

DEMO PROGRAMS FOR NORMAL AND UNIFORM RANDOM VARIABLES TRANSFORMATION

Gordana Jovanovic Dolecek and Fred Harris
Department of Electronics, Institute INAOE, Puebla, Mexico
Department of Electrical Eng., SDSU San Diego, U.S.A.

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Abstract: This paper presents MATLAB-based demo programs for transformation of normal and uniform random variables. Linear as well as nonlinear transforms are considered. It is demonstrated how the transform changes the corresponding probability density function. It is also shown how to use the transformation of given random variable to generate a new desired random variable. The programs can be used as a complement to theoretical classes or alone as a self-study tool.

1 INTRODUCTION

Computer-aided learning has become an important educational research activity in various engineering disciplines and there has been a growing interest in the development of educational software in all areas of study (Fernandez and Sanchez, 2004). As a result many computer packages have been developed to assist learning (Yann and Teng, 2003; Li and Lie, 2004).

It is known that random variable (r.v.) is generally considered as one of the most abstract and conceptually difficult areas in the engineering education and teaching of random variables is one of the subjects that requires more time for its understanding. Our experience shows that the use of demo programs gives students the visual and intuitive representation of the random variables which had traditionally been stated in terms of abstract mathematical description (Jovanovic, 1997; Jovanovic and Champac, 2000). Such software tools must have characteristics like repeatability, capacity to motivate students, interactivity, versatility, easy to use etc.

Presented in this paper are demo programs to teach the transform of uniform and normal random variables. The programs can be used as a complement to theoretical classes or alone as a self-study tool. The programs are interactive, i.e., the user chooses the parameters of the input variables and the corresponding transform.

Each program presents input and output variables and the desired transform as well as the estimated probability densities. The exact densities are also presented.

The rest of the paper is as follows. Next section is devoted to the linear and nonlinear transformations of uniform random variables, while Section 3 demonstrates the linear and nonlinear transforms of normal random variables.

2 UNIFORM VARIABLE

2.1 Demo for Linear Transform

In this program the linear transform (LT) $Y=aX+b$ of the uniform random variable X is discussed. The user chooses the range (R_1, R_2) of the uniform r.v. and the parameters of the linear transform a and b .

The program shows the output variable and the estimation of the input and output probability densities (PDS), thus demonstrating that the linear transform of the uniform r.v. results in an uniform r.v. As an illustration Fig.1 shows the result for $R_1 = -1, R_2 = 4, a = 2,$ and $b = 3$. The estimation of the corresponding PDFs is given in Fig.2.

Figure 3 presents the corresponding uniform PDFs.

It is also demonstrated how this result is useful to generate the desired uniform r.v. from the given uniform r.v. For example, in MATLAB the file *rand*

generates the uniform r.v. in the interval (0, 1). To obtain uniform r.v. in the interval (R_{21}, R_{22}) , we have to apply the linear transform with $a=R_{22}-R_{21}$, where $a=R_{22}-R_{21}$; $b=R_{21}$, to the uniform random variable generated by *rand*. Figure 4 illustrates the generation of the uniform r.v. in the interval (-3, 5).

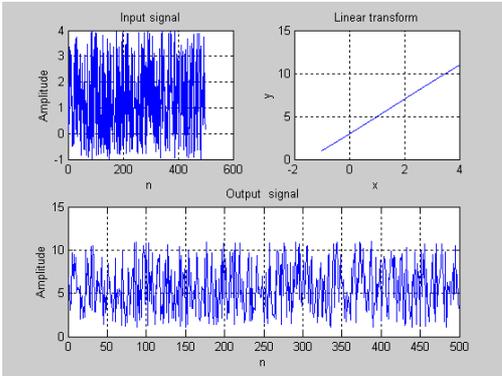


Figure 1: Linear transform of uniform r.v.

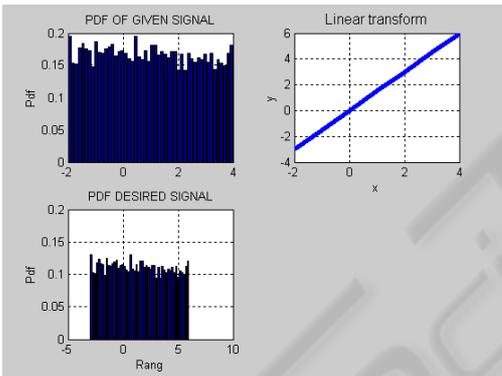


Figure 2: Estimated Input and Output Densities.

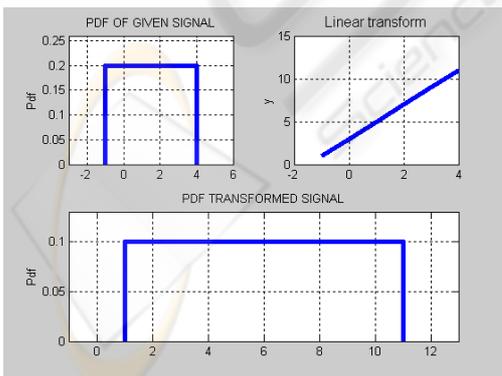


Figure 3: Input and Output Densities.

