PROCESSING OF NON-STATIONARY SIGNAL USING LEVEL-CROSSING SAMPLING

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Abstract: The spectral characteristics of multimedia signals typically vary with time. Preferably, the sampling density of them would comply with instantaneous bandwidth of signal. The paper discusses the level-crossing sampling principle, which provides such capability for analog-to-digital conversion. As the captured samples are spaced non-uniformly, the appropriate digital signal processing is required. The non-stationary signal is characterized by time-frequency representation. Its classical approaches are inspected for applicability to analyze the data obtained by level-crossing sampling. Several enhancements of short-time Fourier transform approach are proposed, which are based on the idea to minimize the reconstruction error not only at sampling instants, but also between them with the same accuracy. Additional benefits are gained if the instantaneous spectral range of analysis is complied with local sampling density: artifacts are removed, complexity of calculations is decreased. The performance of algorithms is demonstrated by simulations. Presented research can be attractive for clock-less designs, which receive now an increasing interest. Their promising advantages can play a significant role in future electronics' development.

1 INTRODUCTION

Conventional digital signal processing techniques often consider the stationarity of a signal within a frame of analysis. It is assumed that the statistical characteristics of signal do not change with time. The concept of stationarity provides the possibility of fixing the sampling rate (it should be at least twice as high as the maximum signal frequency), as well as of constructing effective processing methods, for example the Discrete Fourier transform (DFT). However, natural signals typically are time-varying, and they can be a mixture of events localized both in time and frequency (Akay, 1998).

Intuitively speaking, the non-stationarity of a signal should be reflected in the process of analog-to-digital (A/D) conversion. For example, let us inspect a signal with high instantaneous frequency regions and low instantaneous frequency in other regions. It is more efficient to sample the low frequency regions at a lower rate than the high frequency regions. Consequently, with appropriate non-equidistantly spaced samples one might approximate a signal with fewer samples per interval than in the uniform sampling case, where sampling frequency is defined taking into account only the highest signal component. Two conclusions follow: non-uniform sampling is the natural choice for the discrete representation of a non-stationary signal, and the non-uniformity of sampling process has to be caused by the local properties of signal.

The work presented in this paper is based on the idea of abandoning traditional clock-driven A/D conversion and the uniform digital signal processing, which typically follows it. Instead of that, a clock-less structure of data processing system is suggested, where the A/D conversion is signal-driven. To illustrate the difference, the processing chains of both approaches are illustrated in the Fig.1. Let us emphasize the key benefits of the asynchronous electronics: lower power consumption, absence of the clock screw, reduced heat elimination, lower EMI, automatic adaptation to physical properties, etc. (Hauck, 1995). The popular types of signal-dependent sampling are based on zero-crossing, reference signal crossing, level-crossing or send-on-delta concepts. Each of them has its own advantages and limitations, however joint features are: the signal samples can be spaced non-uniformly, local sampling density depends on local properties of signal, and it is impos-

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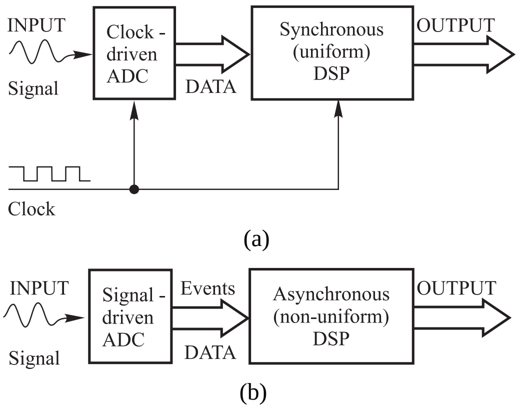

Figure 1: Structures of DSP system based on different paradigm: synchronous (a), asynchronous (b).

