

# EDGE-PRESERVING SMOOTHING OF NATURAL IMAGES BASED ON GEODESIC TIME FUNCTIONS

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**Keywords:** Mathematical morphology, edge-preserving smoothing, nonlinear filtering, geodesic time.

**Abstract:** In this paper, we address the problem of edge-preserving smoothing of natural images. We introduce a novel adaptive approach derived from mathematical morphology as a preprocessing stage in feature extraction and/or image segmentation. Like other filtering methods, it assumes that the local neighbourhood of a pixel contains the essential information required for the estimation of local image properties. It performs a weighted averaging by combining both spatial and tonal information in a single similarity measure based on the local calculation of geodesic time functions from the estimated pixel. By designing relevant geodesic masks, it can deal with specific situation and type of images. We describe in the following two possible strategies and we show their capabilities at smoothing heterogeneous areas while preserving relevant structures in natural - greyscale or multispectral - images displaying different features.

## 1 INTRODUCTION

Image smoothing is a common preprocessing stage used to improve the visual information in an image, and to simplify subsequent image processing stages such as feature extraction, image segmentation or motion estimation (Jähne, 1997). Traditionally, the problem of image smoothing is to reduce undesirable distortions - due to the presence of noise or the quality of the image acquisition process - while preserving important features such as homogeneous regions, discontinuities, edges and textures.

Smoothing techniques have been extensively used in many fields of image processing (Saint-Marc and Medioni, 1991; Winkler et al., 1999; Mrázek et al., 2006; Takeda et al., 2007). The generic continuous expression of a low pass filtering (smoothing) of an image  $f$  is the convolution operation of this image (Jähne, 1997):

$$f(\mathbf{x}) \otimes \mathcal{K}(\mathbf{x}) = \int_{\mathbb{R}^2} \mathcal{K}(\mathbf{x}, \mathbf{y}) f(\mathbf{y}) d\mathbf{y} \quad (1)$$

where  $\mathbf{x} = (x, y)$  are spatial coordinates,  $f(\mathbf{x})$  is the local luminance (greylevel intensity or multispectral value) in  $\mathbf{x}$  and  $\mathcal{K}$  is a kernel function (also called "window function") that is assumed to be normalised:  $\int_{\mathbb{R}^2} \mathcal{K}(\mathbf{x}, \mathbf{y}) d\mathbf{y} = 1$  and shift invariant:  $\mathcal{K}(\mathbf{x}, \mathbf{y}) =$

$\mathcal{K}(\mathbf{x} - \mathbf{y})$ . In practical case, the integral  $\int_{\mathbb{R}^2}$  in Eq. (1) becomes a discrete weighted summation  $\sum_{\mathbb{Z}^2}$  and the support size of the kernel  $\mathcal{K}$  is finite, as it is often desirable to estimate the intensity of a pixel from a local neighbourhood. Depending on the functional form of the kernel, smoothing algorithms are classified into two categories: linear and nonlinear methods (Jähne, 1997), allowing for different weighting of the luminance values. For linear smoothing, local operators are uniformly applied to the image to form the output luminance (van den Boomgaard and van de Weijer, 2003). The most straightforward and the fastest technique consists in using an isotropic kernel, with fixed size and fixed weights, over the image. Such approach yields good results when all the pixels in the window (*i.e.* the kernel support) come from the same 'population' as the central pixel: in image regions corresponding to the interior of an object, it produces a desirable luminance which is representative of the region. However, difficulties arise when the window overlaps a discontinuity: on the boundaries between different regions, it results in averaging of edge values, and therefore significant blurring of the edges. Not only the noise is reduced but also the image structures are smoothed. The problem remains that a fixed kernel  $\mathcal{K}$  is not suited for images featur-

