# SIGNAL DENOISING BASED ON PARAMETRIC HAAR-LIKE TRANSFORMS

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Keywords: Signal denoising, Wavelet, Threshold, Parametric transform.

Abstract: Orthogonal transforms have found considerable interest in signal denoising applications. Recently Parametric Haar-like Transforms (PHTs) have been introduced and shown to be efficient in image denoising and compression applications. PHT is such that it may be computed with fast algorithm in structure a similar to that of classical fast Haar transform and such that its matrix contains a predefined basis vector, called generating vector, as its first row. PHT may be adapted to the characteristics of the input signal or to its parts by a proper selection of the generating vectors. Possibility of adaptation to the input signal may, in principle, be significant source for performance improvement of transform based signal processing algorithms. In this paper, the capability of parametric Haar-like transforms, in 1-D signal denoising application is explored. A new PHT based post-processing algorithm for 1-D signal denoising is proposed, which may be combined with another denoising method in order to improve the quality of the output signal. Experiments were conducted where the basic wavelet thresholding based signal denoising method was complemented with the proposed post-processing algorithm. Simulation results illustrate significant performance improvement due to the use of the proposed algorithm.

## 1 INTRODUCTION

One of the most important problems in signal analysis is noise suppression or denoising where the problem is to find an estimate of a signal that was corrupted by a noise, e.g. additive Gaussian noise. Conventional denoising methods, in particular, Wiener filtering, are based on linear methods. Non-linear methods such as filtering in wavelet transform domain or wavelet-thresholding introduced by Donoho and Johnstone (Donoho *et al*, 1994), (Donoho, 1995), have also been shown be very efficient in signal denoising. Mostly, wavelet denoising was focused on statistical modeling of wavelet coefficients and optimal choice of threshold values (Grace Chang *et al*, 2000). In practice, the most commonly applicable are soft and hard thresholding functions. Recently, another function called customized thresholding was proposed (Yoon *et al*, 2004) that depends on a set of parameters and can be adapted to the input signal. The customized thresholding function combines advantages of traditional soft and hard thresholding, it can become either one of them by setting parameter values. The idea of customized thresholding is similar to that of semi-soft or firm shrinkage (Gao, 1996) and the non-negative garrote thresholding function (Gao, 1998). It was shown that custom thresholding function outperforms the traditional ones and it improves denoising results significantly.

Besides wavelets, different other orthogonal transforms (Egiazarian *et al,* 1999, Pogossova *et al*, 2003, Oktem *et al,* 1999) such as Fourier and DCT, Haar, combination of DCT and Haar, Tree-Structured Haar Transforms (THT), Generalized Lapped transforms and others have been proposed as useful tools in signal denoising applications. In many cases local transform based denoising methods are more profitable than wavelet based denoising methods which are applied to the whole signal.


Minasyan S., Egiazarian K., Astola J. and Guevorkian D. (2006).
SIGNAL DENOISING BASED ON PARAMETRIC HAAR-LIKE TRANSFORMS.
In *Proceedings of the International Conference on Signal Processing and Multimedia Applications*, pages 134-139
DOI: 10.5220/0001570401340139

Recently there has been considerable interest in constructing signal adapted systems for signal denoising, compression, and other applications (Egiazarian *et al,* 1999, Oktem *et al,* 1999, Pogossova *et al*, 2003). Thereby, parametric transforms with matrices described in a unified form involving a set of parameters are of interest nowadays. In this context parametric transform means a wide class of discrete orthogonal transforms (DOTs) that may include classical transforms and an infinite number of new transforms with the possibility to select the desired transform according to parameter values. A unified software/hardware tool can be used to implement the whole class of transforms with the possibility to adjust transform parameters. One can find various methods of synthesizing the parametric transforms in (Agaian *et al*, 1992).

