STABILIZATION OF UNCERTAIN NONLINEAR SYSTEMS VIA PASSIVITY FEEDBACK EQUIVALENCE AND SLIDING MODE

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Abstract: In this paper, a sliding mode controller based on passivity feedback equivalence is developed in order to stabilize an uncertain nonlinear system. It is shown that if the nominal passive system obtained by feedback equivalence is asymptotically stabilized by output feedback, then the uncertain system remains stable provided the upper bounds of the uncertain terms are known. The results obtained are applied to the model of a magnetic levitation system to show the controller methodology design.

1 INTRODUCTION

In the last decade the concept of passivity has been mainly used in the stability analysis of continous-time state-space nonlinear systems (Cai and Han, 2005; Mahmoud and Zribi, 2002) and to analyze the stability properties of nonlinear interconnected systems and special cascaded structures (Byrnes et al., 1991; Ortega, 1991). Besides, an important question arises when the model of the system contains uncertain elements such as constant or varying parameters that are not known or imperfectly known. Under such imperfect knowledge of the model, the feedback that makes the uncertain system passive is no longer robust. Some works using nonlinear adaptive control have been recently devoted to this issue (Su and Xie, 1998; Duarte-Mermoud et al., 2002). On the other hand, the control of nonlinear systems with uncertainties via the sliding mode technique has been widely studied in the literature to attain robust control structures; see, for example the results presented in (Tunay and Kaynak, 1995).

The goal of the present paper is to develop a controller via passivity feedback equivalence and sliding modes that permits to stabilize an uncertain nonlinear system. Stabilization is obtained whenever the passive system associated to the nominal system is asymptotically stabilized by output feedback; a similar approach was presented in (Loria et al., 2001) where a different sliding surface is proposed. The study is completed by means of an example of height distance regulation in the model of a magnetic levitation system.

2 PASSIVITY EQUIVALENCE AND STABILIZATION USING SLIDING MODES

One considers *uncertain* MIMO nonlinear systems described by

$$\Sigma^U : \begin{cases} \dot{x} = f(x) + \Delta f(x) + (g(x) + \Delta g(x))u, \\ y = h(x) \end{cases} \tag{1}$$

where $x \in \Re^n$ is the state vector, $u \in \Re^p$ is the input vector, $y \in \Re^p$ is the output vector. f and the p columns of the matrix g are C^∞ vector fields, and the p components of the vector h are C^∞ functions. Δf and the p columns of the matrix Δg are smooth vector fields defined on $\Re^n$ which represent the model uncertainties. In addition, we suppose, without loss of generality and after a possible coordinates shift, that $f(0) = 0$ and $h(0) = 0$. The MIMO nonlinear system (1) without uncertainties, also referred as the *nominal system*, is described by

$$\Sigma : \begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x). \end{cases} \tag{2}$$

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This is, Σ is given by Σ^U with $\Delta f(x) = 0$ and $\Delta g(x) = 0$ for all x.

Let us now assume that the nominal system Σ has relative degrees $r_1 = 1, \ldots, r_p = 1$, that the matrix $L_g h(0)$ is nonsingular and that it is weakly minimal phase; this is, system (2) is locally equivalent to a passive system (Byrnes et al., 1991). Let $S(y,v) = col\{S_1(y,v), \ldots, S_p(y,v)\}$ be an p dimensional smooth function that we refer as the *switching function* where v is a new input signal. In this work, we set $S(y,v)$ as

$$S(y,v) = y - \int_0^t v(\tau)d\tau. \tag{3}$$

In the sliding mode, $S = \dot{S} = 0$, and the state trajectory of the nominal system is constrained to evolve on the *sliding surface* M_S by the so-called *equivalent control* $u = u_{eq}$. If an initial point does not belong to M_S, the *attractivity condition* $(\dot{S})^T S \leq -\lambda$ with $\lambda > 0$ must be satisfied in a neighbourhood of M_S, so that this surface becomes attractive (Utkin, 1992). The control law which permits to reach the sliding surface can be obtained from the expression $\dot{S} = -F(S)$ where $F(S)$ is, in general, a discontinuous vector function of its arguments.