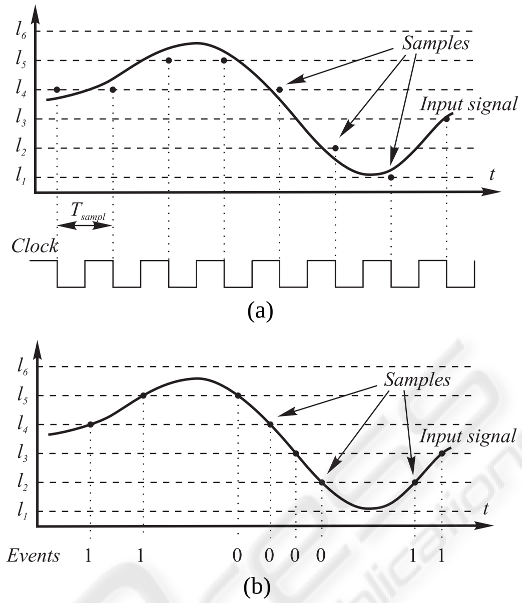

Figure 2: Analog-to-Digital conversion: clock-driven (a) vs. signal-driven level-crossing sampling case (b).

sible to determine the sampling time instants in advance. The paper discusses digital signal processing if the level-crossing sampling scheme is used to capture digital data from a continuous time signal.

2 LEVEL-CROSSING SAMPLING

The principle of uniform sampling is illustrated in Fig.2a: sampling is driven by an external clock with fixed period T_{sampl} that gives the equidistantly spaced samples. The level-crossing sampling (LCS) scheme is based on the principle that samples are captured when the continuous time input signal crosses predefined levels. Typically, the quantization levels are uniformly disposed along the amplitude range of the signal as is shown in Fig.2b.

Such a sampling strategy is not new and has been known at least since the late 1950s (Ellis, 1959). Various terms are used to name it: event-based sampling, level-crossing sampling, magnitude-driven sampling, and sometimes, sampling in the amplitude domain. The variety of existing terminology shows that it is really a generic concept adapted to a broad spectrum of technology and applications. It has been shown that level-crossing sampling has several interesting properties and is more efficient then traditional sampling in many respects (E. Allier and Renaudin, 2003). Classical A/D conversion implements clock-driven sample-and-hold (S/H) operation, which is followed by quantization operation. Considering an ideal clock and an ideal S/H, anyway there is imprecision of conversion due to the limited number of quantization bits L. The Signal-to-(quantization)Noise Ratio (SNR) of classical ADC can be expressed as

$$SNR_{dB} = 1,76 + 6,02L, \quad (1)$$

and it depends only on the resolution of the converter.

In the level-crossing based A/D converter, since a sample is taken only when a level is crossed, the amplitude value of the sample is exact. Due to the fact that samples are spaced non-equidistantly, the application of LCS often requires that the time instant of the sample also be known. In practice, the time interval is measured by a timer that quantizes the time with certain resolution T_{timer}. The SNR in this case can be estimated as (E. Allier and Renaudin, 2003):

$$SNR_{dB} = 10log\left(\frac{3P_x}{P_{x'}}\right) + 20log\left(\frac{1}{T_{timer}}\right), \quad (2)$$

where P_x is power of the random input signal, and $P_{x'}$ is power of it's derivative. In this case SNR does not depend on the number of quantization levels, but depends on the properties of the input signal and on the precision of the timer. Signal-to-noise ratio can be improved simply by decreasing T_{timer}.

The goal of the proposed paper is to explore the use of the level-crossing sampling technique for analysis of a non-stationary signal. In this context, the evaluation of the local sampling density can play a significant role, because it is connected with the local statistical characteristics of a signal. If a signal is changing rapidly, the samples are spaced closer, and conversely - if a signal is varying slowly, the samples are spaced sparsely. The variability of waveform is linked with spectral content, and thereby the local sampling density can be used to estimate the instantaneous maximum frequency of signal.

If the input signal is single sinusoid

$$x(t) = A\sin(2\pi f_0 t + \varphi), \quad (3)$$

where A is the amplitude, f_0 - the frequency and φ - the initial phase, the sampling density can be expressed as

$$\sigma = 2R_{\Delta} f_0, \tag{4}$$

where R_{Δ} is the total number of different levels crossed by the signal.

Determining the sampling density of a broadband process is not as elementary as for a mono-harmonic signal. Analytically it is investigated for band-limited Gaussian process with zero mean and constant spectral density

$$P_x(f) = \begin{cases} S & |f| \leq f_{UP} \\ 0 & \text{otherwise} \end{cases}. \tag{5}$$

The expected number of level l_0 crossings per time unit can be expressed as (Mark and Todd, 1981)

$$\mathrm{E}[\sigma_{l_0}] = \frac{2f_{UP}}{\sqrt{3}} \exp\left(\frac{-l_0^2}{4Sf_{UP}}\right). \tag{6}$$