In particular a family of parametric Haar-like transforms was introduced (Minasyan *et al*, 2001). Parametric Haar-like transform (PHT) is such a DOT that its matrix contains a desired basis function as its first row and such that it may be computed by a fast transform algorithm in structure similar to that of the classical fast Haar transform algorithm. Efficiency of using PHTs in image compression (Minasyan *et al,* 2005) as well as in image denoising (Minasyan *et al,* 2006) has motivated a study of their usefulness also in 1-D signal denoising.

The goal of this paper is to investigate the potential of PHTs in improving the performance of signal denoising. A new signal denoising algorithm is proposed where the corrupted signal is transformed into the transform domain with PHTs that are synthesized according to a signal estimate obtained, *e.g.* by wavelet denoising. The input noisy signal and its estimate are split into small sized windows. For every small window of the estimate one PHT is synthesized such that its matrix has the contents of the window as its first row. This PHT is applied to the corresponding window of the corrupted noisy signal. Next, the transformed coefficients are thresholded by customized threshold (Yoon *et al*, 2004) and transformed back into the original domain by the inverse PHT. Simulations were conducted on several test signals showing significant improvement in reducing the noise as compared to the pure wavelet denoising method.

The paper is organized as follows: Section 2 gives a brief introduction to PHT's. Background on wavelet thresholding methods is given in Section 3. Section 4 is the description of the proposed PHT based denoising algorithm. Section 5 describes simulations and results of experiments. The conclusion is given in Section 6.

## 2 PARAMETRIC HAAR-LIKE TRANSFORM (PHT)

Orthogonal transforms are widely used in signal/image processing, in particular, for signal denoising. In practice, different well-known fixed transforms with fast algorithms such as Discrete Fourier, Cosine, Sine, Haar, and Hadamard transforms are commonly used. Each of these transforms is suitable for a particular type of input signals but none of them performs sufficiently well on different types of input signals. Performance of fixed transforms, in particular, in signal denoising may be increased by making use of parametric, signal adaptive transforms. In a parametric transform based method different transforms may be synthesized and applied to different signals or even to different parts of a signal.

One way of synthesizing parametric transforms is based on unified representations of fast transform algorithms (see Agaian *et al*, 1992), (Minasyan e*t al*, 2001). Such unified representation is based on factorization of transform matrix of an arbitrary order $N$ as a product of block-diagonal sparse matrices and permutation matrices. Blocks of sparse matrices along with permutation matrices play the role of synthesis parameters. One can vary these parameters to synthesize an infinite number of different transforms all a priori possessing fast algorithms for their computation. It is also possible to adjust the parameters to design a transform matrix having some desired features. Good examples of synthesising such parametric transforms are the Haar-like, Hadamard-like transforms which have been proposed in (Minasyan *et al*, 2001) where, in particular, a method was proposed for constructing an orthogonal Haar-like or Hadamard-like transform matrix such that its first row is a predefined normalized vector $h=\left[h_0,\ldots,h_{N-1}\right]$ called generating vector. In (Minasyan *et al*, 2001), one can find the detailed description of constructing a parametric orthogonal Haar-like transform of order $N=2^m$, which involves the generating vector. The transform matrix has such a structure that its first row (column) is the generating vector while the rest of the basis functions are orthogonal to the first row. And, there is a fast algorithm for every Haar-like transform implementation similar to that of classical fast Haar transform algorithm.

It should be noted that the generating vector for the classical discrete Haar transform of order $N=2^m$ is the constant ($1x2^m$)-vector (with all components equal to each other). Using other generating vectors of arbitrary length and arbitrary component values an infinite number of Haar-like transforms, similar in structure to the Haar transform, may be synthesized.

