SIMPLIFIED REPRESENTATION OF LARGE RANGE DATASET

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Abstract: In this paper, we consider two approaches of simplifying medium- and large-sized range datasets to a compact data point set, based on the Radial Basis Functions (RBF) approximation. The first algorithm uses a Pseudo-Inverse Approach for the case of given basis functions, and the second one uses an SVD-Based Approach for the case of unknown basis functions. The novelty of this paper consists in a novel partition-based SVD algorithm for a symmetric square matrix, which can effectively reduce the dimension of a matrix in a given partition case. Furthermore, this algorithm is combined with a standard clustering algorithm to form our SVD-Based Approach, which can then seek an appropriate partition automatically for dataset simplification. Experimental results indicate that the presented Pseudo-Inverse Approach requires a uniform sampled control point set, and can obtain an optimal least square solution in the given control point set case. While in the unknown control point case, the presented SVD-Based Approach can seek an appropriate control point set automatically, and the resulting surface preserves more of the essential details and is prone to less distortions.

1 INTRODUCTION

A range dataset is a picture in which each pixel value encodes not the intensity of a usual 2D image but rather the depth (or range) information. This type of imagery therefore provides direct, explicit geometric information which is useful in many applications. However, this range dataset is usually large-sized, non-uniformly sampled (i.e. the surface is typically irregularly sampled, and exhibits varying sampling densities), and contains noise or unwanted details. The challenging problems include the interpolation of the scattered surface dataset, the removal of the inherent noise from the range dataset and the simplification of this dataset for the large-sized case. At present, the Radial Basis Functions (RBF) are popular for interpolating scattered data since they can effectively interpolate across large, irregular holes in incomplete surface data without constraining the topology of an object or any priori knowledge of the shape (Carr et al. 2003, 2001, 1997, Morse et al. 2001). (Carr et al. 2003), further employed RBF technique to smooth the scattered range data. As an alternative approach, (Fleishman et al. 2005), recently employed the Moving Least-Squares (MLS) technique to handle noisy range datasets. However, since these range dataset usually contains a large data point set, this will bring about a higher computational complexity. For example, computing a RBF interpolation is performed by solving an associated linear system of basis functions of size up to $(N+K)\times(N+K)$, where N is the number of control points and K is the number of a low polynomial coefficients. As this system becomes larger, the amount of computation required to solve it grows as $O(N^3)$. In order to decrease the computational complexity, (Beatson et al. 1999, 2001, 2000) and (Suter 1994) proposed their individual fast evaluation approaches that have a complexity of $O(N\log N)$.

However, the range dataset usually contains redundant information to represent a surface. Considering all the points as control points to fit the surface must lead to a higher complexity, and this is indeed unnecessary. But how to simplify the range dataset is still an open problem. This paper deals with the problem of range dataset simplification, and aims at the simplification of the RBF approximation. To the best of our knowledge, this problem has only been addressed by (Carr, et al. 2001), who proposed a greedy algorithm for reducing the data point set. Their basis idea is to use the fitting accuracy as the criterion to choose the control points. In our works, we prefer to pay attention to the space distribution of the point set, since its inherent geometric properties indicate which points are critical to represent a

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surface. The main contribution of this paper is to present a novel partition-based SVD approach, which can be employed to reduce the dimension of a large-sized symmetric square matrix. Furthermore, we employed this novel tool to the range dataset simplification, and the resulting surface preserves more essential structures and less distortion.

The remainder of this paper is organized as follows. In Section 2, the RBF approach is first introduced briefly. Then, we introduce two approach of simplifying the RBF approximation in Section 3, one is the Pseudo-Inverse Approach and other is the SVD-Based Approach. Experiments and analysis are shown in Section 4. Finally, Section 5 gives our conclusions and future works.

2 RADIAL BASIS FUNCTIONS

An implicit surface is defined by an implicit function, which is a continuous scalar-valued function over the domain, i.e. $f : R^n \to R$. Therein, the function values of points at the implicit surface take on zero, while the function value takes on positive value interior to the implicit surface and is negative outside the surface (or conversely). The magnitude is defined as the distance from a point to the implicit surface. This implicit function is also called a signed distance function. Our goal is to recover the implicit function *f* from a set of dataset (or control point set). Indeed, this is an ill-posed problem, since it has an infinite number of solutions. The standard procedure is to obtain a solution of this ill-posed problem from a variational principle, that is, to minimize the following functional,

$$\min_f \sum_{i=1}^{N} \left(f(\mathbf{x}_i) - f_i' \right)^2 + \alpha \phi[f], \qquad (1)$$

where, $\mathbf{x}_i \in R^n$ is control point coordinate vector, f_i' is the function value of $\mathbf{x}_i$, $\phi[f]$ is a smoothness functional (in general, the thin-plate energy functional (Carr et al. 2001) is adopted), and α is the regularization parameter.