Writting the uncertain system (1) in the new coordinates (y,z), with z being a set of complimentary coordinates, and substituting the feedback

$$u = u_{slid} = b(y,z)^{-1}[-F(S) - a(y,z) + v]. \tag{4}$$

where $b(y,z)$ is nonsingular for all (y,z) near $(0,0)$ and setting $F(S) = \Gamma sign(S)$ where $sign(S) := col\{sign(S_1), \ldots, sign(S_p)\}$ and $\Gamma > 0$, one has

$$\tilde{\Sigma}^U : \left\{ \begin{array}{l} \dot{y} = v - \Gamma sign(S) + \Delta a(y,z) \\ \quad +\Delta b(y,z) b^{-1}(y,z)(-a(y,z) - \Gamma sign(S) + v) \\ \dot{z} = f^*(z) + p(y,z)y + \{\sum_{i=1}^m q_i(y,z)y_i\}v \\ \quad +\Delta p(y,z)y + \{\sum_{i=1}^m \Delta q_i(y,z)y_i\}v \\ \quad +\{\sum_{i=1}^m \Delta r_i(y,z)y_i\}\Gamma sign(S) \end{array} \right. \tag{5}$$

where $p(y,z)$ and the $q_i(y,z)$'s are suitable matrices of appropriate dimensions and $\dot{z} = f^*(z)$ are the so called *zero dynamics* of the nominal system. $\Delta p(y,z)$, the $\Delta q_i(y,z)$'s, and the $\Delta r_i(y,z)$'s are matrices which represent the terms associated to the uncertainties in the z variables. $\Delta a(y,z)$ and $\Delta b(y,z)$ represent the uncertainties associated to the y variable.

Since it is assumed that the nominal system is weakly minimal phase, its zero dynamics are Lyapunov stable with a time-independent and C^2 Lyapunov function $W^*(z)$, and one chooses the signal v as (Byrnes et al., 1991)

$$v = [I + M(y,z)]^{-1}[-(L_{p(y,z)} W^*(z))^T + w] \tag{6}$$

where $M(y,z) = [(L_{q_1} W^*)^T \cdots (L_{q_p} W^*)^T]^T$. This choice makes the closed-loop nominal system $[\dot{y}^T \dot{z}^T] = \bar{f}(y,z) + \bar{g}(y,z)w$ passive from the input w to the output y. Assuming that this passive system is also locally *zero state detectable*[1], its equilibrium $(y,z) = (0,0)$ can be can be made asymptotically stable by the simple output feedback $w = -\phi(y)$ with $\phi(0) = 0$ and $y^T\phi(y) > 0$ for each $y \neq 0$. Let us define define $\xi = (y,z)$ and substitute the assignment (6) together with $w = -\phi(y)$ into the uncertain system (5). The resulting closed-loop system can then be written as

$$\dot{\xi} = \bar{F}(\xi) + \bar{G}(\xi) \tag{7}$$

where

$$\bar{F}(y,z) = \bar{f}(y,z) - \bar{g}(y,z)\phi(y), \quad \bar{G}(\xi) = \bar{G}_1(\xi) + \bar{G}_2(\xi)$$

and

$$\bar{G}_1(y,z) = \begin{bmatrix} \bar{G}_{11}(y,z) \\ 0 \end{bmatrix}, \bar{G}_2(y,z) = \begin{bmatrix} 0 \\ \bar{G}_{22}(y,z) \end{bmatrix} \tag{8}$$

with

$$\begin{array}{rl} \bar{G}_{11}(y,z) = & -\Gamma sign(S) + \Delta a(y,z) \\ & +\Delta b(y,z) b^{-1}(y,z)(-a(y,z) - \Gamma sign(S) \\ & +[I + M(y,z)]^{-1}[-(L_{p(y,z)} W^*(z))^T - \phi(y)]), \end{array}$$

$$\begin{array}{rl} \bar{G}_{22}(y,z) = & \Delta p(y,z)y + \{\sum_{i=1}^m \Delta r_i(y,z)y_i\}\Gamma sign(S) \\ & +\{\sum_{i=1}^m \Delta q_i(y,z)y_i\}[I \\ & +M(y,z)]^{-1}[-(L_{p(y,z)} W^*(z))^T - \phi(y)]. \end{array}$$

We now assume that the uncertain terms satisfy the uniform bounds

$$\| \bar{G}_1(\xi) \| \leq \delta_1, \quad \| \bar{G}_2(\xi) \| \leq \delta_2 \tag{9}$$

for all $\xi \in D$ where $D = \{\xi \in \Re^n : \| \xi \| < r\}$ with $r > 0$ or, equivalently,

$$\| \bar{G}(\xi) \| \leq \delta_1 + \delta_2 = \delta \tag{10}$$

for all D. Notice that $\xi = 0$ is a locally asymptotically equilibrium point of the system $\dot{\xi} = \bar{F}(\xi)$ and one can then assure, by using the Lyapunov approach, that for all bounded initial conditions $\xi(0)$, the solution $\xi(t)$ of the uncertain system (7) is locally ultimately bounded for $t \geq 0$. Moreover, one can show that the sliding surface M_S becomes attractive for any initial point $\xi(0) \in D$ if

$$\begin{array}{l} \Gamma \geq [1 - \| \Delta bb^{-1} sign(S) \|]^{-1}[\| \Delta a \| \\ \quad + \| \Delta bb^{-1}([I + M]^{-1}[-(L_p W^*)^T - \phi] - a) \| + \lambda] \end{array} \tag{11}$$

whenever $\| \Delta bb^{-1} sign(S) \| \neq 1$, with λ being a nonzero positive constant (see, in particular, (Khalil, 1996), Lemma 5.3, Chapter 5, p. 216).

