been linked to *phobic anxiety*, which in itself is a risk factor for *fatal coronary artery disease* which can lead to *sudden cardiac death* (Kawachi et al., 1995).

A high HRV on the other hand, together with other ECG characteristics such as the absence of $\mathcal{P}$ waves, may indicate *atrial fibrillation* (Thaler, 2006).

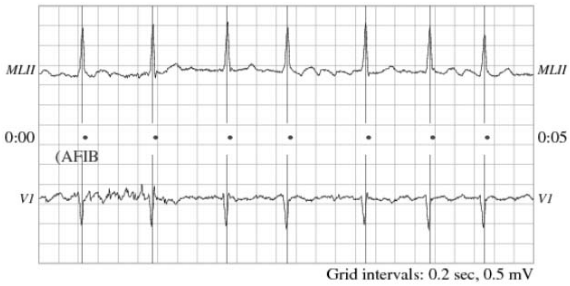


Figure 6: Section of ECG for record 201. Each beat ($\mathcal{R}$ wave peak) is annotated as a dot '•', and the start of atrial fibrillation as '*(AFIB*'.

Figure 6 shows a section of the ECG for record 201 in the MIT-BIH arrhythmia database. The onset of atrial fibrillation is annotated as "*(AFIB*" in the figure. The characteristics of the ECG at this point can be summed up as in (Thaler, 2006), as "irregularly irregular tachycardia" and the absence of $\mathcal{P}$ waves.

Various methods of measuring HRV have been proposed, including time-domain measures, frequency-domain measures and geometric measures. In this section, a combinatorial method is suggested. An algorithm is proposed and presented, together with experimental results of an implementation of the model on the detection of atrial fibrillation.

Problem 3 (**Heart Rate Variability**). *Given a sequence of beats,* $\mathcal{B} = b_1, \ldots, b_n = (t_1, c_1), \ldots, (t_n, c_n)$, *detect irregularities in the* heart rate variability *and report certain types of arrhythmia.*

The definitions in Section 2 serve as the basis for solving this problem. The series of beats $\mathcal{B}$ are transformed into characters d_i from the alphabet Σ. The threshold value ν is important in determining d_i, as is illustrated in the last two columns in Table 2. The data in the table is taken from record 101 in the test database.

Note that the $\mathcal{RR}$ intervals, rr_i, are measured in number of readings of electrical potential per second, and that the sample rate in the test database is $\sigma = 360$. Thus, a $\mathcal{RR}$ interval of 360 is equivalent to 1 sec.

The same data is represented combinatorially in Figure 7. A change of direction is intuitively a point at which an up arrow follows a down arrow, or vice versa.

Within a window of size ω, a range of change of direction indicates a healthy heart; too few ($< \phi$), or too many ($> \xi$) indicate irregularities.

Next an outline of a generic algorithm for solving the heart rate variability problem is presented. Pseudocode for identifying atrial fibrillation based on the generic algorithm follows.

Table 2: Data from record 101.

i	t_i	rr_i	$rr_i - rr_{i-1}$	d_i ($\nu = 5$)	$d_i(\nu = 30)$
427	6:21:964	370	18	C^+	C^0
428	6:23:019	380	10	C^+	C^0
429	6:23:978	345	-35	C^-	C^-
430	6:24:947	349	4	C^0	C^0
431	6:25:928	353	4	C^0	C^0
432	6:26:972	376	23	C^+	C^0
433	6:28:025	379	3	C^0	C^0
434	6:29:064	374	-5	C^-	C^0
435	6:30:092	370	-4	C^0	C^0
436	6:31:042	342	-28	C^-	C^0
437	6:32:017	351	9	C^+	C^0
438	6:33:039	368	17	C^+	C^0

STEP 1

Process series of beats $\mathcal{B}$:

1. Convert into a string on Σ^*.
2. Calculate number of change of direction, for window of length ω.
3. Accumulate absolute total change of direction within window.

STEP 2

If within the window the number of change of direction exceeds ξ, then *report high HRV*, and *do further checks on ECG*.

Else if within the window the number of change of direction is less than ϕ then *report low HRV*, and *do further checks on ECG*.

STEP 3

If within the window the cumulative change of direction is greater than ψ, then *indicate possible arrhythmia* and *do further checks on ECG*.

The type of arrhythmia sought, determines what "do further checks" of Step 2 and 3 is. The program could simply report an irregularity and advise a cardiologist to take a closer look at the ECG configuration, waves and intervals, at this point in time. Alternatively, some automated task can be performed, such as is presented in the pseudocode in Algorithm 2 for the identification of atrial fibrillation.

Algorithm 2 serves as an illustration of the potentials of the model. The detection of atrial fibrillation was chosen because it is one of the most common types of arrhythmias, affecting 1% of the worlds population, rising to 4% in the over 65's (Garratt, 2001). The more important reason for this research paper and model though, is that there is data in the test database from patients presenting this type of arrhythmia, which makes the program meaningful.