An effective expression of the solution of Eq.(1) is in terms of radial basis functions centered at the control points. Radial basis functions are radially symmetric about the control points, which is written as follows,

$$f(\mathbf{x}) = \sum_{i=1}^{N} c_i G(\mathbf{x} - \mathbf{x}_i) + \sum_{k=1}^{K} d_k \varphi_k(\mathbf{x}), \qquad (2)$$

where, $G(x - x_i)$ is a basis function $G : R^n \times R^n \to R$, $\{\varphi_k\}_k^K$ is a basis in the *K*-dimensional null space. In order to determine the coefficients c_i and d_i, one can solve the following linear system.

$$\begin{pmatrix} G & \varphi \\ \varphi^T & 0 \end{pmatrix} \begin{pmatrix} \mathbf{c} \\ \mathbf{d} \end{pmatrix} = \begin{pmatrix} \mathbf{f}' \\ 0 \end{pmatrix}, \qquad (3)$$

where, $\mathbf{f}' = (f_1', ..., f_N')^T$, $G_{ij} = G(\mathbf{x}_i - \mathbf{x}_j)$, $\mathbf{c} = (c_1, ..., c_N)^T$, $\mathbf{d} = (d_1, ..., d_K)^T$. The basis function is of the form $G(\cdot) = G(\|\cdot\|_2)$, which includes biharmonic spline, triharmonic spline and multiquadric (refer to (Carr et al. 2001) for details). The $\sum_k^K d_k \varphi_k(\mathbf{x})$ in Eq.(2) is usually a degree one polynomial since the thin-plate energy consist of second order derivatives.

In the scattered data interpolation case, it is straightforward to employ all the data points to construct the coefficient matrix of Eq.(3) and directly solve the coefficient vectors **c** and **d**. Herein, all the data points are regarded as control points. For the small-sized range datasets, this direct approach is very useful for the direct solution of the interpolation problem. But in the moderate- and large-sized cases, the coefficient matrix of Eq.(3) would exceed the computational capability of the usual machine. Thus, the control point set has to be a subset of the whole range dataset. Indeed, the range dataset is usually redundant with respect to the representation of a surface. It is unnecessary to regard all the data points as control points.

3 SIMPLIFICATION OF RANGE DATA

Consider the medium- and large-sized range datasets case. In this section, we devise two approaches in order for the solution of the implicit function *f* to account for the control point set and non-control point set.

3.1 Pseudo-Inverse Approach

In this case, the control point set is known in advance. This means that the basis functions have been determined and the basis of the functional space of the implicit function *f* are fixed. Herein, we only need to select an appropriate set of coefficients c_i and d_i for Eq.(2). This can be achieved through minimizing the following functional,

$$\min_{f(\mathbf{x},\mathbf{c},\mathbf{d})}\left(\sum_{i}^{N}\left|f(\mathbf{x}_i,\mathbf{c},\mathbf{d})-f_i\right|^2+\sum_{j}^{M}\left|f(\mathbf{x}_j,\mathbf{c},\mathbf{d})-f'_j\right|^2\right), \quad (4)$$

where, N is number of control points, M is number of non-control points.

Considering Eq.(2), Eq.(3) and Eq.(4) together, one can convert the above minimization problem as follows,

$$\begin{pmatrix} G_N & \varphi_N \\ \varphi_N^T & 0 \\ G_M & \varphi_M \end{pmatrix}\begin{pmatrix}\mathbf{c}\\ \mathbf{d}\end{pmatrix}=\begin{pmatrix}\mathbf{f}'_N\\ 0\\ \mathbf{f}'_M\end{pmatrix} \quad (5)$$

where, $\mathbf{f}'_N$ and $\mathbf{f}'_M$ are respectively the vectors of function values of control points $\mathbf{x}_i$ and non-control points $\mathbf{x}_j$, G_N and φ_N are constructed by the control points, which are described in the same manner as in Eq.(3), G_M and φ_M are constructed by the non-control points, which can simply be determined using Eq.(2) with the same control points that are used for G_N. G_N is a symmetric matrix of size $N\times N$ while G_M is a asymmetric matrix of size $M\times M$. Through the pseudo-inverse of the coefficient matrix in Eq.(5), we can obtain a solution of **c** and **d** in a least square sense.