ing real structures on various scales and with different shapes: there is a trade-off between localisation accuracy and noise sensitivity. Nonlinear smoothing has been developed to overcome these shortcomings, which tend to preserve important features along with noise removal during smoothing (Narendra, 1981; Brownrigg, 1984; Perona and Malik, 1990; Nitzberg and Shiota, 1992). In this context, the need for an adaptive approach to cope with inhomogeneities in images is well recognised (Saint-Marc and Medioni, 1991; Gomez, 2000; Grazzini et al., 2004). Strong relations have been established between a number of widely-used adaptive filters for digital image processing (Mrázek et al., 2006). The most common strategy is to locally vary the kernel over image regions according to their contents (Takeda et al., 2007): around a pixel  $\mathbf{x}$  to be updated, one uses a kernel  $\mathcal{K} = \mathcal{K}_{\mathbf{x}}$  with proper weights depending on the actual image variability in the neighbourhood of  $\mathbf{x}$ . A critical issue is then how to measure image variability for generating the weights of the kernel. A possible approach consists in adapting the local effect of the filter by using both the location of the nearby samples and their luminance values. That is, the proposed kernel  $\mathcal{K}_{\mathbf{x}}$  takes into account two factors: spatial distances  $|\mathbf{x} - \mathbf{y}|$  and tonal distances  $|f(\mathbf{x}) - f(\mathbf{y})|$ . Introducing a tonal weight, the mixing of different luminance 'populations' is prevented. Such approach is known as the *bilateral filtering* technique introduced by (Tomasi and Manduchi, 1998) as an intuitive generalisation of the Gaussian convolution. In fact, it is clear from the literature that there is a direct relationship between this technique and other nonlinear smoothing methods (van den Boomgaard and van de Weijer, 2003; Mrázek et al., 2006): anisotropic diffusion (Barash, 2002), local-mode finding (van den Boomgaard and van de Weijer, 2002) or mean-shift analysis (Comaniciu and Meer, 2002).

Following the line of thought of (Tomasi and Manduchi, 1998), this paper introduces a new algorithm for edge-preserving smoothing of natural images as a preprocessing stage in feature extraction and/or image classification. The present method exploits an image-dependent approach derived from concepts known in mathematical morphology (Soille, 2004). It is based on the definition of appropriate *geodesic masks* and the local estimation of pairwise *geodesic time functions* within these masks (Lantuéjoul and Maisonneuve, 1984; Soille, 1994a; Soille, 1994b; Soille and Grazzini, 2007). Likewise bilateral filtering, the main idea is to associate with each pixel a weighted convolution of sample points within an adaptive neighbourhood, where the weights depend not only on the spatial distances but also on the tonal distance to the

centre pixel. With respect to other nonlinear techniques, which often involve iterative operations (Perona and Malik, 1990; Comaniciu and Meer, 2002), this approach presents the advantage of not depending upon any termination time. Moreover, it enables to determine adaptively, directly from the unsmoothed input data, the neighbouring sample points and the associated weights. Namely, by designing relevant geodesic masks, the geodesic time functions computed from a single pixel provide a similarity measure of the twofold spatial and tonal information in the local neighbourhood of this pixel.

The rest of the paper is organised as follows. In the next section, we recall the fundamental notions of geodesic path, geodesic mask and geodesic time known in mathematical morphology. In section 3, we introduce a new filter for image smoothing based on the estimation of local geodesic time on the gradient magnitude. In section 4, we propose an alternative algorithm based on the calculation of the geodesic time on image variations able to deal with the presence of noise. In section 5, we present and discuss some results and also compare the approach with other existing techniques. A conclusion regarding the current results and a description of future foreseen developments are presented in section 6.

## 2 GEODESIC TIME ON GREYLEVEL IMAGES

Geodesic transforms are classical operators in image analysis (Lantuéjoul and Maisonneuve, 1984; Verwer et al., 1989). The *geodesic distance* between two pixels of a connected set (typically, a binary image) is defined as the length of the shortest path(s) linking these points and remaining in the set (the so-called *geodesic paths*). This idea can be generalised to greylevel images (images with single-valued luminance) using the *geodesic time on geodesic mask* (Soille, 1994b; Ikonen and Toivanen, 2007). In such case, the image is then treated as a 'height map', *i.e.* a surface embedded in a 3D space, with the third coordinate given by the greylevel values. The geodesic paths are then constrained to the surface of the height map: typically, the path between two close pixels can be long, if there is a high 'ridge' in the greylevel map between them.