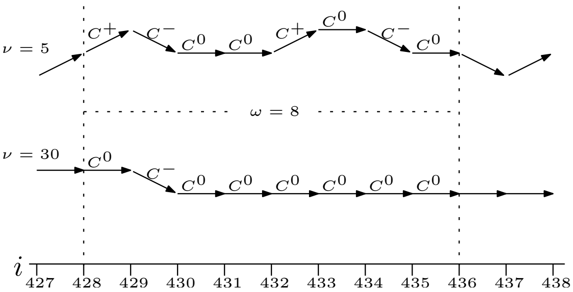


Figure 7: Combinatorial depiction of HRV for data from Table 2.

As in Algorithm 1, only normal beats are taken into account; the implementation details are omitted from the pseudocode for reasons of simplicity.

The algorithm is an online algorithm and is O(n), except for Line 13. This check however, can be done in $O(\omega)$ time, using, for example, the algorithm in (Portet, 2008), and given that ω is constant, the algorithm remains $O(n)$.

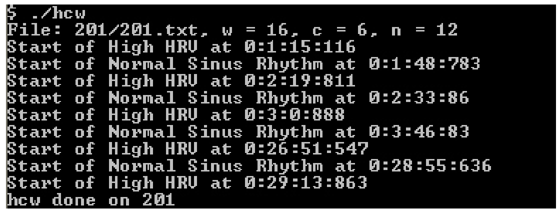
Algorithm 2. Identify Atrial Fibrillation.

1: $cod\_count \leftarrow 0$ ▷ change of direction within window
2: $i \leftarrow 1$
3: **while** not end of $\mathcal{B}$ **do** ▷ for each beat
4: $(t_i, c_i) \leftarrow \mathcal{B}[i]$
5: **if** $c_i =$ 'N' **then** ▷ if beat is normal
6: determine d_i by Equations 1 and 2
7: **if** $d_{i-\omega} \not\approx d_{i-\omega-1}$ **then** ▷ equivalence test
8: $cod\_count \leftarrow cod\_count - 1$
9: **if** $d_i \not\approx d_{i-1}$ **then**
10: $cod\_count \leftarrow cod\_count + 1$
11: **if** $cod\_count > \xi$ **then** ▷ high HRV
12: report high HRV
13: **if** $\mathcal{P}$ waves absent in window **then**
14: report ATRIAL FIBRILLATION
15: increment i ▷ next beat

3.3.1 Experimental Results

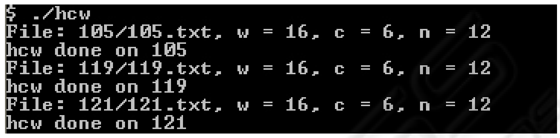
Algorithm 2 was implemented and tests were carried out on the records that present atrial fibrillation (201, 202, 203, 210, 217, 219, 221, 222), and randomly chosen records (100, 104, 105, 119, 121, 205, 208, 230), that don't present this type of arrhythmia.

Figure 8 shows the output on the program on record 201 and for parameters $\omega = 16, \xi = 6$ and $\nu = 12$. The program outputs the time that high HRV is first observed. The time normal sinus rhythm resumes is also displayed. Checking the ECG for the absence of $\mathcal{P}$ waves at this point in time is a strong indicator of atrial fibrillation. Figure 9 shows the program run on records 105, 119 and 121 which do not exhibit atrial fibrillation, for the same parameters as the test above.



```
$ ./hcw
File: 201/201.txt, w = 16, c = 6, n = 12
Start of High HRV at 0:1:15:116
Start of Normal Sinus Rhythm at 0:1:48:783
Start of High HRV at 0:2:19:811
Start of Normal Sinus Rhythm at 0:2:33:86
Start of High HRV at 0:3:0:888
Start of Normal Sinus Rhythm at 0:3:46:83
Start of High HRV at 0:26:51:547
Start of Normal Sinus Rhythm at 0:28:55:636
Start of High HRV at 0:29:13:863
hcw done on 201
```

Figure 8: Output of program for Problem 3, on record 201. Abnormal HRV is detected.



```
$ ./hcw
File: 105/105.txt, w = 16, c = 6, n = 12
hcw done on 105
File: 119/119.txt, w = 16, c = 6, n = 12
hcw done on 119
File: 121/121.txt, w = 16, c = 6, n = 12
hcw done on 121
```

Figure 9: Output of program for Problem 3, on records 105, 119 and 121. These records do not demonstrate unusual HRV and none is detected.

The ideal input parameters for the program depend on each individual subject. Thus, the learning period of the algorithm should be used to estimate these optimal values. In the case of the test database, this learning period is the first 5 mins of each record. In the case of a real-world situation, a reasonable learning period could be combined with user-defined parameters and the length of the period itself may be a user-defined parameter.