Let us consider an example of synthesizing a Haar-like transform of order $N$=8 with the generating vector $\mathbf{h}=\left(1/\sqrt{204}\right)\cdot[1,2,3,4,5,6,7,8]$ on its first row. The matrix $H_8$ of the desired transform is supposed to be presented as:

$$H_8 = P^{(4)}H^{(3)}P^{(3)}H^{(2)}P^{(2)}H^{(1)}P^{(1)},$$

where we define $P^{(1)} = P^{(4)} = I_8$. Then, we define

$$H^{(1)} = \frac{1}{\sqrt{5}}\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}\oplus\frac{1}{5}\begin{bmatrix}3 & 4\\ 4 & -3\end{bmatrix}\oplus\frac{1}{\sqrt{61}}\begin{bmatrix}5 & 6\\ 6 & -5\end{bmatrix}\oplus$$
$$\oplus\frac{1}{\sqrt{113}}\begin{bmatrix}7 & 8\\ 8 & -7\end{bmatrix}.$$

With this matrix we obtain the result of the first stage:

$$\mathbf{x_1} = H^{(1)}\mathbf{h} = \left(1/204\right)\cdot\left[\sqrt{5},0,5,0,\sqrt{61},0,\sqrt{113},0\right]^T.$$

We then define the permutation matrix $P^{(2)} = P^{sh}(8)$ to be the perfect shuffle of order 8. Applying $P^{(2)}$ to $\mathbf{x_1}$ results in

$$P^{(2)}\mathbf{x}_1 = (1/\sqrt{204})\cdot\left[\sqrt{5},5,\sqrt{61},\sqrt{113},0,0,0,0\right]^T.$$

Now we define $H^{(2)}$ as:

$$H^{(2)} = \frac{1}{\sqrt{30}}\begin{bmatrix}\sqrt{5} & 5\\ 5 & -\sqrt{5}\end{bmatrix}\oplus\frac{1}{\sqrt{174}}\begin{bmatrix}\sqrt{61} & \sqrt{113}\\ \sqrt{113} & -\sqrt{61}\end{bmatrix}\oplus I_4.$$

Applying this matrix to $P^{(2)}\mathbf{x_1}$ yields:

$$\mathbf{x_2} = H^{(2)}P^{(2)}\mathbf{x_1} = 1/\sqrt{204}\left[\sqrt{30},0,\sqrt{174},0,0,0,0\right]^T.$$

Taking $P^{(3)} = P^{(sh)}(4)\oplus I_4$ and defining

$$H^{(3)} = \frac{1}{\sqrt{204}}\begin{bmatrix}\sqrt{30} & \sqrt{174}\\ \sqrt{174} & -\sqrt{30}\end{bmatrix}\oplus I_6,$$

we will find

$$\mathbf{x_3} = H^{(3)}P^{(3)}\mathbf{x_2} = [1,0,0,0,0,0,0,0]^T.$$

Substituting the defined matrices into the factorization of $H_8$ we obtain the desired matrix:

$H_8 \approx$

$$\frac{1}{\sqrt{204}}\cdot\begin{bmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8\\ 2.4 & 4.8 & 7.2 & 9.6 & -2.1 & -2.5 & -2.9 & -3.3\\ 5.8 & 11.7 & -3.5 & -4.7 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 7.4 & 8.8 & -5.6 & -6.4\\ 12.8 & -6.4 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 11.4 & -8.6 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 10.9 & -9.1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 10.7 & -9.4\end{bmatrix}$$

PHT is an input-adapted transform that may be adjusted to the input signal to improve the performance of fixed transforms in different applications. It has recently been shown that PHT may efficiently be used in image compression applications (Minasyan *et al,* 2005) and also in image denoising (Minasyan *et at,* 2006). This motivated us to study the PHT also in signal denoising.

## 3 WAVELET THRESHOLDING FUNCTIONS

Let $\mathbf{y}$=$\mathbf{x}$+$\mathbf{z}$ be a *(1xN)* input noisy signal, $\mathbf{x}$ be corresponding noiseless signal and $\mathbf{z}$ be Gaussian white noise with $N(0, \sigma^2)$.

Transform–based approach to noise reduction problem consists of following steps:

1. Transform the noisy signal into the corresponding transform domain;
2. Apply some thesholding to the resulting coefficients by zeroing out the coefficients lower than a certain amplitude;
3. Transform back to the original domain, performing the inverse transform.