If we consider a greylevel image as an integrable function  $g$  (designed as the *geodesic mask*), the time  $\tau_g(\mathcal{P})$  necessary for travelling on a path  $\mathcal{P}$  defined on the domain of definition of  $g$  is expressed as the integral of  $g$  along  $\mathcal{P}$  (Soille, 1994b):

$$\tau_g(\mathcal{P}) = \int_{\mathcal{P}} |g(\mathbf{s})| ds. \quad (2)$$

The designation 'time' is justified by considering units equal to the inverse of that of a speed for the image  $g$ . Following, the geodesic time  $\tau_g(\mathbf{p}, \mathbf{q})$  separating two points  $\mathbf{p}$  and  $\mathbf{q}$  is the smallest amount of time allowing to link  $\mathbf{p}$  to  $\mathbf{q}$  in  $g$ :

$$\tau_g(\mathbf{p}, \mathbf{q}) = \min\{\tau_g(\mathcal{P}) \mid \mathcal{P} \text{ is a path linking } \mathbf{p} \text{ to } \mathbf{q}\}.$$

In the discrete case, a path  $\mathcal{P}$  of length  $l$  going from  $\mathbf{p}$  to  $\mathbf{q}$  is defined as a  $l$ -tuple  $(\mathbf{x}_1, \dots, \mathbf{x}_l)$  of pixels such that  $\mathbf{x}_1 = \mathbf{p}, \mathbf{x}_l = \mathbf{q}$ , and  $(\mathbf{x}_{i-1}, \mathbf{x}_i)$  defines adjacent pixels for all  $i \in [2, l]$ . Therefore, the greylevel values intuitively represent the cost of traversing the pixels: the lower the greylevel value, the faster the propagation. More precisely, the cost  $c_i$  of travelling from a pixel  $\mathbf{x}_i$  to an adjacent pixel  $\mathbf{x}_{i+1}$  is <sup>1</sup>:

$$c_i = \frac{1}{2}(g(\mathbf{x}_i) + g(\mathbf{x}_{i+1})) \cdot |\mathbf{x}_i - \mathbf{x}_{i+1}| \quad (3)$$

where the spatial distance  $|\mathbf{x}_i - \mathbf{x}_{i+1}|$  can be either the Euclidean distance or the optimal Chamfer propagating weights in a binary  $3 \times 3$  mask (Borgefors, 1986). The time  $\tau_g(\mathcal{P})$  necessary to cover a discrete path  $\mathcal{P}$  then refers to the sum of the greylevel values of the pixels along  $\mathcal{P}$ ; the geodesic time  $\tau_g(\mathbf{p}, \mathbf{q})$  finds the path with the lowest sum of greylevel values along all possible discrete paths linking  $\mathbf{p}$  to  $\mathbf{q}$ . This concept is closely related to the notion of grey weighted distance transform defined in (Levi and Montanari, 1970) and to the continuous formulation of the eikonal equation (Cohen and Kimmel, 1997).

### 3 SMOOTHING FILTER DERIVED FROM LOCAL GEODESIC TIME ON GRADIENT MAGNITUDE

In many adaptive smoothing techniques, a way to circumvent mixing the values from different populations is to introduce a similarity measure between the pixels of the image. We observe in particular that calculating the tonal weight in bilateral filtering (Tomasi and Manduchi, 1998) implicitly introduces an estimate of the local gradient between neighbouring values, then using this estimate to weight the respective measurements. Accordingly, we propose to combine both spatial and tonal information into a single similarity measure by estimating locally the geodesic time

<sup>1</sup>A reason why we do not use the term 'distance' for designing this geodesic function is precisely Eq. (3): indeed, it appears obvious for the cost  $c_i$  to be null between two adjacent pixels.

with the magnitude of the spatial gradient of the image as the geodesic mask. A related concept was described in (Sumengen et al., 2006) within the continuous framework of the eikonal equation for image segmentation. The underlying idea is that the geodesic paths associated to this function define the intrinsic neighbourhood relationship between the sample points when the 2D image is projected onto the 3D spatial-tonal domain (the 'height map' described earlier).