2.2 Demo for Nonlinear Transform

The Demo program for the absolute value of uniform r.v. illustrates that the transform does not change the type of the uniform r.v., as shown in example in Fig.5, where the input r.v. is uniform in the interval (-5, 5). The estimation of the input and output PDFs are given in Fig.6.

The following demo shows the transformations of the uniform r.v. X in the range (0, 1).

The demo illustrated in Figs 7 and 8 shows how to generate an exponential random variable by applying transform

$$Y = -\frac{1}{\lambda} \log(1 - X), \quad (1)$$

to a uniform variable X .

Figure 9 illustrates the demo for the transformation of the uniform random variable X ,

$$Y = \sqrt{-2\sigma^2 \ln X}. \quad (2)$$

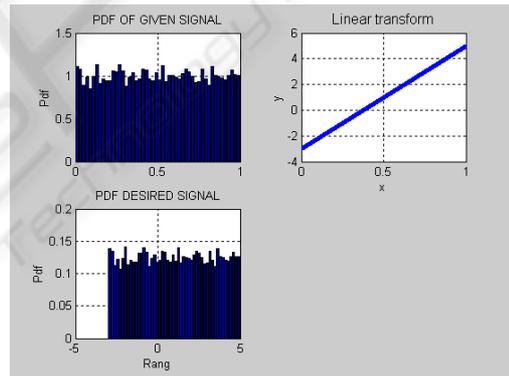


Figure 4: Generation of the desired uniform r.v.

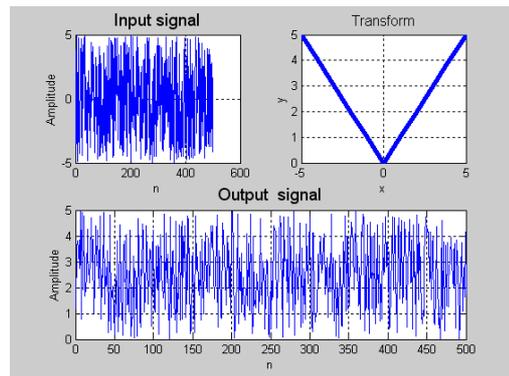


Figure 5: Absolute value of uniform r.v.

The resulting random variable is Rayleigh as shown in Fig.9.

3 NORMAL VARIABLE

3.1 Demo for Linear Transform

This program demonstrates that the linear transform of the normal r.v. gives again the normal r.v. User chooses the mean value and variance of the normal variable, as well as the parameters of the linear transform a and b . Figure 10 illustrates the demo for the mean value 2, variance 4, and $a=3$, $b=4$. The figure shows the estimation of the output PDF demonstrating that the output random variable is also the normal variable. The same is confirmed in Fig.11.

MATLAB file *randn* generates the normal r.v. with the zero mean value and the variance 1. The normal random variable with mean m and the variance σ^2 is obtained by applying the following transform

$$Y = \sigma X + m. \tag{3}$$

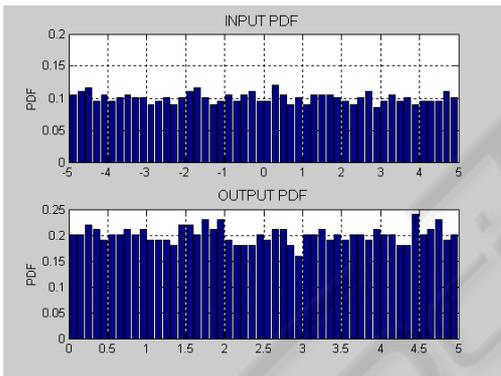


Figure 6: Estimation of the corresponding densities.

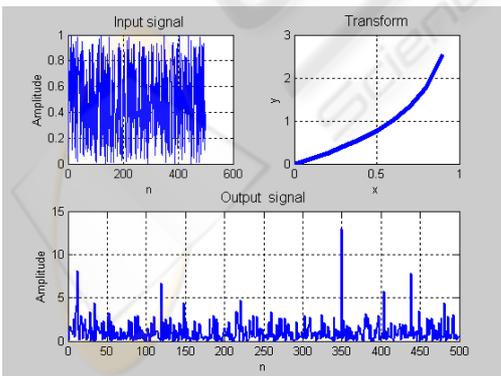


Figure 7: The generation of exponential r.v.

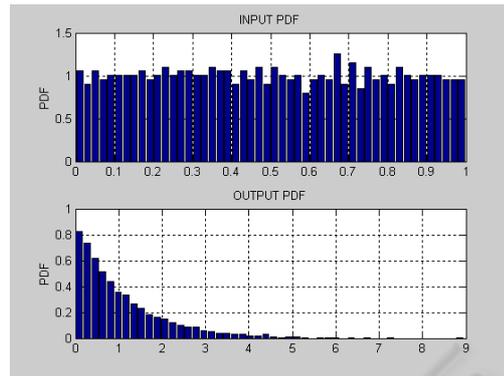


Figure 8: Estimation of the corresponding densities.

