POINT CLOUD DENOISING USING ROBUST PRINCIPAL COMPONENT ANALYSIS

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Keywords: Point Cloud Denoising, PCA, Robust Estimation, Shrinkage, Smoothing.

Abstract: This paper presents a new method for filtering noise occurring in point cloud sampled data. The method smoothes the data set whereas preserves sharp features. We propose a new weighted variant of the principal component analysis method, which instead of using exponential weighting factors inversely proportional to the Euclidean distance to the mean, which is computationally expensive, uses weighting factors assignment by inversely proportional repartition of the sum of distance to the mean. The determination of weighted factors by means of inverse proportional repartition makes our variant robust to outliers. Additionally, we propose a simple solution to the problem of data shrinkage produced by the linear local fitting of the principal component analysis. The proposed method is simple, easy to implement, and effective for noise filtering.

1 INTRODUCTION

Data filtering is a task of vital importance in areas like signal processing, statistical analysis, computer vision and 3D object reconstruction. In these areas there is a huge information quantity, represented by discrete samples, which need to be processed for reducing the noise produced by devices in the acquisition process. Though 3D reconstruction applications have been growing due to the improvements in the 3D scanner technology, there is a remaining problem: the raw data produced by 3D scanners are noisy and are far away of being used directly into the 3D reconstruction process without a previous processing.

The development of robust point clouds denoising algorithms has received much attention in last years. The goal of such algorithms is either to remove or to reduce the noise in the data, whereas preserving sharp features on the original surface model. So far, researches in the field of digital image filtering have been adapted for point clouds filtering algorithms. Nevertheless, the adaptation of algorithms for digital image filtering to point clouds filtering is not direct, due to three main reasons: irregularity, shrinkage, and drifting. The irregularity refers to the irregular sampling density of the point clouds. The shrinkage and the drifting refer to the volume reduction and the spatial displacement suffered by the points, which are produced by the use of the mean instead of the data points (Fleishman, 2003).

Principal Component Analysis (PCA), initially used for digital image processing, has been adapted for point clouds processing (Gumhold, 2001, Pauly, 2002). This adaptation has a drawback: PCA is highly sensitive to the outliers present in the point clouds. There are several variants that fix this drawback (Rousseeuw, 1999, De la Torre, 2001, Skokal, 2002, Hubert, 2005), almost all of them are based on robust statistics (Huber, 1981). Such variants, called robust or weighted PCA, improve the PCA, making it less sensitive to the outliers. However, the robust PCA variants use exponential weighting factors to correct the outlier problem, which is computationally expensive.

This paper presents a new method for point clouds denoising. Our method first calculates a weighted mean. The weighting factors assignment of the mean is achieved by inversely proportional distributing the sum of distances to the mean. Then, using the weighting factors and the mean, the method constructs a covariance matrix and realizes an eigen-analysis of such matrix. In this way it is obtained a fitting plane expanded by the eigenvectors corresponding to the largest eigenvalues, and a normal vector to the plane, which

A. Leal Narváez E. and Eduardo Leal Narváez N. (2006).
POINT CLOUD DENOISING USING ROBUST PRINCIPAL COMPONENT ANALYSIS.
In *Proceedings of the First International Conference on Computer Graphics Theory and Applications*, pages 51-58
DOI: 10.5220/0001358900510058

is oriented in the direction of the third eigenvector corresponding to the smallest eigenvalue. Then, a displacement of the neighbourhood mean along the normal vector is achieved in order to preserve sharp features.

The main contributions of this work are: i) a simple method for point clouds denoising that does not require a either previous mesh representation, nor local polynomial fitting, ii) a simple approach to prevent the shrinkage problem, and iii) a mechanism for bias reduction. Our method is robust to outliers, fast and easy to implement.

The remainder of this paper is organized as follows. In section 2, related work dealing with mesh and point clouds denoising algorithms are presented. In section 3, a short review of principal component analysis is presented. In section 4, the stages of our method are explained. In section 5, the results of our method are shown. In section 6, conclusions and future work are discussed.

2 RELATED WORK

Point clouds have become a primitive for surface representation and geometric modelling; however such point clouds are noisy due to the inherent noise of the acquisition devices. Point clouds should be noise free for using in 3D reconstruction. Recently, a great research effort has been done in mesh and point clouds denoising and smoothing, producing numerous algorithms.

Taubin (Taubin, 1995) applies in mesh smooting a discrete version of the Laplacian Operator, which is taken from signal processing. The method is linear in both time and memory.

Desbrum et al. (Desbrum, 1999, Desbrum, 2000) and Bajaj (Bajaj, 2003) successfully use anisotropic diffusion over meshes, in order to improve the smoothing in reasonable time.

Peng et al. (Peng, 2001) use locally adaptive Wiener filtering for denoising geometric data represented as semiregular mesh. The algorithm allows interactive local denoising.

Pauly and Gross (Pauly, 2001) apply Wiener filtering to restore surfaces from point clouds in presence of blur and noise.

Fleishman et al. (Fleishman, 2003) and Jones et al. (Jones, 2003) have independently proposed the use of Bilateral filtering based on robust statistics for features preserving and mesh smoothing.

Mederos et al. (Mederos, 2003) follows the same approach that Fleishman and Jones, by modifying a high order fitting method, called Moving Least Squares (MLS), to preserve sharp features. Their approach also considers optimization techniques for reducing the execution time of the algorithm.

