

CALCULATION OF OPTIMAL TRAJECTORY IN 3-D STRUCTURED ENVIRONMENT BY USING GEODESY AND MATHEMATICAL MORPHOLOGY

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Abstract: A new method for obtaining the optimal path to disassembly an object in a 3-D structure is presented in this paper. To obtain the optimal path, we use an extension of the mathematical morphology and the geodesic distance to 3-D sets. The disassembly algorithm is based on the search for a path of minimum cost by using the wave-front of the geodesic distance. Cost is considered to be the number of changes in trajectory required to be able to remove the object. The new method will be applied to disassembly objects in several 3-D environments. The result path for removing an object in a concrete 3-D set will be shown.

1 INTRODUCTION

A method for obtaining the optimal trajectory from an initial point to a final point in a 3-D space is presented in this paper. When the trajectory is computed from the initial point to the final destination, the following factors must be taken into consideration:

- Avoiding collisions with the environment: as there is a structured environment, the path between the two points must avoid any collisions between the moving object and the environment.
- The shortest path: when this is taken into consideration, the displacement is reduced to a minimum. It must be noted that there may be more than one path with the same distance between the two points. Such paths are all correct and must be taken into consideration.
- Minimizing changes in direction: From all the possible paths, only those that require the minimum number of direction changes along the path will be chosen.

Another characteristic of the proposed trajectory is that it allows paths in the 3-D space, which implies an important characteristic gain over the 2-D trajectories, which are the ones used in traditional robot navigation. In the traditional navigation of robots, the movement is defined in a 2-D

environment although the real movement is in 3-D. Furthermore, in the case of air robots or submarine navigation, a trajectory in a 3-D environment is necessary, since the robots can move with more freedom.

Another area where the use of 3-D trajectories is necessary is in manipulator robots. When a manipulator needs to move within a complex environment with obstacles, it has to define the trajectory to be followed from the starting point to the target, as for example, in the pick and place application in a structured environment (McAvoy et al, 2000). Furthermore, this type of trajectory is necessary when we must have access to the inside of a product to manipulate it, and we must move the manipulator between the other components of the product without colliding with them (Puente et al, 2003).

The techniques for the generation of trajectories can be divided into three categories: global, local or mixed (Puente, 2002), according to the environment in which the solution is required. The global techniques are the most interesting ones, since a complete representation of the work area is used.

In this paper, geodesic techniques for computing the trajectory between two points are used (Belta and Kumar, 2002) (Tchon and Duleba, 1993). The geodesic techniques used have been extended to work in 3-D spaces. The use of geodesic techniques affords a simple way of defining security parameters

for avoiding collisions between the object to be moved from the initial to the target points and its environment. To do so, a dilation of the environment is used, taking the size of the object to be separated into consideration.

The organization of this paper is as follows: Section 2, we extend the geodesic transformations to 3-D environments. The algorithm for calculating the optimal path in the 3-D structured environment is presented in Section 3. Examples of the uses of the algorithm in 3-D structured environments are presented in Section 4. Finally, our conclusions and the bibliography are outlined in the final section.

2 3-D GEODESIC DISTANCE

The geodesic distance in three-dimensional sets is a generalization of the two-dimensional distance. Geodesy in 3-D brings nothing really new into play, as it only requires the choice of a digital unit ball of the structuring element (Serra, 2002). Let i and j be coordinates in Z^3 . As such, the geodesic dilation of size n of i in A corresponds to the geodesic distance of i in A up to j :

$$f : D_f \subset Z^3 \rightarrow \{0,1\} \quad (5)$$

In Fig. 2, we show the geodesic distance $d_A(i,j)$ between the points i and j in a 3-D structure. We can observe that the geodesic distance avoids all the obstacles between the initial and final points.

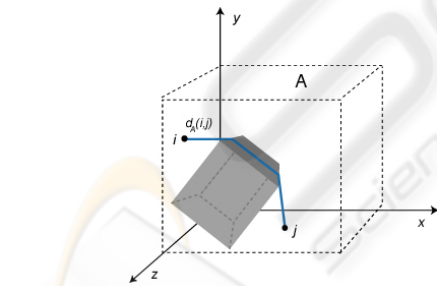


Figure 2: Geodesic distance in a 3-D setting.

To summarize, in digital Z^3 , the geodesic distance function starts from marker i and progresses according to a connectivity: connectivity 6, connectivity 12 (cube-octahedron), or connectivity 26, for example, as we can see in Fig. 3.