The values of the magnitude of the spatial gradient  $|\nabla f|$  of the image  $f$  are regarded as the cost of crossing a pixel. By replacing the function  $g$  by  $|\nabla f|$  in Eq. (3), it implies that the time necessary to travel between two pixels separated by high gradient values is higher than the time necessary to travel between two pixels separated by low gradient values. In the case of multivalued image, the calculation of the geodesic time must however be considered carefully, either processing channel by channel (*i.e.* calculating local gradient and local geodesic time separately for each band), or processing simultaneously the information provided by the different channels of the image (*i.e.* using an estimation of the local multispectral gradient). The major advantage of the second approach is that it is taking into account the actual multispectral edge information, so further smoothing will be more efficient along edges, and, thus, edges will be better preserved. A way to estimate the gradient magnitude of a multichannel image  $f$  with components  $f_n, n = 1, \dots, N$  is by means of the eigenvalue analysis of the image squared differential proposed in (Di Zenzo, 1986), expressed by the so-called  $2 \times 2$  matrix called first fundamental form (Scheunders and Sijbers, 2001):  $\left[ \sum_n \left( \frac{\partial f_n}{\partial x} \right)^2, \frac{\partial f_n}{\partial x} \frac{\partial f_n}{\partial y}, \sum_n \frac{\partial f_n}{\partial x} \frac{\partial f_n}{\partial y}, \sum_n \left( \frac{\partial f_n}{\partial y} \right)^2 \right]$ . The direction of maximal and minimal change are given by the eigenvectors of this matrix while the corresponding (positive) eigenvalues  $\lambda_1 \geq \lambda_2$  denote the rate of change. In particular, for greylevel images ( $N = 1$ ), it is verified (Scheunders and Sijbers, 2001) that the largest eigenvalue is given by the squared gradient magnitude:  $\lambda_1 = |\nabla f|^2$  and the corresponding eigenvector lies in the direction of the gradient. Therefore, taking into account these observations, we select the locally defined function  $\lambda_1$  (or indifferently  $\sqrt{\lambda_1}$ ) as the natural estimate for the gradient magnitude of the image<sup>2</sup>.

For each pixel  $\mathbf{x}$ , the similarity measure to all other pixels in its neighbourhood can then be com-

<sup>2</sup> $\lambda_1$  is naturally used as being the derivative energy in the most prominent direction, but other measure can be considered, *e.g.*,  $\lambda_1 - \lambda_2$  which is similar to  $\lambda_1$  corrected for the energy contributed by noise (Sapiro and Ringach, 1996).

puted as a (monotonically) decreasing function  $\mathcal{K}$  of the geodesic time  $\tau_g(\mathbf{x}, \cdot)$  from  $\mathbf{x}$  within the geodesic mask  $g(\mathbf{x}) = \lambda_1(\mathbf{x})$ . Classically, a Gaussian function  $G_\sigma$  with standard deviation  $\sigma$  will provide desirable results:

$$\mathcal{K}(\mathbf{x}, \mathbf{y}) = G_\sigma(\tau_g(\mathbf{x}, \mathbf{y})) = \exp\left\{-\frac{\int_{\mathbf{x}}^{\mathbf{y}} \lambda_1(\mathbf{s}) d\mathbf{s}}{\sigma^2}\right\} \quad (4)$$

but other kernels are not excluded. We can finally perform a weighted average of local samples by applying Eq. (1) with this kernel (figure 1, top). This way, the effective sampling procedure of the pixels  $\mathbf{y}$  used for estimating the value of the central pixel  $\mathbf{x}$  is adapted locally to image features such as edges. Indeed, larger weight should be assigned to pixels  $\mathbf{y}$  that involve low gradient values along the minimal geodesic paths from  $\mathbf{x}$ , and vice versa. In particular, luminance values from across a sharp feature are given less weight because they are penalised by the time on the gradient magnitude. In practise, due to memory and computational limitations, the support of  $\mathcal{K}$  is limited to a fixed size, *i.e.* sample pixels  $\mathbf{y}$  that are further away (in the spatial domain) than a fixed distance  $\omega$  (*e.g.* given by an analysing window  $\Omega$ ) to the central pixel  $\mathbf{x}$  are not considered. Moreover, we introduce a parameter  $\alpha$  which controls the global strength of the smoothing: the geodesic mask is refined and replaced by  $g(\mathbf{x}) = \alpha \cdot \lambda_1(\mathbf{x})$ . When  $\alpha$  increases, the geodesic time on the embedded surface becomes more sensitive to image deformations: the cost of crossing a pixel increases.  $\alpha$  controls the relative influences of tone and space in the calculation of the similarity measure of neighbour pixels.