One of the best known denoising methods is based on using a discrete wavelet transform at Step 1 (and corresponding inverse discrete wavelet transform at Step 3). The most commonly used thresholding functions at Step 2 are the hard-thresholding and soft-thresholding functions. Recently, the custom thresholding function (CTF) was introduced .

*a)* The hard-thresholding function selects (significant) wavelet coefficients that are greater than the given threshold λ and sets the others to zero:

$$f_h(\mathbf{x}) = \begin{cases}\mathbf{x}, \text{ if } |\mathbf{x}| \geq \lambda\\ 0, \text{ otherwise.}\end{cases} \quad (1)$$

The hard-thresholding function is discontinuous at threshold λ, e.g. at |**x**| = λ. That is why the artifacts, known as Gibbs phenomena, near the discontinuities appear in the denoised signal.

*b)* The soft-thresholding function, which is called also wavelet shrinkage function, shrinks the wavelet coefficients by threshold λ towards zero:

$$f_s(\mathbf{x}) = \begin{cases} \mathbf{x}-\lambda, & \text{if } \mathbf{x} \geq \lambda \\ 0, & |\mathbf{x}| < \lambda \\ \mathbf{x}+\lambda, & \text{if } \mathbf{x} \leq -\lambda. \end{cases} \tag{2}$$

*c)* The custom-thresholding function, (CTF):

$$f_c(\mathbf{x}) = \begin{cases} \mathbf{x} - sgn(\boldsymbol{x})(1-\alpha)\lambda, & \text{if } |\mathbf{x}| \geq \lambda \\ 0, & \text{if } |\mathbf{x}| \leq \gamma \\ \alpha\lambda\left(\frac{|\mathbf{x}|-\gamma}{\lambda-\gamma}\right)^2 \left\{(\alpha-3)\left(\frac{|\mathbf{x}|-\gamma}{\lambda-\gamma}\right)+4-\alpha\right\}, & \text{otherwise} \end{cases} \tag{3}$$

where γ is the cut-off value, below which the wavelet coefficients are set to zero, $0< \gamma < \lambda$ , and α is the parameter that decides the shape of the thresholding function $f_c(\mathbf{x})$, $0 \leq \alpha \leq 1$. This function is continuous at λ and can be adapted to the signal characteristics.The customized thresholding function may be considered as a linear combination of soft-thresholding function and hard thresholding function $f_c(\mathbf{x}) = \alpha \cdot f_h(\mathbf{x}) + (1-\alpha) \cdot f_s(\mathbf{x})$ that is continuous around the threshold λ. By varying the parameters α, γ and λ, it is possible to vary the CTF between the soft and hard thresholding functions or just to switch from one function to another one.

# 4 PHT-BASED POST ROCESSING ALGORITHM FOR SIGNAL DENOISING

The proposed denoising algorithm belongs to the general class of transform based denoising algorithms described in Section 3 where we use the Parametric Haar-like Transforms (PHT) as the invertible transform of Step 1 (and its inverse at Step 3). The idea of the proposed algorithm is to use the signal adapted PHTs instead of fixed orthogonal transforms in order to better distinguish between signal and noise in the transform domain. The main point consists in finding the suitable generating vectors for PHT synthesis. In an ideal case, if the generating vectors would be taken from the original uncorrupted signal, then the whole energy of the corrupted signal in the transform domain would be concentrated in only the first transform coefficient. By zeroing out all the rest coefficients would remove almost all the noise while would preserve the original signal untouched. However, since in reality the original signal is unknown, we may only use its estimate to form the generating vectors for PHT synthesis. As such an estimate we use the result of wavelet denoising. It has been shown that VisuShrink tends to oversmooth the signal, leading to loss of details and increase estimation error. Taking this into account we use the CTF.