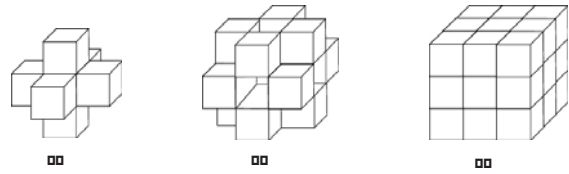


Figure 3: Different connectivities in 3-D sets.

The real physical movement of the marker object through the 3-D mask influences the choice of the connectivity. One consideration to be taken into account in the calculation of the geodesic distance is the diagonal movement. Diagonal movements in the Z^3 digital structure cannot be carried out physically, since collisions with obstacles or edges of the mask can occur. This can be seen in the example in Fig. 4.

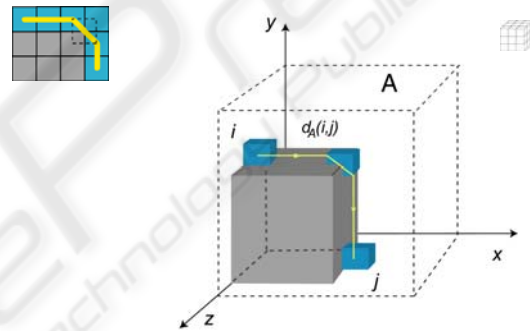


Figure 4: Diagonal movements in connectivity 26.

3 ALGORITHM FOR CALCULATING THE OPTIMAL PATH

The optimal path can be obtained from the geodesic distance. It is obvious that from the 3-D connectivity defined, different optimal paths can be obtained. Furthermore, as the wave-front can have many points, different mechanisms that limit the search must be taken into account. To avoid collisions with the environment during movements, connectivity 6 has been defined.

In our algorithm, the structure or 3-D setting through which the object to be removed must move, will be defined as the mask. The marker will be the object situated at the original point. To reduce calculations, we only compute the geodesic distance from the centre of the marker. Since we must ensure that the object can move through the structure, we

will dilate the 3-D mask with a structuring element that is equal to the form of marker. Once the geodesic distance has been calculated from the initial to the final points, the marker is redirected from the final to the initial point through the wave-front.

There are many different paths with the same geodesic distance. To create the shortest path, a process of “Branch-and-bound”, which limits the search for valid paths, is carried out. From all the valid paths, the one with the minimum cost is chosen. Minimum cost is understood as the fewest changes in directions. As such, in the search process, paths that surpass a certain threshold of changes in direction, or which have co-ordinates that have already been reached by a path of a lower cost, will be eliminated. Depending on the length of the path under study, a given threshold of changes in direction must not be surpassed. Each new path under construction (from final to origin) is inserted into a queue in which the priority level corresponds to the changes of direction up to that moment (e-path_add(path, priority level)). The first path in the queue having the lowest priority level is then extracted from the queue. The algorithm detailed hereafter in pseudocode is:

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• inputs:
    mask3d as 3-D mask
    marker3d as 3-D marker
    origin as point
    final as point
• outputs: path as point vector
• Data structures:
    - wave-front as 3-D wave-front
    - pix as present co-ordinate
    - e-paths as structure of paths
    - newpath as array of co-ordinates
• Initialisation:
    wave-front ← 3-D geodesic distance
    from origin to final
    pix ← final point
    newpath_add ← final point
    e-paths_add ← (newpath,1)
• Path propagation from final to origin in
    the wave-front, according to the neighbours
    in the valid connectivity of the pix in the
    wave-front. The pix is the last co-ordinate
    of the optimal path up to the moment.
while (pix<>origin) do
    ∀ first level of priority p' ∈ e-path do
        pix ← (e-paths_first(p'))_last
        ∀ co-ordinate c' ∈ Ng(pix)do
            newpath_add(c')
            prioritylevel ← direction-changes(newpath)
            if(prioritylevel) > max-direction-changes
            (newpath_long) then
                newpath_reject(c')
            else
                e-paths_add(newpath, prioritylevel)
            end if

```