To calculate the sampling density, it is necessary to sum up the sampling instants of all the quantization levels l_k

$$\mathrm{E}[\sigma] = \sum_{k=1}^{2^L-1} \sigma_{l_k}. \tag{7}$$

One more of the main parameters describing the sampling process is the time interval between two adjacent samples $\Delta t_n = t_{n+1} - t_n$. The mean value of the interval is tied with sampling density as $\overline{|\Delta t_n|} = \frac{1}{\sigma}$. The exact Δt_n values can be estimated analytically only for special cases, i.e., for the mono-harmonic signal (3). If the signal crosses the level l_k at the time instant t_n and the level l_{k+1} at the time instant t_{n+1}, the Δt_n can be calculated as

$$\Delta t_n = \frac{1}{2\pi f_0} \left| \left(\arcsin\left(\frac{l_k}{A}\right) - \arcsin\left(\frac{l_{k+1}}{A}\right)\right)\right|. \tag{8}$$

Around extremes the signal crosses the same level twice and the distance between crossings is

$$\Delta t_n = \frac{1}{\pi f_0} \left(\frac{\pi}{2} - \arcsin\left(\frac{|l_{min \mid max}|}{A}\right)\right). \tag{9}$$

If Δt_n cannot be estimated analytically, the upper and lower bounds of time interval can be evaluated based on the signal parameters. The minimum distance is determined as

$$T_{min} \geq \frac{\Delta l_{min}}{max(|x'(t)|)}, \tag{10}$$

where Δl_{min} is the minimal distance between two quantization levels, and $x'(t)$ is first derivative of the signal. The case, where the signal crosses the same level twice, is distinct, because $\Delta l = 0$ and T_{min} can reach zero. The upper bound of Δt_n is infinity, because the level-crossing sampling might not be triggered if the signal waveform is located between two consecutive quantization levels. To avoid this, the distance between quantization levels has to be less than the amplitude of the signal.

In addition, the following facts should be noted - if a signal waveform has some regularities, the sample flow has the same regularities as well. This effect often leads to a problem that the methods, which are derived for deliberately non-uniform sampling, do not always work satisfactorily for a particular case - level-crossing sampling, which provides signal-dependent non-uniform data. The level-crossing based analog-to-digital conversion is asynchronous in the sense that it does not have the clock that determines the positions of samples. That leads to a drastic change in the standard signal and data processing and initiates a new research area - asynchronous signal processing.

3 CLASSICAL TFRs AND NON-UNIFORM SAMPLING

The non-stationary signal is characterized by time-frequency representation (TFR). As the signal samples captured according to the level-crossing principle are spaced non-uniformly, the appropriate digital signal processing is required. In this section, the applicability of classical TFR approaches to analyze LCS data is inspected. The time-frequency representation is characterized by points on a time-frequency gram. For practical applications it is assumed that a finite duration Θ of bandlimited to Ω signal is observed. The traditional approaches for TFR calculations are based on Short-time Fourier transform (STFT) (Gabor, 1946), joint time-frequency distribution (Cohen, 1995) and wavelet transform (WT) (Chui, 1992).

3.1 Short-time Fourier Transform

The classical method for analyzing non-stationary signals is short-time Fourier transform. It was proposed by Gabor (Gabor, 1946). STFT is based on the well known Fourier transform. The basic idea of STFT is to introduce a time window, which is moved along the signal, and in such a way the time indexed spectrogram of $x(t)$ is defined as

$$STFT(t,f) = \int_{-\infty}^{\infty} x(\tau) w^*(t-\tau) \exp(-j2\pi f\tau) d\tau, \tag{11}$$

where $w(t)$ is a time window and $\cdot^*$ denotes the complex conjugates.

In the case of finite number of discrete samples $x_n = x(t_n)$, $n = \overline{1,N}$ (N is a number of samples

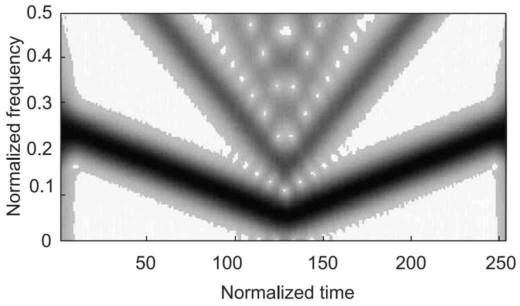

Figure 3: STFT based time-frequency representation of test-signal sampled by crossing 7 levels.