The advantages of this approach are several. First, the algorithm is easily implemented since only the linear system of Eq.(5) needs to be solved. Second, the error converges since we can get a least square solution from Eq.(5). However, its deficiencies are also obvious. From a theoretical viewpoint, the control point set is redundant. As we knew, the basis functions $G(\mathbf{x}-\mathbf{x}_i)$ were non-compactly supported. This means that the control point set is redundant with respect to the representation of a surface. Furthermore, because the control points are fixed, the resulting implicit function of Eq.(2) is bounded in the functional space spanned by the basis function $G(\mathbf{x}-\mathbf{x}_i)$. It is impossible to deform the implicit function f to go beyond this original functional space. This means that the control points should be uniform samples over the whole domain but not the local samples. Therefore, it is necessary to investigate the choice of the control points. From the perspective of an application, in large-sized range data case, due to a large number of data points, the coefficient matrix of Eq.(5) would quickly exceed the computational capability of the usual machine. In the following section, we will try to devise a novel approach to overcome these two problems.

3.2 SVD-Based Approach

In this case, the control point set is unknown. Our basic idea is to apply the Singular Value Decomposition (SVD) to the coordinate set of range data points to simplify the data points so as to obtain some principal control points. Obviously, the first problem we encountered will be the conflict between the large-sized point coordinate set and the usual limited computational capability. It is straightforward to partition the large-sized set into a series of subsets, and then deal with them individually. In the following, we first explain our novel partition-based SVD approach, then apply it to the large-sized data point set for point simplification.

In the RBF approximation, the coefficient matrix of Eq.(3) is real symmetric and positive semi-definite. Our goal is to exploit these properties in order to reduce the dimension of this square matrix. Without loss of generality, we first give the following two basic propositions (see (Yu et al. 2005) for a formal proof).

Proposition 1

Let a real symmetric matrix $\mathbf{A}\in R^{n\times n}$ be SVD decomposed as $\mathbf{A}=\mathbf{U}\Lambda\mathbf{V}^T$, where $\Lambda=diag(\lambda_1,...,\lambda_n),\lambda_1\geq...\geq\lambda_n\geq 0$. If $r\leq rank(\mathbf{A})$ and let $\mathbf{A}'=\mathbf{U}'\Lambda'\mathbf{V}'^T$, where $\mathbf{U}',\mathbf{V}'\in R^{n\times r},\Lambda'\in R^{r\times r}$, then, $\left|\mathbf{A}-\mathbf{A}'\right|_2=\lambda_{r+1}$.

Proposition 2

Partitioning columns of **A** into r block submatrices $\mathbf{A}=(\mathbf{A}_1,...,\mathbf{A}_r)$, where $\mathbf{A}_i\in R^{n\times p_i}, i=1,...,r$, $n=\sum_{i=1}^{r}p_i$, we have, $rank(\mathbf{A}_i)=rank(\mathbf{A}_i^T\mathbf{A}_i)$.

Proposition 1 implies that $\mathbf{A}'$ is an optimal approximation of **A** among all rank r matrices in a 2-norm sense. Furthermore, if $rank(\mathbf{A})=r$ and $r<n$, this implies that **A** can be partitioned into r block submatrices and each block submatrix is expected to be of rank 1.

Due to the real symmetry property of **A**, each block partitioned along column (or row) must correspond to a block partitioned along row (or column). For convenience, each strip block can be compressed to a real symmetric matrix. Proposition 2 implies that this compressed symmetric matrix $\mathbf{A}_i^T\mathbf{A}_i$ has the same rank as $\mathbf{A}_i$. Therefore, we are able to expect

the square matrix $\mathbf{A}_i^T\mathbf{A}_i$ to be of rank 1. This procedure is illustrated in Fig.1.

$$\begin{array}{c} \\ \mathbf{A}_1^T \\ \mathbf{A}_i^T\mathbf{A} = \\ \mathbf{A}_r^T \end{array} \begin{array}{c} \begin{array}{cccc} \mathbf{A}_1 & & \mathbf{A}_i & \mathbf{A}_r \end{array} \\ \begin{pmatrix} * & \cdots & * & \cdots & * \\ \vdots & \ddots & & & \vdots \\ * & & * & & * \\ \vdots & & & \ddots & \vdots \\ * & \cdots & * & \cdots & * \end{pmatrix} \end{array} \rightarrow \mathbf{B} = \begin{pmatrix} \mathbf{A}_1^T\mathbf{A}_1 & & & 0 \\ & \ddots & & \\ & & \mathbf{A}_i^T\mathbf{A}_i & \\ & & & \ddots \\ 0 & & & \mathbf{A}_r^T\mathbf{A}_r \end{pmatrix}$$

Figure 1:Real symmetric matrix **A** is converted to a quasi-diagonal form **B**.