```

done
done
done
• List of optimal paths
if(pix = origin)then
    ∀ first level of priority p' ∈ e-path do
        path ← e-path_first(p')
    done
end if

```

4 EXPERIMENTAL RESULTS

Below, an example of the application of the previous algorithm for calculating the optimal path to be followed to remove an object (marker) from the initial to the final position in a 3-D setting (mask) is presented. The marker occupies a cubic space (3x3x3). Inside the mask there are different obstacles of different sizes. Once the algorithm has been applied, three different trajectories of the same cost (4 changes in direction) are obtained. In Figs 5, 6 and 7, different views of the paths follow by the marker within the mask can be seen, it is represented by spheres. The initial and final points are indicated in the figure by 1 and 2, respectively.

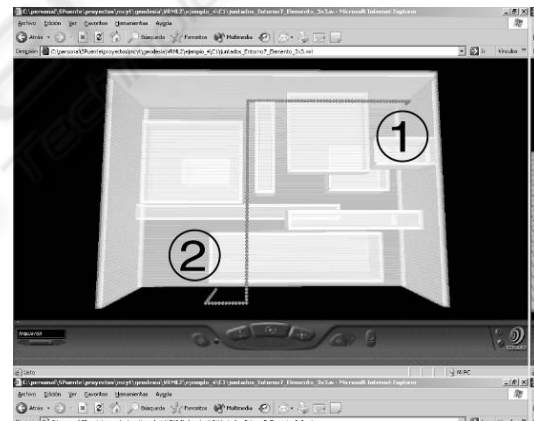


Figure 5: View of the path to be followed in a 3-D setting with obstacles, using a marker of 3x3x3. The initial and the target points are indicated by 1 and 2 respectively.

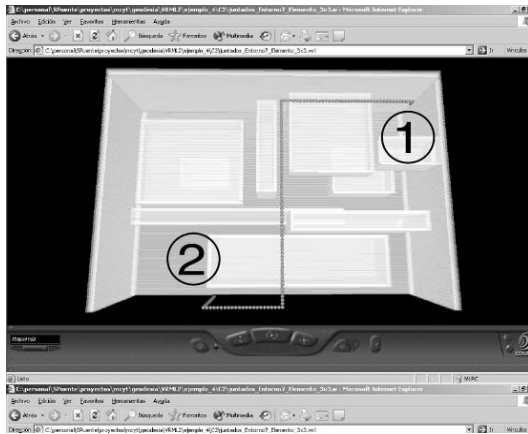


Figure 6: View of second possible path within the same setting and with the same marker.

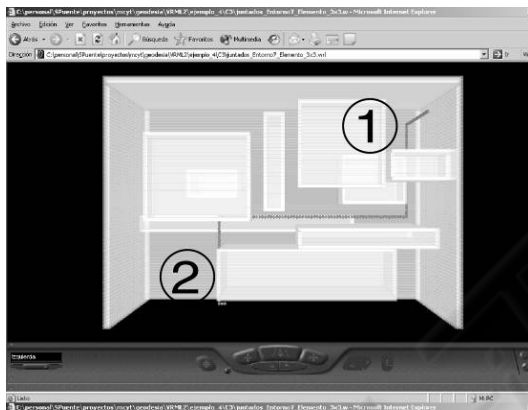


Figure 7: View of a third possible path within the same setting and with the same marker.

5 CONCLUSIONS

In this paper, we have presented a new method for obtaining the optimal path that an object to be disassembled in a 3-D structure should follow. The novelty of this algorithm is that it is based on the use of the mathematical morphological geodesic distance. In this case, all of the operations have been adapted to three-dimensional spaces. The disassembly algorithm is based on the search for a path of minimum cost. Cost is considered to be the number of changes in trajectory required to be able to remove the object. After having implemented the algorithm, an example of the application has been presented. It has been observed that different solutions of the same cost can be obtained. Having several different paths for carrying out the disassembly task by a manipulator, allows us to

choose any of them that the manipulator's restrictions permit.

Furthermore, the proposed algorithm allows navigation within a 3-D setting, considering the volume of the object to be transported within the space.

We would like to continue researching in this field, calculating the optimal path in a dynamic way and including new information about the setting while the path is being followed.

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REFERENCES

- Belta, C., Kumar, V., 2002. Trajectory design for formations of robots by kinetic energy shaping. *19th IEEE International Conference On Robotics and Automation*. pp. 2593-2598.
- McAvoy, B., Sangolola, B., Szabad, Z., 2000. Optimal trajectory generation for redundant planar manipulators. *IEEE Int. Conf. On Systems, Man & Cybernetics*. pp. 3241-3246.
- Puente, S.T., Torres, F., Aracil, R., 2003. Non-Destructive Disassembly Robot Cell for Demanufacturing Automation. *IFAC Workshop on Intelligent Assembly and Disassembly - IAD'2003. Bucharest*, pp. 126-131.
- Puente, S.T., 2002. Desensamblado automático no destructivo para la reutilización de componentes. Aplicación al desensamblado de PC's. *Ph.D. Thesis, University of Alicante*.
- Serra, J., 2002. Course on Morphological Operators, *Centre de Morphologie Mathématique, Ecole des Mines de Paris, France*.
- Tchon, K., Duleba, I., 1993. On inverting singular kinematics and geodesic trajectory generation for robot manipulators, *Journal of Intelligent & Robotic Systems*, 8 (3) pp. 325-359.