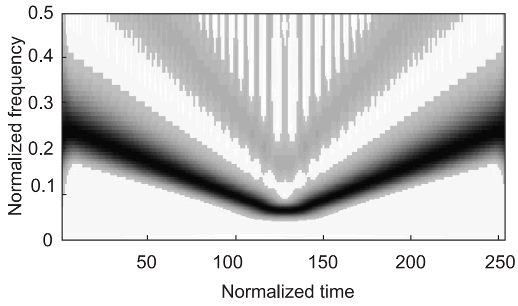

Figure 4: WT based time-frequency representation of test-signal sampled by crossing 7 levels.

within time interval Θ), the STFT based TFR on the uniformly spaced time-frequency grid with frequency step $\frac{1}{\Theta}$ and time step $\frac{1}{2\Omega}$ can be calculated as

$$TFR_{STFT}(k,m) = \left| \sum_{n=1}^{N} x_n w^*(k/2\Omega - t_n) \exp(-j2\pi t_n m/\Theta) \right|. \quad (12)$$

The expression (12) uses the general form of DFT, in which the restriction, that requires the uniform spacing of samples $x_n = nT$, can be ignored. To examine what happens if this expression is used for analysis of level-crossing samples, the single chirp (parameters of it will be described in the Section 6) is chosen as a test-signal. The Fig.3 illustrates the fact that in addition to the true component, spurious components appear at the higher odd harmonics. These artifacts are due to the use of LCS approach for signal with regularities in the waveform. The additional source of artifacts can be the absence of the orthogonality of transformation functions $\exp(-j2\pi t_n m/\Theta)$ if t_n are not placed uniformly.

A well-known problem inherent in STFT is the inverse relationship between time and frequency resolutions. Extension of the window's $w(t)$ length improves the frequency resolution, but at the same time degrades the temporal selectivity. To overcome this difficulty of short time Fourier transform, alternative methods of time-frequency analysis have been developed. The two most popular of them are a wavelet transform and a Wigner-Ville distribution.

3.2 Wavelet Transform

The wavelet transform of a continuous-time signal $x(t)$ is defined as

$$WT(t,a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(\tau) h^* \left(\frac{t-\tau}{a} \right) d\tau, \quad (13)$$

where a is the scaling factor and $h(t)$ is the so-called analyzing wavelet. The time-frequency version is obtained by making the substitution $a = f_0/f$. The analysis can be viewed as a filter bank comprising bandpass filters with bandwidths proportional to frequency. The multi-resolution nature of wavelet analysis leads to some limitations. Wavelet transform uses a scaling profile such that frequency resolution decreases at high frequencies, and temporal resolution decreases at low frequencies. While this choice of scaling leads to nice mathematical structures and algorithms, there is no physical reason to assume that it corresponds to natural structure behavior. For discrete WT, in order to get the best performance of analysis, the time- and scale-sampling grid often should be considerably over-sampled, that introduces the redundancy in the TFR.

The general form of time-frequency representation based on discrete wavelet transform can be expressed as

$$TFR_{WT}(k,m) = \left| \frac{1}{\sqrt{f_0\Theta/m}} \sum_{n=1}^{N} x(t_n) h^* \left(\frac{k/2\Omega - t_n}{f_0\Theta/m} \right) \right|. \quad (14)$$

Such a notation enables the processing of both uniformly and non-uniformly sampled data. The nice mathematical feature of WT for equidistantly spaced samples states: for any k and $a = 2^m$ $(k, m \in \mathbb{Z})$ the $\{h(t_h)\}_{(m,k)}$ is a subset of one discreet wavelet, which is uniformly sampled at the sampling frequency of the signal. In the case of non-equidistantly spaced samples this property is lost, because the values of wavelet $h(t)$ have to be calculated at different points set $\{t_h\}_{(m,k)}$ for each scaling factor a (or frequency of analysis $f = f_0/a$). Due to this fact, the computation complexity of WT in the non-uniform sampling case considerably exceeds the complexity of the uniform sampling case.

The time-frequency representation obtained by (14), if level-crossing sampling is used, is demonstrated in Fig.4. It shows the reduction of the temporal resolution in the low frequency region and diminished spectral resolution in the high frequency region. The additional artifacts appear as well.