[1] A system (2) is locally zero-state detectable if there exists a neighbourhood U of 0 such that, for all $x \in U$, $y(t) = h(x(t)) \equiv 0$ implies that $x(t) \to 0$ as $t \to \infty$. It is said to be locally zero-state observable if there exists a neighbourhood U of 0 such that, for all $x \in U$, $y(t) = h(x(t)) \equiv 0$ implies that $x(t) = 0$.

3 APPLICATION TO THE MODEL OF A MAGNETIC LEVITATION SYSTEM

In this work we consider the single-axis levitation system described in (Cho et al., 1993) (see Fig. 1). A force balance analysis leads to a state space representation of the system with state $x = (x_1, x_2) = (d - d_0, \dot{d} - \dot{d}_0)$ and control input $u = V_c - V_{c0}$ where d is the the distance of the ball from the reference line and V_c is the control voltage applied to the amplifier; d_0 and $\dot{d}_0$ are equilibrium points for a given nominal control voltage V_{c0}. The state space representation is given by

$$\dot{x} = f(x) + g(x)u, \quad y = h(x) = x_2 \tag{12}$$

with

$$f(x) = \begin{bmatrix} x_2 \\ \hat{b}(x_1)V_{c0}/m - g \end{bmatrix}, \; g(x) = \begin{bmatrix} 0 \\ \hat{b}(x_1)/m \end{bmatrix} \tag{13}$$

where m is the mass of the ball, g is the gravity and $\hat{b}(x_1) = 1/[a_1(x_1 - d_0)^2 + a_2(x_1 - d_0) + a_3]$, with a_1, a_2 and a_3 being real constant parameters. Since $L_g h(x) = \hat{b}(x_1)/m \neq 0$, the system has a relative degree $r = 1$. Thus, in the coordinates $\xi = (y, z) = (x_2, x_1)$, the levitation system (12),(13) takes the form

$$\begin{aligned} \dot{y} &= [\hat{b}(z)V_{c0}/m - g] + [\hat{b}(z)/m]u, \\ \dot{z} &= y. \end{aligned} \tag{14}$$

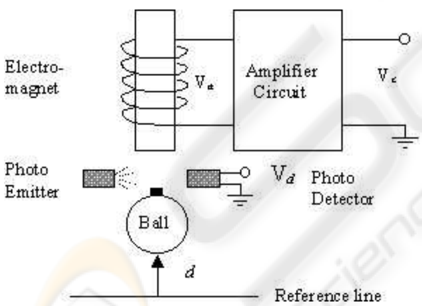

Figure 1: Schematic diagram of the magnetic levitation system.

The system's zero dynamics are then described by the first order differential equation $\dot{z} = f^*(z) = 0$ for which the quadratic positive definite function $W^*(z) = (1/2)z^2$ satisfies $L_{f^*} W^*(z) = 0$, and the system is weakly minimum phase. One then has that the feedback

$$u = \frac{m}{\hat{b}(z)}[-\frac{\hat{b}(z)}{m}V_{c0} + g - z + w] \tag{15}$$

makes the system (14) feedback equivalent to a C^2 passive system from w to y with a C^2 storage function $V = W^*(z) + (1/2)y^2$. Even more, the resultant closed-loop system is a loosless one because of the fact that $\dot{V} = yw$. One can also verify that this closed-loop system is zero-state observable, thus the additional feedback

$$w = -ky, \tag{16}$$

with $k > 0$, can make the origin $(y, z) = (0, 0)$ of the system

$$\dot{\xi} = \begin{bmatrix} \dot{y} \\ \dot{z} \end{bmatrix} = \bar{F}(\xi) = \begin{bmatrix} -k & -1 \\ 1 & 0 \end{bmatrix} \xi = \bar{A}\xi. \tag{17}$$

asymptotically stable.

In (Cho et al., 1993) it is noticed that the solenoid characteristics change with temperature, and a change of $\pm 20\%$ can appear in $\hat{b}(x_1)$ when the levitation system has been operated for a short period of time. Thus, the actual force-distance relationship, denoted by $b(d)$, may be expressed as

$$b(d) = \hat{b}(d) + \Delta\hat{b}(d) \tag{18}$$

where $\Delta\hat{b}(d)$ is an unknown modeling error which can be as high as 20% of $\hat{b}(d)$. The uncertain model associated to the nominal model (14) can then be written, also in the coordinates $\xi = (y, z)$, as

$$\begin{aligned} \dot{y} &= [\hat{b}(z)V_{c0}/m - g] + [\Delta\hat{b}(z)V_{c0}/m] \\ &\quad + ([\hat{b}(z)/m] + [\Delta\hat{b}(z)/m])u, \\ \dot{z} &= y. \end{aligned} \tag{19}$$

This is, the uncertainties are given by $\Delta a(y, z) = \Delta\hat{b}(z)V_{c0}/m$ and $\Delta b(y, z) = \Delta\hat{b}(z)/m$.