Algorithms based on priority queue data structures (Soille, 1994b), implementing Dijkstra algorithm, enable an efficient implementation of the local geodesic time with a computational complexity of  $O(\omega^2 \log \omega^2)$  (Ikonen, 2007). They take advantage of the fact that the analysing windows  $\Omega$  are finite and totally ordered, and they guarantee that pixels that effectively contribute to the output are processed only once. Thus, running the algorithm calculating the local geodesic time from each pixel in the image results in a total complexity of  $O(w \cdot h \cdot \omega^2 \log \omega^2)$  where  $w$  and  $h$  are respectively the width and the height of the input image.

#### 4 DENOISING FILTER DERIVED FROM LOCAL GEODESIC TIME ON IMAGE VARIATIONS

It is well-known that discontinuities in an image likely correspond to important features. However, noise cor-

ruption can generate discontinuities as well. Therefore, how to measure significant discontinuities is a nontrivial problem. In particular, spatial gradient is known to be quite sensitive to noise. Owing to over-locality, it is inadequate to detect significant discontinuities from a noisy image, which usually causes adaptive smoothing algorithm based on gradient information to yield poor results. To tackle this problem, measures of higher order differentiations have been proposed (Nitzberg and Shiota, 1992). However, these techniques involve higher computational complexity since usually, in those approaches, a discontinuity measure has to be used in each iteration of adaptive smoothing.

In order to have both a strong denoising effect and a sharper image, we define a geodesic time that accounts for both the distance between pixels and the roughness of the 'height map', *i.e.* a measure of the shortest path drawn on the projection of the 2D image onto the spatial-tonal domain. For this purpose, we express the geodesic time in the continuous case by:

$$\tau_g(\mathcal{P}) = \int_{\mathcal{P}} \left| \frac{dg(\mathbf{s})}{ds} \right| ds. \quad (5)$$

so that the cost  $c_i$  of crossing pixels becomes:

$$c_i = \frac{1}{2} |g(\mathbf{x}_i) - g(\mathbf{x}_{i+1})| \cdot |\mathbf{x}_i - \mathbf{x}_{i+1}| \quad (6)$$

with the distance  $|\mathbf{x}_i - \mathbf{x}_{i+1}|$  defined as before. The similarity measure is then estimated by setting the geodesic mask to the input noisy image:  $g(\mathbf{x}) = f$ . This definition derives from the so-called weighted distance on curves space of (Ikonen and Toivanen, 2007), where the multiplication in Eq. (6) is replaced by an addition. Its intuitive interpretation is that it represents the minimal amount of ascents and descents to be travelled to reach a neighbouring pixel. This notion also expresses a similar geometric notion as the path variation defined in (Arbeláez and Cohen, 2003). Note that in fact the geodesic mask associated to the time  $\tau_g(\mathcal{P})$  of Eq. (5) is constantly updated through the propagation of the geodesic time (Ikonen and Toivanen, 2007; Ikonen, 2007). For multichannel images  $f$ , the norm in Eq. (6) must be understood as a multispectral norm, *e.g.* the  $L^\infty$  norm on the different channels; in such case, we have, when estimating the minimal path,  $|g(\mathbf{x}_i) - g(\mathbf{x}_{i+1})| \leq t$  if and only if  $|f_n(\mathbf{x}_i) - f_n(\mathbf{x}_{i+1})| \leq t$  for all  $n = 1, \dots, N$ . As a consequence, this approach depends, like bilateral filtering (figure 1, bottom right) and contrary to the one presented in the previous section, on the dimension of the tonal space. Whereas the formulation with Eq. (2) was going through the lowest values of the spatial gradient (the mask being defined as a function  $g(\mathbf{x}) \sim |\nabla(\mathbf{x})|$ ), the geodesic time defined with