It can be noted that the compressed matrices $\mathbf{A}_i^T\mathbf{A}_i$ construct a block diagonal matrix **B**. In other words, the partition of **A** is transformed to a block diagonal matrix **B**. This transform effectively simplifies the computation burden, since the future numerical analysis of **A** can be fulfilled through the individual analysis of each symmetric block $\mathbf{B}_i = \mathbf{A}_i^T\mathbf{A}_i$.

Applying SVD decomposition to $\mathbf{B}_i$, one can get $\mathbf{B}_i = \mathbf{A}_i^T\mathbf{A}_i = \mathbf{V}_i\Lambda_i^2\mathbf{V}_i^T$, where the SVD of $\mathbf{A}_i$ is $\mathbf{A}_i = \mathbf{U}_i\Lambda_i\mathbf{V}_i^T$. When we let $rank(\mathbf{B}_i) = rank(\mathbf{A}_i^T\mathbf{A}_i) = 1$, the approximation of $\mathbf{B}_i$ can therefore be written as, $\mathbf{B}_i' = \lambda_1^{(i)^2}\mathbf{v}_1^{(i)}\mathbf{v}_1^{(i)^T}$, where $\lambda_1^{(i)^2}$ is the maximum singular value of Λ_i^2, $\mathbf{v}_1^{(i)}$ is the column singular vector of $\mathbf{V}_i$ corresponding to $\lambda_1^{(i)^2}$. Consequently, **B** can be reconstructed by the approximations $\mathbf{B}_1',...,\mathbf{B}_r'$ as follows,

$$\mathbf{B}' = \begin{pmatrix} \mathbf{B}_1' & & 0 \\ & \ddots & \\ 0 & & \mathbf{B}_r' \end{pmatrix} = \begin{pmatrix} \mathbf{v}_1^{(1)} & & 0 \\ & \ddots & \\ 0 & & \mathbf{v}_1^{(r)} \end{pmatrix} diag\left(\lambda_1^{(1)^2},...,\lambda_1^{(r)^2}\right) \begin{pmatrix} \mathbf{v}_1^{(1)^T} & & 0 \\ & \ddots & \\ 0 & & \mathbf{v}_1^{(r)^T} \end{pmatrix} \quad (6)$$

For each block $\mathbf{B}_i$, we know that $\mathbf{B}_i$ is indeed the Gram matrix of $\mathbf{A}_i$, which is symmetric and positive definite. The diagonal difference between $\mathbf{B}_i$ and its approximation $\mathbf{B}_i'$ indicates which column (or row) vectors of $\mathbf{A}_i$ are principal basis vectors of $\mathbf{A}_i$. This is due to the fact that each diagonal entry of $\mathbf{B}_i$ is the inner product of each column (or row) vector of $\mathbf{A}_i$. If the *i*th column (or row) vector is a principal vector of $\mathbf{A}_i$, its inner product can be approximated by the *i*th diagonal entry of the approximation $\mathbf{B}_i'$. One can conveniently preserve a column (or row) of $\mathbf{A}_i$ by evaluating $\left|b_{j,j}^{(i)} - b_{j,j}'^{(i)}\right|_2, b_{j,j}^{(i)} \in \mathbf{B}_i, b_{j,j}'^{(i)} \in \mathbf{B}_i', j = 1,..., p_i$ so that these selected columns (or rows) construct an *r*-dimensional matrix $\tilde{\mathbf{A}}_{r\times r}$, which is a dimension reduced version of **A**.

Up to now, we established a partition-based SVD approach for the large-sized real symmetric matrix case. It can be summarized as follows.

Partition-Based SVD Approach:

1) Convert the partitioned $\mathbf{A} = (\mathbf{A}_1,...,\mathbf{A}_r)$ to a block diagonal matrix **B**;
2) Compute the approximation $\mathbf{B}_i'$ of each symmetric subblock $\mathbf{B}_i$ in **B** through SVD, in which $rank(\mathbf{B}_i') = 1$;
3) Compute the diagonal difference between $\mathbf{B}_i$ and $\mathbf{B}_i'$;
4) Select the principal vector of $\mathbf{A}_i$ by evaluating $\left|b_{j,j}^{(i)} - b'_{j,j}{}^{(i)}\right|_2, b_{j,j}^{(i)} \in \mathbf{B}_i, b'_{j,j}{}^{(i)} \in \mathbf{B}_i'$ to construct the reduced matrix $\tilde{\mathbf{A}}$ of size *r*×*r*.