The proposed algorithm (Wavelet-PHT denoising algorithm) may be described in four steps:

1. The input signal is denoised by wavelet transform to find an estimate of an uncorrupted signal.
2. The input signal is transferred window by window (which are non-overlapping) into the transform domain by PHTs that are synthesized on the base of the estimate of the original signal in the corresponding window obtained at Step 1. Thus, both the original signal and the estimate are divided into non-overlapping windows, for instance, of length 8. For each window of the estimate the PHT containing the corresponding window content as its first row is synthesized. Then, it is applied to the window of the original signal at the same location.
3. The customized thresholding is applied to each transformed window. The parameters α, λ and γ of the thresholding function were determined empirically.
4. Then, each thresholded window is transformed back with inverse PHTs. Note that the direct and inverse PHTs may be computed with fast algorithms in structure similar to that of Haar transform.

# 5 SIMULATION RESULTS

The proposed method was tested on different artificial test signals such as *Blocks, Bumps, HeavSine, Doppler, Cusp* of length 256 taken from Matlab's WaveLab toolbox. The signals were corrupted by additive Gaussian noise with SNR (signal-to-noise-ratio) 7 and then denoised by the proposed algorithm. In all the experiments bellow the Daubechies asymmetric wavelet with 8 vanishing moments and 8 decomposition levels was used at Step 1 of the algorithm. The results of the experiments were averaged over 30 runs.

Table 1 presents the results of one set of experiments. In this experiment soft and hard threshoding with the universal threshold $\lambda = \sigma\sqrt{2\log N}$ were used both in wavelet denoising and in PHT post-processing where $N$ is a length of a signal in the case of wavelet denoising and $N$=$w$=8 is a window size in PHT post-processing. The second and third columns correspond to soft thresholding of both wavelet coefficients and PHT coefficients. The fourth and fifth columns correspond to the case of hard thresholding. One can see that in the most of the cases the proposed method (third column) reduces significantly noise in the sense of MSE comparing with MSE of soft thresholded estimate.

Table 1: Comparative results of MSE averaged over 30 runs: denoising with the universal threshold $\lambda = \sigma\sqrt{2\log N}$ .

| Signal | Soft | PHT soft | Hard | PHT hard |
|---|---|---|---|---|
| Blocks | 4.18 | 0.96 | 1.33 | 0.82 |
| Bumps | 4.62 | 0.94 | 1.16 | 0.76 |
| Doppler | 2.34 | 0.65 | 0.67 | 0.63 |
| Cusp | 0.70 | 0.18 | 0.29 | 0.39 |
| Heavi Sine | 1.09 | 0.24 | 0.35 | 0.41 |

Table 2 presents the results of another set of experiments where the soft and customized thresholding functions were applied with empirically optimized thresholding parameters in order to explore potential of the proposed method.

The second column of Table 2 represents the MSEw values of wavelet denoising using soft thresholding with optimized threshold values $\lambda_w$ given in the fourth column of the table.

In the third column the values MSEwp obtained after PHT-based post-denoising are given. Again window size was chosen $w$=8 but now CTF with optimized parameter values $\lambda_{wp}$ (see column 5), and fixed $\alpha = 0.97$ and $\gamma = 0.9 \cdot \lambda$ were used (these values were experimentally found as the optimal for all the experimented signals).

Table 2: Comparative results of MSE averaged over 30 runs: denoising with empirically found optimal thresholds.

| Signal | MSEw | MSEwp | $\lambda_w$ | $\lambda_{wp}$ |
|---|---|---|---|---|
| Blocks | 0.84 | 0.57 | 0.9 | 6 |
| Bumps | 0.81 | 0.47 | 0.7 | 5 |
| Doppler | 0.63 | 0.48 | 1.3 | 3 |
| Cusp | 0.50 | 0.19 | 2.7 | 7 |
| Heavi Sine | 0.88 | 0.29 | 3 | 3 |

The experiments have shown that for each signal the optimal ranges of threshold values $\lambda_{wp}$ which have been used in proposed denoising method for signals *Blocks*, *Bumps*, *Doppler*, *Cusp* and *HeaviSine* are 4~7.5, 5~7, 2.5~3.5, 3~8 and 3~3.8, respectively.