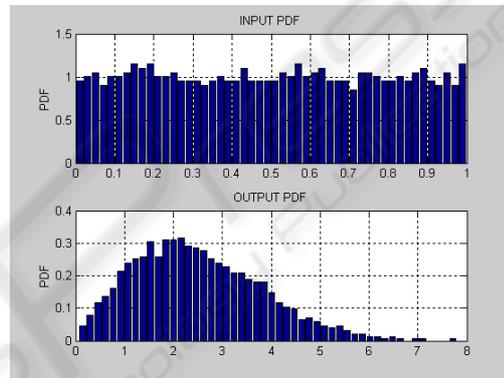


Figure 9: Estimation of the Rayleigh density.

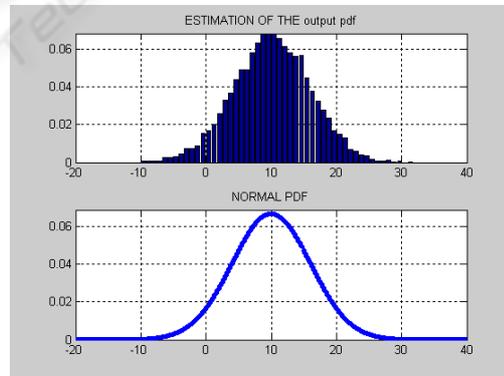


Figure 10: Estimation of the output density.

3.2 Demo for Nonlinear Transform

Figure 12 illustrates the absolute value of the normal r.v. with zero mean and the variance 4. The resulting random variable is the one-sided normal variable. The exponential transform

$$Y = e^X \tag{4}$$

of the normal variable is illustrated in Figs.13 and 14.

4 CONCLUSIONS

This paper describes the educational software for transformations of the uniform and normal random variables. The linear as well as nonlinear transforms are considered. It is also described how to use the transformations to generate different random variables.

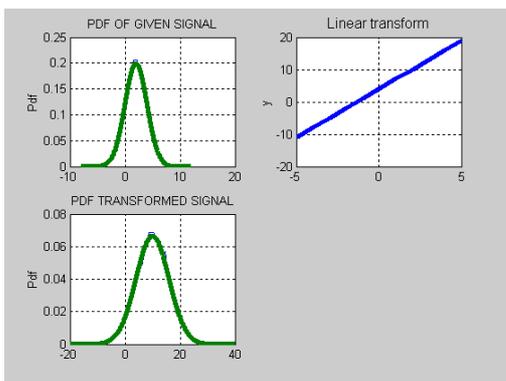


Figure 11: Input and output densities.

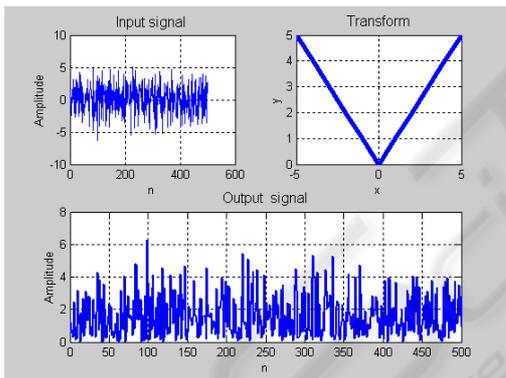


Figure 12: Absolute value of normal r.v.

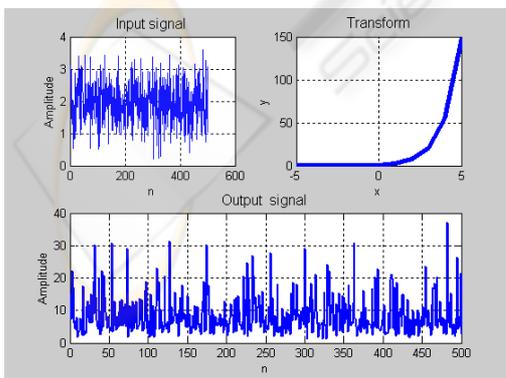


Figure 13: Exponential transform.

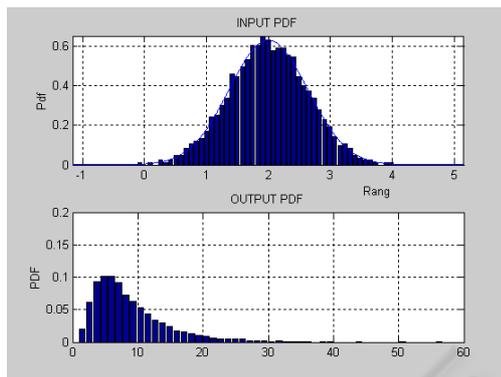


Figure 14: Estimation of the densities.

The programs are written in MATLAB, although any previous knowledge of MATLAB is not required. The programs can be used as a complement to theoretical classes or alone as a self-study tool for teaching basic course on random signals.

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