However, it can be noted that the approximation error of **B** can be evaluated through 2-norm of $\left|\mathbf{B} - \mathbf{B}'\right|_2$. Moreover, the upper bound of error can be estimated as $\left|\mathbf{B} - \mathbf{B}'\right|_2 \le \max_i \lambda_2^{(i)^2}$ (for details, refer to [10]). Indeed, a good partition approach can be deduced by minimizing the error of $\max_i \lambda_2^{(i)^2}$. It can also be noted that $\max_i \lambda_2^{(i)^2}$ is a variable for various partition approaches. It can be further proven that for the singular values of each column of **A**, $\sigma_1,...,\sigma_n$ and $\sigma_1 \ge ... \ge \sigma_n \ge 0$, there exists a partition (i.e. the dimension of **A** is reduced to *r*) such that, $\max_i \lambda_2^{(i)^2} \le (\sigma_r^2 + \sigma_{r+1}^2 + ... + \sigma_n^2)/2$ (for details, refer to (Yu et al. 2005)). This implies that $(\sigma_r, \sigma_{r+1},..., \sigma_n)$ correspond to the non-principal columns (or rows) of **A** respectively, and in order to further reduce the error of $\max_i \lambda_2^{(i)^2}$, we have to seek a column (or row) combination of **A** (i.e. all the non-principal columns or rows are left in $\mathbf{A}_r$) so as to obtain the minimum of $\left(\sigma_r^2 + \sigma_{r+1}^2 + ... + \sigma_n^2\right)$. Of course, if the non-principal columns or rows are distributed in each partition $\mathbf{A}_i$, this minimization problem will be described as follows,

$\min \max_i \left(\sigma_1^{(i)2} + ... + \sigma_{p_i}^{(i)2}\right)$.

However, seeking the optimal partition of **A** is indeed an NP-complete problem. We will present below our partition scheme in the case of range data simplification.

Consider a coordinate set of range data points. One can note that the distribution of the point cloud formed by data points is not uniform. The geometric distribution of data points is redundant with respect to the topological structure of a point cloud. In order to obtain a compact data point set, we can directly apply the SVD technique to the basis functions of Eq.(3). Indeed, the sub-matrix *G* in Eq.(3) consists of the basis functions $G(\mathbf{x}_i - \mathbf{x}_j)$, which is a real symmetric and positive semi-definite matrix. If there existed a partition for *G*, applying the above presented partition-based SVD approach to *G*, one could easily simplify *G* to get a dimension reduced version of *G*. But, how to partition *G* is still an open problem that is covered in this section.

Considering *G*, one can note that the basis function $G(\mathbf{x}_i - \mathbf{x}_j)$ is a function of the Euclidean distance $\|\mathbf{x}_i - \mathbf{x}_j\|$, and $G(\mathbf{x}_i - \mathbf{x}_j) = G(\mathbf{x}_j - \mathbf{x}_i)$. The *i*th row (or column) of *G* is a vector of $(G(\mathbf{x}_1 - \mathbf{x}_i),...,G(\mathbf{x}_N - \mathbf{x}_i))$ about the *i*th point $\mathbf{x}_i$. Herein, the geometric meaning of the singular values of the columns (or rows) of *G* is that the singular value σ_i of column (or row) *i* of *G* is a measurement of the divergence of the data point set to the *i*th data point $\mathbf{x}_i$, i.e. the bigger the singular value σ_i is, the farther the data point set departs from the *i*th data point $\mathbf{x}_i$. In terms of proposition 2, the *r* columns (or rows) of *G* with the first *r* maximum singular values of columns should be put into *r* different partitions of *G*. It is clear that the resulting partition of *G* through the selection of points with the larger divergence as the centers of the different partitions can minimize the residual error between *G* and its approximation G'.

However in our case, *G* is not approximated by the same dimension of G' but its dimension is reduced (i.e. the number of control points needs to be reduced). Obviously, the reduction of the control points will lead to the loss of some details. If only the points with the largest divergence are considered, many details will have to be abandoned. This can be demonstrated in Fig.2. If a point of set *A* has a larger divergence than the points of set *B* as illustrated in Fig.2, then, each point of its neighbourhood in *A* have similar divergence that are more than divergences of the points of *B*. Thus, the control points will be selected from set *A* rather than set *B*. It is clear that in this manner, the distribution of the selected control points will not be uniform over the original data point set. Therefore, it is necessary to further require that each control point should hold a large divergence with respect to all the control points $A \cup B$, so that the control points can be distributed over the original data point set as uniformly as possible. The divergence of each point $\mathbf{x}_i$ is computed as follows,