Besides, for each signal the confidence intervals (CI) for MSEwp values were obtained during the experiments. In particular, CI for *Blocks* is [0.42, 0.69], CI for *Bumps* is [0.38, 0.57], CI for *Doppler* is [0.32, 0.66], CI for *Cusp* is [0.13, 0.30] and CE for *HeaviSine* is [0.20, 0.53].

It can be seen that by applying the proposed denoising the remaining noise is reduced significantly in the sense of MSE.

Fig. 1 illustrates the performance of the proposed algorithm on the example of the signal *Bumps*. One can see that the result of the proposed algorithm (Fig.1,d) is significantly closer to the uncorrupted signal (Fig.1,a) as compared to the result of the pure wavelet denoising (Fig.1,c). Performance improvement is similar for other signals as well.

## 6 CONCLUSION

A method based on parametric Haar-like transforms for improving wavelet denoising is presented here. It employs a parametric Haar-like transform that can be custom designed for each part (window) of the signal. The simulation results verify the efficiency of the proposed method. Similar algorithm may be extended also to image denoising application.

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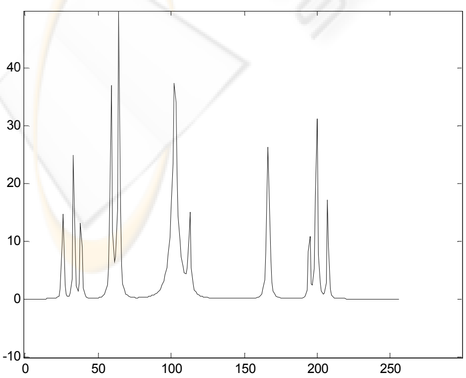
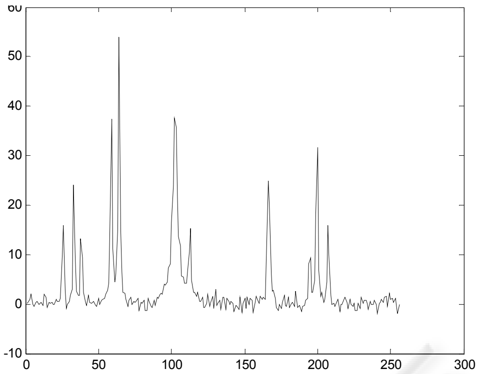


a)



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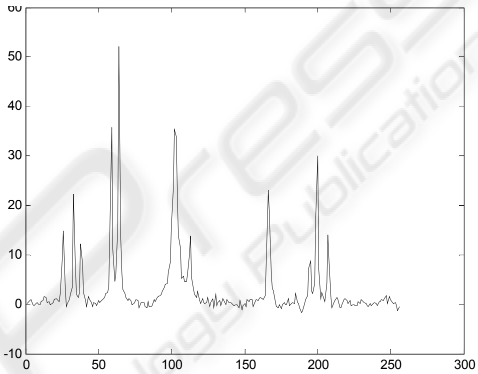

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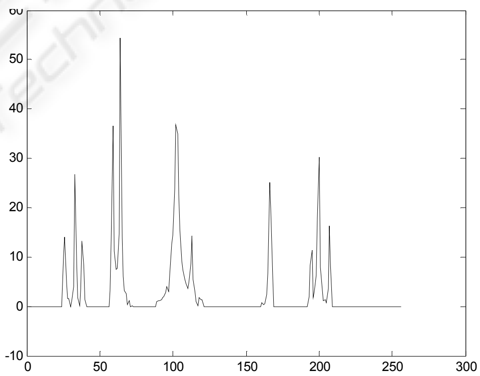

d)

Figure 1: Denoising of signal *Bumps*: a) Original signal, b) noisy signal (SNR=7dB), c) wavelet denoised signal (soft thresholded, MSEw=0.7111), and d) signal denoised by proposed method (thresholded by CTF, *w*=8, MSEwp=0.3735).