3.3 Wigner-Ville Distribution

Time-frequency analysis, based on the use of Wigner-Ville function, is defined as

$$WVD(t,f) = \int_{-\infty}^{\infty} x\left(t+\frac{\tau}{2}\right) x^*\left(t-\frac{\tau}{2}\right) \exp(-j2\pi f\tau)d\tau. \quad (15)$$

It provides high-resolution representation in time and in frequency for mono-component signals. However, if the signal consists of several subcomponents, additional interference or cross-terms appear due to the quadratic nature of kernel and non-linear properties of it. In order to mitigate this deleterious effect, a variety of modified kernels have been introduced. One way to remove the interference is by smoothing the time-frequency plane, but this will be at the expense of decreased resolution in both time, and frequency. A promising approach of how to suppress cross-terms and improve resolution is the use of signal-dependent kernels (Baraniuk and Jones, 1993).

A discrete form of the Wigner-Ville distribution (WVD) can be expressed as

$$TFR_{WVD}(k,m) = 2\left|\sum_{n=1}^{N} x(k/2\Omega + t_n)x^*(k/2\Omega - t_n) \cdot \exp(-j4\pi t_n m/\Theta)\right|. \quad (16)$$

The necessity of knowing signal values at time instants $\tau + t_n$ and $\tau - t_n$ for all $n = \overline{1,N}$ leads to the fact that the expression (16) can be used only for uniform and specifically regular sampling series. Therefore it is impossible to use the WVD approach for processing data captured by the level-crossings.

4 ENHANCEMENTS OF DFT

It can be concluded from the discussion above, that the most useful approach for practical applications using the level-crossing sampling is based on STFT. However it has to be enhanced and adjusted to the LCS to suppress the presence of spurious components.

The key operation of discrete STFT is the DFT algorithm, which is applied to the windowed signal samples. Thus the STFT enhancement can be reduced to the development of DFT-like methods, which take into account LCS features. The level-crossing sampling principle provides not only samples at certain events, but also the rule that the signal between two sampling instants does not cross any quantization level. This information can be exploited in the processing. The proposed idea is to minimize the error between the original signal and that reconstructed by the Fourier series, not only at sampling time instants, but also between them with the same accuracy. The problem lies in the fact that the reconstruction error can be obtained only at the time moments in which the signal samples are known.

Basing on the Fourier series the signal waveform can be reconstructed from its spectral estimates by the following formula

$$\hat{x}(t) = \sum_m X_m \exp(j2\pi t f_m), t \in [0,\Theta], \quad (17)$$

where X_m are Fourier coefficients at frequencies $f_m = m/\Theta$. If the original continues-time signal is $x(t)$, the reconstruction error is

$$\varepsilon(t) = x(t) - \hat{x}(t), \quad (18)$$

and the following minimization task

$$\int_0^{\Theta} |\varepsilon(t)|^2 \, dt \to \min \quad (19)$$

can be established on the understandings, that the signal values are known only at sampling points, and the reconstructed signal is defined by (17). The problem (19) has to be resolved with respect to the coefficients $\{X_m\}$. Two approaches can be considered: the first one is based on setting up the continuous time signal by interpolation of known samples, while the second approach, which minimizes the continuous time reconstruction error, is based on the interpolation of error samples.

4.1 Signal Interpolation

If signal samples $\{x_n\}$ are interpolated within the time interval $[0,\Theta]$, the problem (19) can be rewritten as

$$\int_0^{\Theta} \left|\tilde{x}(t) - \sum_m X_m \exp(j2\pi t f_m)\right|^2 dt \to \min, \quad (20)$$

where $\tilde{x}(t)$ is the interpolated signal. To find the minimum, all the individual derivatives of X_m have to be considered as being equal to zero. Taking into account that $\{\exp(j2\pi f_m t)\}$ is a set of orthogonal functions into interval $[0,\Theta]$ if frequencies $f_m = m/\Theta$, after some algebra the following formula for $X_m^{(x)}$ ($^{(x)}$ denotes that signal samples are interpolated) estimation can be obtained:

$$X_m^{(x)} = \frac{1}{\Theta}\int_0^{\Theta} \tilde{x}(t)\exp(-j2\pi t f_m)dt. \quad (21)$$

The expression (21) is similar to the formula for the calculation of the Fourier series coefficients for signal $\tilde{x}(t)$.

Signal interpolation can easily be done by connecting the samples with polynomials $p_n^k(t)$ of order k as $\tilde{x}(t) = \sum_n p_n^k(t)$, or a band-limited interpolation can be performed using a sum of time-shifted sinc functions.