$$\begin{cases} div1_i = tr\left(\sum_{\mathbf{x}\in S}(\mathbf{x}-\mathbf{x}_i)(\mathbf{x}-\mathbf{x}_i)^T\right) \\ div2_i = tr\left(\sum_{\mathbf{y}\in S_0}(\mathbf{y}-\mathbf{x}_i)(\mathbf{y}-\mathbf{x}_i)^T\right) \end{cases}, \quad (7)$$

where, *S* is the data point set while S_0 is the control point set. The control point $\mathbf{x}_c$ is selected using $\mathbf{x}_c = \arg\max_i (div1_i + div2_i)$, where $\arg(\cdot)$ extracts the point $\mathbf{x}_i$ which yields the maximum of $(div1_i + div2_i)$.

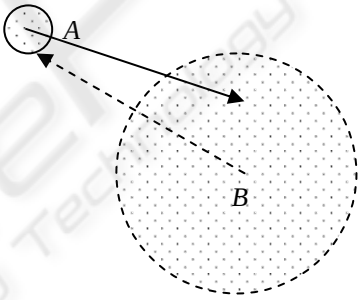


Figure 2: The divergence of different point sets. The points in *A* have a similar divergence that is larger than the divergence of the points in *B*.

However, the implementation of Eq.(7) is time-consuming. In practice, we prefer to use a standard clustering algorithm (Duda et al. 2001) instead. For clarity, some concepts need to be first defined as follows.

- Centre of a partition P_i is regarded as clustering centre, which is defined as, $\mathbf{y}_i = \arg\max_j tr\left(\sum_{\mathbf{x}\in S}\left(\mathbf{x}-\mathbf{x}_j^{(i)}\right)\left(\mathbf{x}-\mathbf{x}_j^{(i)}\right)^T\right), \mathbf{x}_j^{(i)} \in P_i, \mathbf{y}_i \in S_0$;
- Measure between two points, $d_{ij} = tr\left((\mathbf{x}_i - \mathbf{x}_j)(\mathbf{x}_i - \mathbf{x}_j)^T\right), \quad \mathbf{x}_i, \mathbf{x}_j \in S$;

Based on the above definitions, our **Partition Algorithm** can be stated as follows:

1) {Clustering}

{Initializing}: Input data point set *S*, centre set S_0 and an initial partition $\bigcup_{i=1}^{r} P_i = S$;

{Merging, Splitting and Deleting}: These standard clustering operations are carried out based on the above definitions of the measure d_{ij} and the selected centre $\mathbf{y}_i \in S_0$;

Iteration until there is no change in each P_i;

2) {Partitioning G}
Loop: from i=1 to r
Partition G in terms of P_i;
EndLoop;

The proposed partition algorithm is a divergence-based iterative approach. The initial centre set S_0 is usually the point set with the first r maximum singular values of columns of G. The measure between two points is indeed another representation of the Euclidean distance. It can make the centers of partitions as divergent as possible. But the clustering algorithm is only an approximation of Eq.(7). Thus, our partition algorithm can only approximate the global optimal partition.

In short, the presented partition-based SVD approach and partition algorithm constitute our SVD-based approach for dataset simplification. It is clear that data redundancy and computational complexity of the large-sized range dataset can be effectively amended in this SVD-based approach. The highlight property of this approach is that the control points are not fixed in advance. This means that the basis functions can be modified adaptively in terms of the change of range dataset. The resulting solution of Eq.(2) would be a global least square solution.

4 EXPERIMENTS AND ANALYSIS

An intuitional way to evaluate the data point simplification is to visualize the resulting implicit surface. Our experiments of simplifying dataset are first carried out on a range dataset of human faces for a detailed analysis. The original range dataset includes about 35,000 points, which is meshed and illustrated in Fig.3a.