Eq. (5) (and  $g(\mathbf{x}) \sim f(\mathbf{x})$ ) minimises the changes in luminance values. Image denoising is finally performed using a kernel like Eq. (4), and introducing similarly a multiplicative control parameter  $\alpha$  on the smoothing (figure 1, bottom left).



Figure 1: Results of smoothing applied on the image of the boat *La Cornouaille*. Top: smoothing using the filter based on the magnitude of the gradient as described in section 3 with a control parameter  $\alpha = 20$  (left, soft smoothing effect) and  $\alpha = 5$  (right, strong smoothing). Bottom: smoothing using the filter described in section 4 based on the image variations with  $\alpha = 20$  (left) and the output of some bilateral filtering (right). For both 'geodesic' filters, the parameters  $\sigma = 1$  and  $\omega = 21$  were fixed.

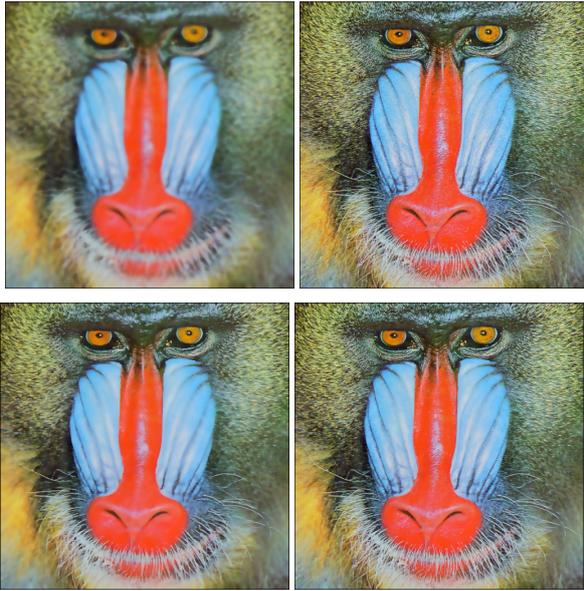
## 5 EXPERIMENTS AND COMPARISON WITH OTHER TECHNIQUES

We show in this section the results of the new proposed algorithms applied on different types of images<sup>3</sup>: greylevel or multispectral, displaying different typical features: texture, edges or noise. Depending on the input data and the final purpose, we choose to employ one of the two approaches: smoothing and image simplification with the algorithm described in section 3, denoising with the algorithm of section 4.

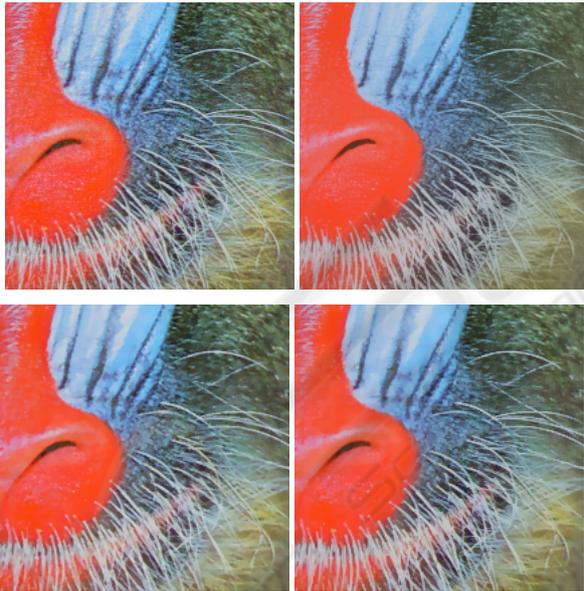
<sup>3</sup>Consult the online USC database <http://sipi.usc.edu/database/database.cgi?volume=misc> and the webpage [http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL\\_COPIES/GOMEZ1/results.html](http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/GOMEZ1/results.html) of (Gomez, 2000) for the original sample images.