If signal samples are interpolated with zero-order polynomials (piece-wise constant line changing value at midpoints between samples):

$$\begin{aligned} X_m^{(x0)} &= \sum_{n=1}^{N} x_n \int_{\frac{t_n+t_{n-1}}{2}}^{\frac{t_n+t_{n+1}}{2}} \exp(j2\pi f_m t)dt \\ &= \frac{j}{2\pi f_m} \sum_{n=1}^{N} x_n \exp(j2\pi f_m t_n) \\ &\quad \cdot (1 - \exp(-j2\pi f_m \Delta t'_n)), \end{aligned} \tag{22}$$

where $\Delta t'_n = (t_{n+1} - t_{n-1})/2, t_0 = 0, t_{N+1} = \Theta$.

For piece-wise linear interpolation the polynomial $p_n^1(t) = \alpha_n(t - t_n) + x_n$ can be used, where $\alpha_n = \Delta x_n / \Delta t_n$, $\Delta x_n = x_n - x_{n-1}$, $\Delta t_n = t_n - t_{n-1}$, which gives:

$$\begin{aligned} X_m^{(x1)} &= X_m^{(x0)} + \frac{1}{(2\pi f_m)^2} \sum_{n=1}^{N} \alpha_n \exp(j2\pi f_m t_n) \\ &\cdot (1 - \exp(-j2\pi f_m \Delta t_n)) + \frac{j}{2\pi f_m} \sum_{n=1}^{N} \alpha_n \Delta t_n \\ &\cdot \exp(j2\pi f_m t_n) \exp(-j2\pi f_m \Delta t_n). \end{aligned} \tag{23}$$

Band-limited interpolation of samples can be described as:

$$\tilde{x}^{(sinc)}(t) = \sum_{k=0}^{K} c_k \mathrm{sinc}(2\Omega t - k). \tag{24}$$

In this case DFT transform gives:

$$X_m^{(sinc)} = \sum_{k=0}^{K} c_k \exp(-j\pi f_m k/\Omega), \tag{25}$$

where c_k are coefficients that can be found from a linear equation system

$$x_n = \sum_{k=0}^{K} c_k \mathrm{sinc}(2\Omega t_n - k). \tag{26}$$

Such an approach, besides the complexity of DFT, also requires the solution of linear system with N equations and with $K+1$ unknowns. Interpolation by sinc functions can be effectively done for the stationary signal and if the gaps between samples do not exceed $1/2\Omega$. In this case the appropriate width of function can be fixed. However, for the non-stationary signal, the sinc functions should be stretched and time-shifted in accordance with instantaneous signal bandwidth and local sampling density.

4.2 Error Interpolation

Like the interpolation of signal samples, the continuous-time reconstruction error function $\tilde{\varepsilon}(t)$ can be constructed from its values $\varepsilon_n = x_n - \hat{x}_n$, and the problem (19) can be interpreted as minimization of area under the function $|\tilde{\varepsilon}(t)|^2$.

Using zero-order polynomial interpolation the minimization task becomes:

$$\sum_{n=1}^{N} \left| x_n - \sum_{m} X_m^{(\varepsilon 0)} \exp(j2\pi f_m t_n) \right|^2 \Delta t'_n \to \min. \tag{27}$$

After the derivation and some algebra the solution can be expressed in the matrix form:

$$\mathbf{X}^{(\varepsilon 0)} = \mathbf{\Psi x}(\mathbf{\Phi \Psi}^T)^{-1}, \tag{28}$$

where $\phi_{mn} = \exp(j2\pi f_m t_n)$, $\psi_{mn} = \phi_{mn} \Delta t'_n$, and $\cdot^T$, $\cdot^{-1}$ denotes the transpose and inverse operation of matrix respectively.

The first-order polynomial interpolation of error samples provides the problem, which looks like a sum of two zero-order interpolation tasks:

$$\frac{1}{2} \left(\sum_{n=1}^{N-1} |\varepsilon_n|^2 \Delta t_n + \sum_{n=2}^{N} |\varepsilon_n|^2 \Delta t_{n-1} \right) \to \min. \tag{29}$$

The solution is similar to the expression (28):

$$\mathbf{X}^{(\varepsilon 1)} = (\mathbf{\Psi}'\mathbf{x}' + \Psi'' x'')(\mathbf{\Phi}'\mathbf{\Psi}'^T + \mathbf{\Phi}''\mathbf{\Psi}''^T)^{-1}, \tag{30}$$

where $\mathbf{\Phi}'$, $\mathbf{\Psi}'$, $\mathbf{x}'$ and $\mathbf{\Phi}''$, $\mathbf{\Psi}''$, $\mathbf{x}''$ matrices are formed from $\mathbf{\Phi}$, $\mathbf{\Psi}$, $\mathbf{x}$ by using indexes $n' = \overline{1, N-1}$ and $n'' = \overline{2, N}$ respectively.