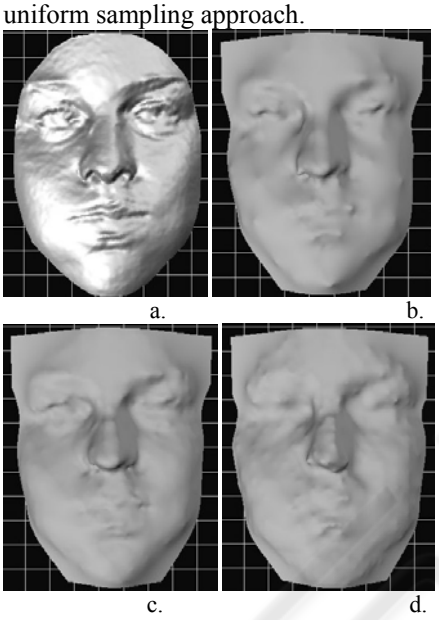
In the first experiment, we apply the pseudo-inverse approach to this dataset. About 2,100 control points are chosen uniformly over the whole dataset. The resulting surface is shown in Fig.3b. Due to the reduction of the control points, many details of the face are lost. However, it can be noted that the essential structures are preserved. Clearly, the control points determine the essential structures of the face in the resulting implicit surface. When the control points are chosen uniformly over the whole original dataset, the essential features can be unbiasedly chosen as the control points. Indeed, the influence of the non-control points in Eq.(5) is very limited. We also show the resulting surface only based on the control points in Fig.3c (i.e. non-control points are not used). Obviously, the variance between Fig.3b and Fig.3c is very small. Furthermore, reducing the number of control points to about 1,100, we fit the surface on the basis of a selected control point set. The resulting surface is shown in Fig.3d. It can be noted that there are no distinct details lost in Fig.3d compared with Fig.3c. This indicates that uniform sampling can preserve the essential structures of the face. But it can also be noted that distortions are also visible around the nose in Fig.3d.

In the second experiment, we apply the SVD-based approach to the same range dataset as in Fig.3a. In this approach, the partition of G dominates the quality of the resulting surface. Thus, the two criteria of Eq.(7) become highlighted. In our experiment, we first consider the first criterion of Eq.(7), i.e. $div1_i = tr\left(\sum_{x\in S}(\mathbf{x}-\mathbf{x}_i)(\mathbf{x}-\mathbf{x}_i)^T\right)$, as the criterion of partition. Herein, the Partition Algorithm described in section 3.2 is simply replaced by sorting $\{div1_i\}$. The control points are reduced to 30,000, 10,000 and 6,000 points respectively. The resulting surfaces are shown in Fig.4. It can be noted that due to the non-uniformity of the control points, many structures are lost. Clearly, some regions contain few or no control points (such as the nose area) while others contain an excess of control points (such as the cheek area). However, the essential facial outlines are still retained. Moreover, when we consider the two criteria of Eq.(7), i.e. use the partition algorithm to obtain an appropriated partition of G, it can be noted that some details of the face can also be preserved even if the control points are further reduced. The resulting surfaces with the different numbers of control points are shown in Fig.5.

Furthermore, comparing our SVD-Based Approach with the uniformed down-sampling approach, we can compare Fig.5c and Fig.5d with Fig.3c and Fig.3d. This is because in Fig.3c and Fig.3d, we uniformly down-sampled the control points over the original range dataset, and the other non-control points were discarded. In addition, the number of control points in Fig.3c and Fig.3d are similar to the number of control points in Fig.5c and Fig.5d respectively. It can be noted that Fig.5c and Fig.5d appear more distinct and preserve more details than

Fig.3c and Fig.3d, and there are comparatively less distortions in Fig.5c and Fig.5d.

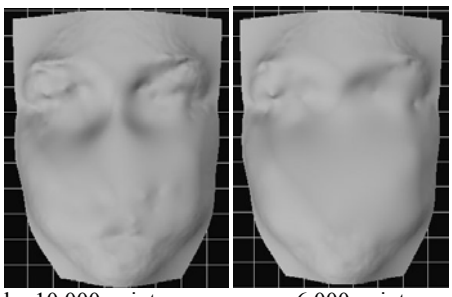
In order to further verify the efficiency of the SVD-Based Approach, we subsequently apply the SVD-Based Approach to 5 range datasets with different facial expressions. The original numbers of data points are in the range of 32,000-35,000, and are reduced to 1,100 through our SVD-Based Approach. For comparison, we also apply the uniform sampling approach to these 5 range datasets, and their point numbers are also reduced to about 1,100. All the resulting surfaces are shown in Fig.6. It can be noted that the resulting surfaces obtained by the SVD-Based Approach preserve more details, and have less distortions compared to the ones obtained by the uniform sampling approach.



a. b. c. d.