Both filtering techniques result in visually satisfying smoothed versions of the original images (figure 1). Indeed, the generic approach enables to conserve features through the combined spatial and tonal actions represented in the similarity measure based on the geodesic time functions. Both filters nicely smooth homogeneous areas while preserving important structures such as the boundaries of objects. Close inspection to the images also shows they are good at enhancing subtle texture regions and they suppress small elements corresponding to the main heterogeneity (figure 2). The image structures are not geometrically damaged, what might be fatal for further processing like classification or segmentation. Indeed, it creates homogeneous regions instead of points or pixels as carriers of features which should be introduced in further processing stages. The amount of smoothing can be controlled by the parameter  $\alpha$  (figure 2(a)). With a small  $\alpha$  value, the estimated image will be much smoother so that details have been sacrificed to the effect of denoising, producing a gaussian-like blurring effect. High values of  $\alpha$  preserve almost all contrasts, and thus lead to filters with little effect on the image. For intermediate values of  $\alpha$ , the filtering procedures result in a diffusion effect, amplifying or attenuating the given local contrast in parts of an image. The filter based on gradient's magnitude show higher capability at blurring small discontinuities and sharpening edges when applied on noise-free images (figure 2(b)), whereas the filter based on image variations performs better when dealing with noisy images (figure 3). The intrinsic dependence of the latter filter on the luminance differences allows one to give less influence to outlier pixels when denoising (suppressing peaks in luminance distribution). A possible improvement when smoothing an image also regards the input central pixel value  $f(\mathbf{x})$ . As underlined in (van den Boomgaard and van de Weijer, 2002), using this value as the 'reference' for the estimation of the local geodesic time assumes that it is more or less noise free. This is naturally a questionable assumption when building a noise suppression filter and may have effect on the result. Following the authors' suggestion we could replace the central pixel's value with some estimate of the true value, e.g. the median value in an window of size  $3 \times 3$ .

As discussed in the introduction (section 1), the bilateral filter of (Tomasi and Manduchi, 1998) uses, as a simple and intuitive choice of the adaptive kernel, separate terms for penalising the spatial and tonal distances. In practise, Gaussian influence weighting functions are used. Our filters and the bilateral one result in very similar smoothed images (fig-



(a) Influence of the control parameter  $\alpha$  on the 'geodesic' filters, *resp.* based on the gradient's magnitude (first row, with  $\alpha = 1$  and  $\alpha = 20$ ), and on image variations (second row, with  $\alpha = 5$  and  $\alpha = 10$ ); the parameters  $\sigma = 1$  and  $\omega = 11$  were fixed.



(b) Outputs of smoothing performed on a detail of the mandrill (top left) with some bilateral filter (top right) and the filters based *resp.* on the gradient's magnitude (bottom left) and image variations (bottom right); the parameters for both 'geodesic' filters are  $\alpha = 10$ ,  $\omega = 11$  and  $\sigma = 1$ .

Figure 2: Smoothing applied to the mandrill image.

ure 2(b)). However, breaking the filtering kernel into spatial and tonal terms weakens the estimator performance since it limits the degrees of freedom and ig-

nores correlations between the positions of the pixels and their values. Using a twofold similarity measure like the geodesic time, defined on either the spatial gradient or the image variation, enables to account for these correlations. A smoothing technique described in (Gomez, 2000) is based on adaptive Gaussian filters. It adjusts locally the smoothing scale (*i.e.* the standard deviation of the Gaussian kernel) in a scale-space framework, through a minimal description length criterion. This way, the resulting smoothed data values intuitively share, at every location, a similar signal-to-noise ratio. Likewise the approach described in section 4, it is not based on partial derivatives. However, this approach results in blurred images when used for denoising (figure 3). Compared to iterative schemes like anisotropic diffusion (Perona and Malik, 1990), our approach is a non-iterative estimation technique, which makes it more efficient and more stable (figure 3).