The switching function $S(y, v)$ is given by (3) with $v = -z + w$. Such a choice leads to the control law

$$u = u_{slid} = \frac{m}{\hat{b}(z)}[-\Gamma sign(S) - \frac{\hat{b}(z)}{m}V_{c0} + g - z + w], \tag{20}$$

with $\Gamma > 0$, which allows to reach the sliding surface in a finite time. By selecting the additional output feedback (16), we obtain the closed-loop system

$$\dot{\xi} = \bar{A}\xi + \bar{G}(\xi) \tag{21}$$

where

$$\bar{G}(\xi) = \bar{G}_1(\xi) = \begin{bmatrix} [-\Gamma sign(S) + \frac{\Delta\hat{b}(z)V_{c0}}{m} \\ + \frac{\Delta\hat{b}(z)}{\hat{b}(z)}[\frac{\hat{b}(z)V_{c0}}{m} + g - \Gamma sign(S) \\ -z - ky]] \\ 0 \end{bmatrix} \tag{22}$$

From the size of the modelling error $\Delta\hat{b}(z)$ one can verify, after some computations, that the uncertainty term $\bar{G}_1(\xi)$ satisfies the uniform bound $\| \bar{G}_1(\xi) \| \leq \delta$ for a constant δ. It then follows that the solution $\xi(t)$ of the uncertain system (21) is ultimately bounded for $t \geq 0$.

The magnetic levitation system described by equations (12),(13) was simulated together with the passivity based sliding mode controller (3),(20). The nominal value of the ball's mass m and the constant coefficients used in the force-distance relationship $\hat{b}(z)$ were selected as in (Cho et al., 1993), this is $m = 2.206\ gr$, $a_1 = 0.0231/mg$, $a_2 = -2.4455/mg$, $a_3 = 64.58/mg$. In fact, as it is noted in (Cho et al., 1993), the validity of the $\hat{b}(x_1)$ is constrained to the range of 35 mm and 48 mm. By choosing the nominal value of the control applied to the amplifier circuit to be $V_{c0} = 4.87\ volts$, we obtained the equilibrium point $(d_0, \dot{d}_0) = (38.2\ mm, 0\ mm/sec)$. The initial conditions of the magnetic levitation system were fixed to $x_1(0) = 44.2\ mm$ and $x_2(0) = 0\ mm/sec$, while the controller parameters were selected as $\Gamma = 10$ and $k = 2$. In order to diminish the effect of chattering due to the discontinuity of the sign function, a saturation function given by

$$sat(S) = \begin{cases} 1, & if \ \ S > \varepsilon \\ S/\varepsilon, & if \ \ -\varepsilon \leq S \leq \varepsilon \\ -1, & if \ \ S < -\varepsilon \end{cases}$$

with $\varepsilon > 0$, was used instead of the *sign* function. In order to evaluate the performance of the control scheme, a variation of 20% in the value of the function $\hat{b}(z)$ was introduced at $t = 7\ sec$ in all the simulations. The time closed-loop plot corresponding to the distance d is shown in Figures 2 for $\varepsilon = 0.001$. From this plot, we can notice that the distance of the ball to the reference line is always regulated to the equilibrium point $d_0 = 38.2\ mm$ with no overshoot.

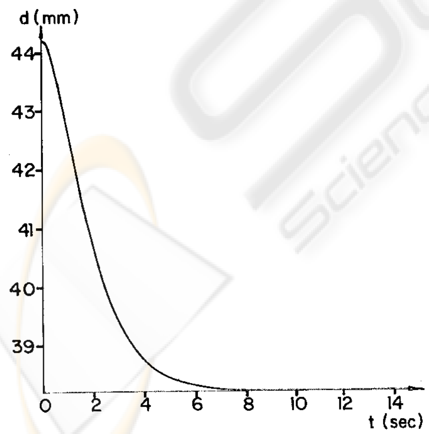

Figure 2: Closed-loop response of the distance, d; $\varepsilon = 0.001$.

4 CONCLUSIONS

In this paper, a passivity-based sliding mode controller design that allows to stabilize an uncertain nonlinear system has been presented. The proposed controller has also been applied to the model of a magnetic levitation system in order to regulate the height of a levitated ball around at one of its equilibria.

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