5 PROPOSED APPROACH

The proposed approach is based on the same time windowing principle as in the STFT case. However, instead of general DFT more sophisticated methods are used, which have been described in the Section 4. Enhanced algorithms have increased mathematical complexity, particularly the error interpolation case, because the solving of linear system with N equations and M unknowns is required. M represents a number of frequencies in the Fourier series. The equation system can be solved correctly, if the number of samples is equal or greater than the number of frequencies. The greater the N/M ratio, the higher the stability of the solution. It has been shown, that, using the level-crossing sampling approach, the number of samples depends on the signal properties. Relationships between the local sampling density and the instantaneous upper spectral frequency of signal have been derived. Performing the time-frequency analysis, these interdependencies can be exploited from an other point of view. The bandwidth of analysis can be limited using information about the local sampling density. The number of frequencies, as well as the dimensions of matrices vary with the time. For simulations, which will follow in the next section, the analysis bandwidth is selected as a minimum value of two frequencies: total bandwidth Ω or highest signal frequency estimated from the sampling density:

$$\Omega_a(t) = \min\left(\frac{N_w(t)}{2R_\Delta T_w} + \Omega_\Delta, \Omega\right), \qquad (31)$$

where $N_w(t)$ is the number of signal samples in the time interval with length T_w, and Ω_Δ is necessary to ensure the coverage of actual signal bandwidth. The frequencies of analysis are $f_m = m/\Theta : |f_m| \leq \Omega_a$.

6 SIMULATION RESULTS

The computer simulation has been carried out to demonstrate the performance of approaches, which have been developed for time-frequency analysis of data captured by level-crossings. As a test-signal a chirp has been selected, which in the first half of observation diminishes from middle frequency to low frequency region (down to the normalized frequency 0.05), while in the second half rises back to the normalized frequency 0.25. Seven quantization levels have been placed equidistantly to cover the input range of the test-signal. The observation time is $\Theta = 256$, and 536 samples in total are obtained.

Time-frequency representations calculated by STFT and WT approaches have been already illustrated in the Section 3 (see Fig.3 and Fig.4). Let us inspect the enhanced algorithms, which are based on interpolation (expressions (22) and (23)). Fig.5 shows TFR obtained in the case, where time windowed test-signal samples are interpolated by zero-order polynomials. The spurious components are attenuated, however the presence of them is still observable. If the first-order polynomials are used for interpolation, the result is slightly improved, but the complexity of calculations is higher. Fig.6 shows time-frequency representation of test-signal if error samples are interpolated by zero-order polynomials (28). The spurious harmonics are completely suppressed, however other artifacts are observable in the time region, where the frequency of chirp is low. Reduction of the instantaneous frequency results in the decreasing of local sampling density. As the grid of temporal analysis and length of time window $w(t)$ are fixed, the number of the significant samples can fall below the number of frequencies of analysis. Such a situation causes the problem of matrix inversion quality and leads to the appearance of artifacts. The use of interpolation by first-order polynomials does not have an impact on this effect.

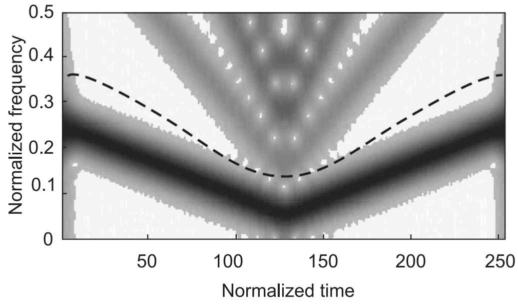

Figure 5: STFT approach in combination with zero-order interpolation of signal samples (dashed line shows instantaneous bandwidth estimate from local sampling density).