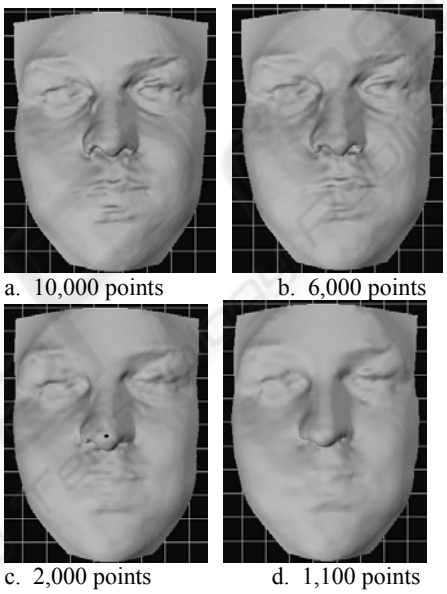
Figure 3: The resulting surfaces through the Pseudo-Inverse Approach. a) original model, b) result using pseudo-inverse approach, c) result using uniform sampling approach, d) result using uniform down-sampling approach, in which the number of points is reduced to about 1,100.

a. 30,000 points



b. 10,000 points c. 6,000 points

Figure 4: The facial model is simplified by the SVD-based approach with the first criterion of Eq.(7). Non-uniform sampling leads to the quality of the resulting surfaces decreasing quickly.

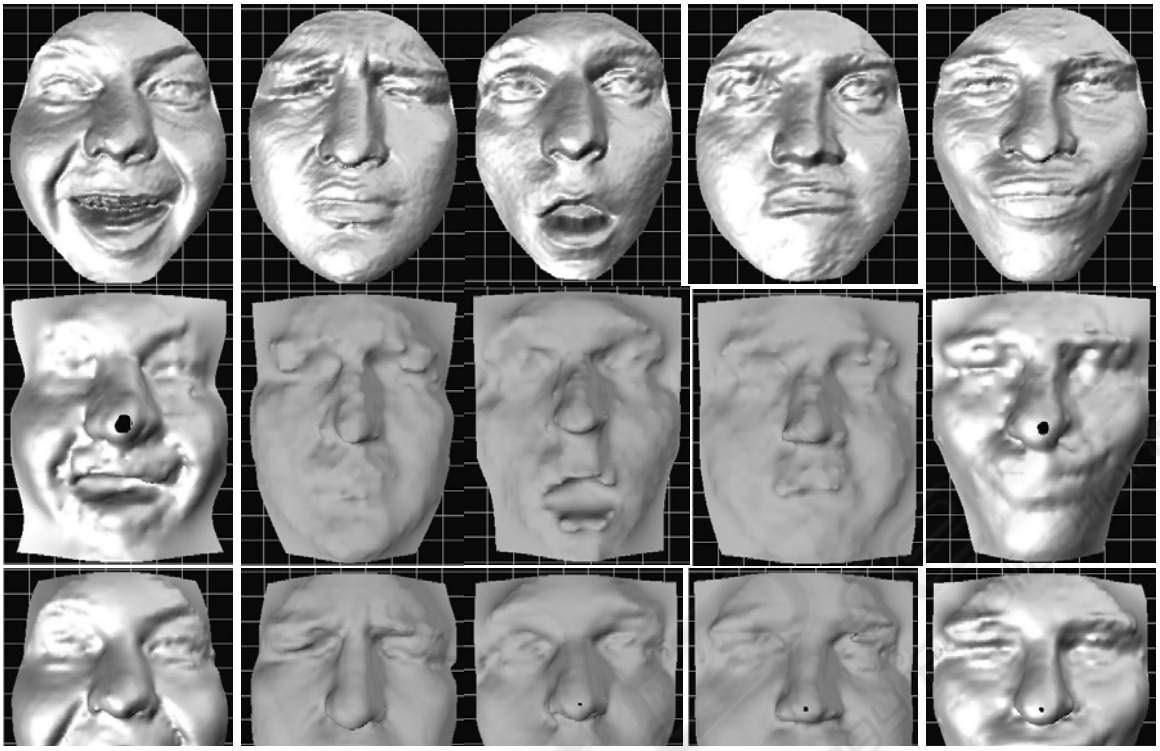


a. 10,000 points b. 6,000 points c. 2,000 points d. 1,100 points

Figure 5: The facial model is simplified by the SVD-based approach with the two criterion of Eq.(7). Considering the uniform sampling preserves many details effectively.

5 CONCLUSIONS

In this paper, we presented two approaches of computational simplification for medium- or large-sized range dataset, one is the Pseudo-Inverse Approach, and the other is the SVD-Based Approach. The novelty in this paper is that we devised a novel partition-based SVD algorithm, which can effectively reduce the dimension of a symmetric square matrix in a given partition case. We combined the partition-based SVD algorithm with a standard clustering algorithm to form our SVD-Based Approach. Experimental results indicate that in a given control point set case, the Pseudo-Inverse Approach can give an optimal solution in a least square sense. While in the case of an unknown



control point, the SVD-Based Approach can seek an appropriate control point set automatically, and the resulting surface can preserve more details and generate less distortions.

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