4 CONCLUSIONS & FURTHER WORKS

The combinatorial formalization of some practical ECG analysis and interpretation problems has been attempted in this paper. The suggested algorithmic solutions presented, are online and use pattern matching and simple statistical techniques to determine a subjects heart rate and heart rate variability (HRV).

The algorithms were implemented and certain types of arrhythmia are reported on data from the MIT-BIH arrhythmia database. The experimental results were promising, a fact that suggests that further investigation and development of the model and programs are desirable.

Improvements can be made to the existing algorithms and the program can be extended to identify further types of arrhythmia. HRV measures more accurately identify irregularities on longer recordings than the 30 minutes in the records in the test database. This suggests that the model would possibly better be used in ambulatory (Holter) devices, and telemetry communication could be employed for sending crucial data to the lab or clinic.

An additional improvement would be the application of more sophisticated statistical analysis algorithms, employed on the data in the learning period, to determine 'better' parameters as input to the test period of the program.

Another use of the model is for prerecorded ECG data. Holter monitors are often used to gather ECG data for an extended period of time, for later analysis. Using the combinatorial model, this data could be indexed using for example a suffix tree or suffix array (Crochemore et al., 2007; Gusfield, 1997). For an ECG of length n, this indexing can be done in $O(n)$ time.

Finally, further areas of investigation is to extend the modeling of the ECG data to include more than one lead, as well as to implement the beat classification algorithm of Section 3.1.

REFERENCES

Afonso, V. X., Wieben, O., Tompkins, W. J., Nguyen, T. Q., and Luo, S. (1997). Filter bank-based ecg beat classification. In *Proceedings of the 19th Annual International Conference of the IEEE*, volume 1, pages 97–100. Engineering in Medicine and Biology Society.

Aho, A. V. and Corasick, M. J. (1975). Efficient string matching: an aid to bibliographic search. *Commun. ACM*, 18(6):333–340.

ANSI/AAMI (1998a). *EC38: Ambulatory Electrocardiographs*. Association for the Advancement of Medical Instrumentation.

ANSI/AAMI (1998b). *EC57: Testing and Reporting Performance Results of Cardiac Rhythm and ST Segment*. Association for the Advancement of Medical Instrumentation.

Chazal, P. D., O'Dwyer, M., and Reilly, R. B. (2004). Automatic classification of heartbeats using ecg morphology and heartbeat interval features. *IEEE Transactions on Biomedical Engineering*, 51:1196–1206.

Conover, M. B. (2002). *Understanding Electrocardiography*. Mosby.

Crochemore, M., Hancart, C., and Lecroq, T. (2007). *Algorithms on Strings*. Cambridge University Press.

Garratt, C. (2001). *Mechanisms and Management of Cardiac Arrhythmias*. BMJ Books, London.

Goldberger, A. L., Amaral, L. A. N., Glass, L., Hausdorff, J. M., Ivanov, P. C., Mark, R. G., Mietus, J. E., Peng, G. B. M. C. K., and Stanley, H. E. (2000). PhysioBank, PhysioToolkit, and PhysioNet: Components of a new research resource for complex physiologic signals. *Circulation*, 101(23):e215–e220. Circulation Electronic Pages: http://circ.ahajournals.org/cgi/content/full/101/23/e215.

Gusfield, D. (1997). *Algorithms on Strings, Trees and Sequences: Computer Science and Computational Biology*. Cambridge University Press.

Hu, Y. H., Palreddy, S., and Tompkins, W. J. (1997). A patient-adaptable ecg beat classifier using a mixture of experts approach. *Biomedical Engineering, IEEE Transactions on*, 44(9):891–900.

Iliopoulos, C. S. and Michalakopoulos, S. (2009). A combinatorial model for ecg interpretation. In *Proceedings of ICMIBE 2009, International Conference on Medical Informatics and Biomedical Engineering*. World Academy of Science.

Kawachi, I., Sparrow, D., Vokonas, P., and Weiss, S. (1995). Decreased heart rate variability in men with phobic anxiety (data from the normative aging study). *The American journal of cardiology*, 75(14):882–885.

Massachusets Institute of Technology (1999). Mit-bih ecg database. Available: http://ecg.mit.edu/.

Portet, F. (2008). P wave detector with pp rhythm tracking: evaluation in different arrhythmia contexts. *Physiological Measurement*, 29(1):141–155.

Springhouse (Editor) (2007). *ECG Interpretation Made Incredibly Easy!* Lippincott Williams & Wilkins, 4^{th} edition.

Stein, E. (2000). *Rapid Analysis of Arrhythmias*. Lippincott Williams & Wilkins, 3^{rd} edition.

Task Force of the European Society of Cardiology the North American Society of Pacing Electrophysiology (1996). Heart Rate Variability : Standards of Measurement, Physiological Interpretation, and Clinical Use. *Circulation*, 93(5):1043–1065.

Thaler, M. S. (2006). *The Only EKG Book You'll Ever Need*. Lippincott Williams & Wilkins, 5^{th} edition.