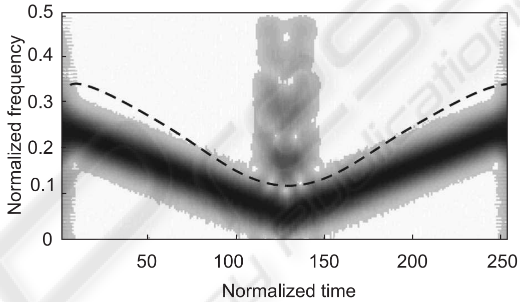

Figure 6: STFT approach in combination with zero-order interpolation of error samples (dashed line shows instantaneous bandwidth estimate from local sampling density).

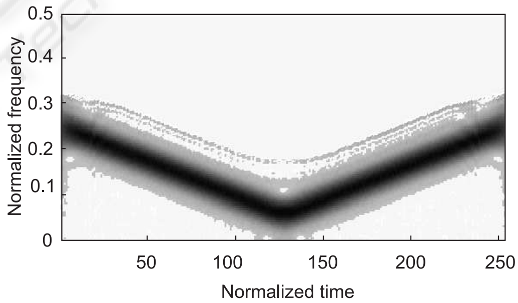

Figure 7: TFR of test-signal if approach of varying the range of analysis is used.

To improve the quality of TFR in the region, where sampling density is low, the bandwidth of analysis has been cut down according to the expression (31). The estimated bandwidth of signal is illustrated in Fig.6 and Fig.5 by dashed line ($\Omega_\Delta = 0.1$). The coherence between the sampling density of a signal and the frequency range of an analysis gives several benefits - the stability of the algorithm is increased, the complexity of calculations is decreased and the presence of artifacts is eliminated. Fig.7 demonstrates the time-frequency representation obtained by the algorithm based on the expression (28) in the case, where the number of analysis frequencies are varied according to the sampling density. The chirp can be tracked without any presence of artifacts.

7 CONCLUSION

The processing of non-stationary signal using level-crossing sampling approach has been investigated. On the one hand, such a sampling strategy provides several interesting properties - signal to quantization noise ratio does not depend on the number of quantization bits, local sampling density reflects the instantaneous bandwidth of signal, etc. On the other hand, the captured samples are placed non-uniformly and that requires rethinking of the processing methodology. The classical approaches of time-frequency analysis have been discussed. Time-frequency representations have been obtained using general forms of them, which are suitable also for processing of non-uniformly sampled signals. The simulation shows that the main drawback of STFT is the appearance of spurious components, while wavelet transform gives low spectral resolution at high frequencies and low temporal resolution at low frequencies.

Several enhancements have been proposed, which are based on the idea of minimizing the error between the original signal and that reconstructed by the Fourier series, not only at sampling time instants, but also between them with the same accuracy. The problem lies in the fact that the original signal values are known only at sampling instants. One solution is based on the consideration, that the continuous time signal is constructed by interpolation of known signal samples. The expressions for zero-order and first order polynomial interpolation as well as for band-limited interpolation with sinc functions have been established. The other approach is to interpolate the error samples in the same manner.

Simulation results show the improvement of TFRs if enhanced algorithms are used instead of classical ones. Additional benefits can be gained if the bandwidth of analysis is varied along the time axis according to changes in local sampling density: the artifacts are removed, the complexity of calculations can be decreased. The common drawback of STFT based methods is restrictions on the resolution. Extension of the windows $w(t)$ length improves the frequency resolution but at the same time degrades the temporal selectivity. To overcome this rule, the signal-dependent transformation described in (Greitans, 2005) can be used. Due to the limited size of the paper, this method is not discussed above, however the TFR obtained by

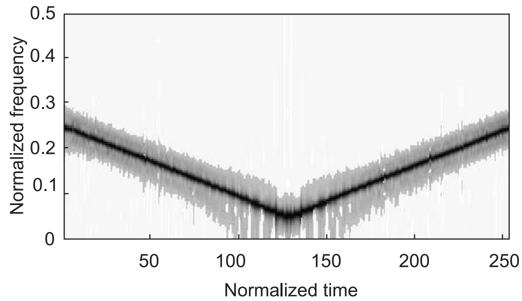

Figure 8: TFR of test-signal if signal-dependent transformation is used.

signal-dependent algorithm is shown in the Fig.8 for the illustration. The increased resolution is achieved by adapting the transformation functions to the local spectral characteristics of the signal. As it is being done in an iterative way, the mathematical complexity is higher than for STFT based algorithms.

The proposed approach of processing non-stationary signals using level-crossing sampling is attractive for clock-less designs, which are now receiving increasing interest. Their advantages can play a significant role in future electronics' development.

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