## 6 CONCLUSIONS

In this paper, we explore the use of geodesic time functions for edge-preserving smoothing of natural images. The basic idea is similar to that of spatial-tonal filtering approaches, which consist in employing both geometric and luminance closeness of neighbouring pixels. The originality of the approach we propose lies in the definition of a new similarity measure combining both spatial and tonal information and based on the local estimation of some geodesic time functions. We show that, by designing relevant geodesic masks, we can define new smoothing filters enable at simplifying and/or denoising images, depending on the input data and on the target purpose. Finally, the proposed techniques show good results in different situations and images, where they were able to preserve the main structures, such as edges and textures, while smoothing other homogeneous parts. Likewise other spatial-tonal based techniques, the degree of smoothing in the image can also be controlled in order to adjust the fidelity to the original image.

Current research is geared towards improving and extending the present work. Improvements regard mainly the parameters' selection. There is in particular an issue regarding the spatial extent of the window used for estimating the local geodesic time functions. In this paper, we used a finite spatial window  $\Omega$  with size  $\omega$  for limiting the calculations. An alternative approach would be to limit the weighting average to the sample pixels reached from the central pixel with a time inferior to a given threshold value. This way, it would be possible to determine when the neighbour

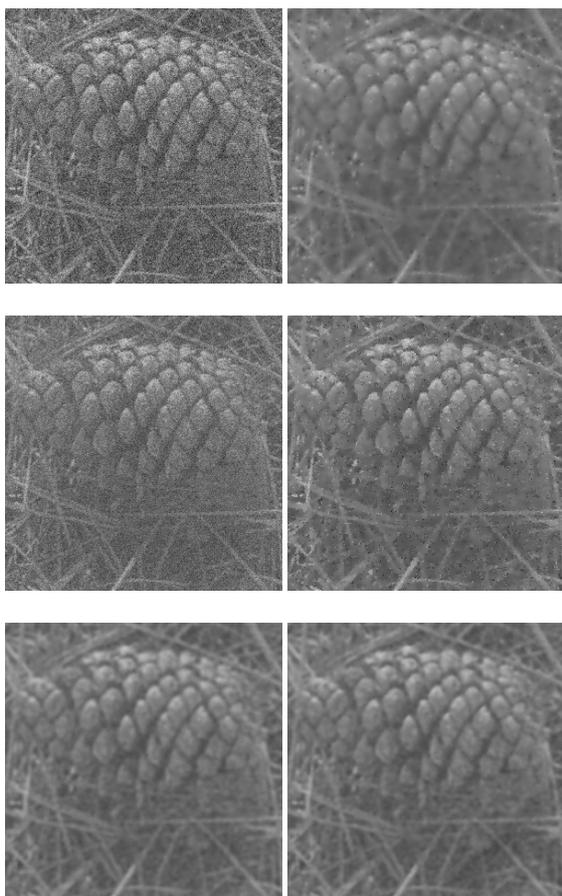


Figure 3: The results of smoothing of the pine cone image corrupted with Gaussian noise (detail, top left) are displayed, in this order, for: the method of (Gomez, 2000), some bilateral filter according to (Tomasi and Manduchi, 1998), anisotropic diffusion following (Perona and Malik, 1990) and the 'geodesic' filters based on gradient magnitude and image variations.

pixels must be considered as outliers. The selection of the smoothing control parameter  $\alpha$  should also be investigated. Another issue regards directly the estimation of the local geodesic time: the calculation could be refined by normalising locally the local cost of pixel crossing, *i.e.* considering  $c_i / \max \{c_i \mid \mathbf{x}_i \in \Omega\}$  in the geodesic propagation instead of  $c_i$  currently. A natural extension to our approach is to consider median filtering instead of weighted averaging of the pixels in local neighbourhoods. Selecting the (weighted) median rather than the mean helps to prevent outlier pixels from unduly distorting the result.

We believe that the proposed techniques are of particular interest for filtering data for which a discrete approach should be adopted, instead of a continuous one, in order to avoid creating spurious artifacts through diffusion-like processes. We foresee in par-

ticular potential applications in the fields of medical imaging and remote sensing.

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