Choudhury and Tumblin, (Choudhury, 2003) present a single-pass nonlinear filter for edge preserving and smoothing. The method is called Trilateral filtering and it is an evolution of the Bilateral filtering. The filter produces better outlier rejection and strong noise reduction than Bilateral filtering.

Schall et al. (Schall, 2005) have proposed a probabilistic method which consists of using a kernel density estimation technique. It associates to each point a local measure of probability to locate the point into the surface. The method achieves effectiveness result in filtering and robustness in outliers detection.

3 PRINCIPAL COMPONENT ANALYSIS

Principal Component Analysis (PCA) is a statistical method that tries to explain the covariance structure of data by means of various components expressed as linear combinations of the original variables (Hubert, 2005).

The first component of PCA corresponds to the direction in which the projected data have the largest variance. The second component is orthogonal to the first component, and maximizes the variance of the data points projected on it.

PCA is applied widely in bias identification into data sets. It is used for data dimensionality reduction and visualization (Jolliffe, 1986), data clustering (Pauly, 2002) and pattern recognition (De la Torre, 2001). Despite the versatility of PCA, it is sensitive to outliers present in data. Figure 1a shows a set of points mainly concentrated at the low part of the figure. Three of them, which are considered outliers, are enclosed in red circles. The first component of PCA, blue line, should indicate the main direction of data dispersion, but it is observed a bias produced by the outliers. Figure 1b shows the correction by robust PCA.

The PCA, first take a set of neighbors $N(p_i)$ around a point p_i, next the neigborhood mean $\overline{p}_i$ is estimated using (1). Finally, using (2), the covariance matrix CM is obtained from the points

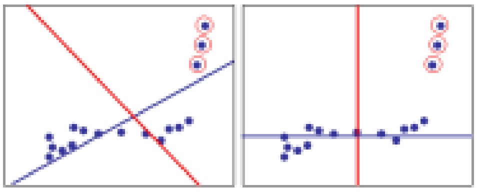


Figure 1: Principal components of a dataset. (a) PCA (b) Robust PCA.

and the neigborhood mean.

$$\overline{p}_i = \frac{1}{n}\sum_{j=1}^{n} p_j\,, \quad p_j \in N(p_i) \qquad (1)$$

$$CM = \frac{1}{n}\sum_{j=1}^{n}(p_j - \overline{p}_i)(p_j - \overline{p}_i)^T \qquad (2)$$

It is observed that CM is a symmetric positive semi-definite 3x3 matrix. The CM eigen-decomposition produce the principal components with their associated three eigenvectors v_1, v_2, v_3 and three real eigenvalues $\lambda_1 \le \lambda_2 \le \lambda_3$. The eigenvalues measure the variation of the points in the neighborhood along of the directions of their corresponding eigenvectors. The orthogonal eigenvectors v_2, v_3 define the directions of highest variation and expand a fitting plane on the neighborhood. The eigenvector v_1 approximates the normal vector at p_i.

4 PROPOSED METHOD

The method has four stages: weights assignment for the PCA by inversely proportional repartition, features preservation, shrinkage prevention and bias correction.

Starting from a noisy point set $P = \{p_i \in R^3\}$, close to an unknown and smooth two dimensional manifold boundary surface *S*, we want to determine a noise free point set *P'* that preserves the sharp features of the surface from which they were sampled. The main idea is to use a robust version of PCA, which allows determining a local fitting plane to a neighbourhood $N(p_i)$, close enough to the surface *S*. Such fitting plane is not influenced by the outliers due to the weights assignment by inversely proportional repartition. In addition, the PCA establish an orthogonal unit vector to the plane in the point p_i, which is an estimation of the true normal at p_i. Once we have the normal vector and the fitting plane, we apply the operator $p' = p + tn_p$ (Alexa, 2001, Fleishman, 2003) to preserve sharp features of *S*. Then, our method prevents the shrinkage and the drifting by shifting the neigborhood mean projection along the tangent plane. The bias produced by the linear approximation of PCA is reduced by applying the bootstrap method.

4.1 PCA and Inverse Repartition

The weights assignment by inversely proportional repartition is the key of the robustness of our robust PCA variant. The weights assignment is done according to (3). In that way, we punish with small weights the points farthest respect to the mean and we recompense with large weights the points near to the mean. Using inversely proportional repartition, the outliers influence over the mean is reduced and the principal components are not biased by them. This weights estimation is neither computationally expensive nor dependent of user parameters.

$$w_i = \begin{cases} 1, & d_i = 0 \\ \dfrac{1}{d_i \cdot \sum D_{mean}(N(p_j))}, & \text{in other cases} \end{cases} \qquad (3)$$

In (3) $\sum D_{mean}(N(p_j))$ is the sum of the distances between the points in the neighborhood of p_j (including p_j) and the neighborhood mean, d_i is the Euclidian distance between the neighborhood mean and a point p_i in the neighborhood. To estimate the weighted mean $\overline{p}_w$, robust to outliers, we use (4). The weights are estimated for each point in $N(p_j)$, and we use this weights and the weighted mean to compute the weighted covariance matrix, using (5).

$$\overline{p}_w = \frac{\sum w_i p_i}{\sum w_i} \qquad (4)$$

$$CM_w = \frac{1}{n}\sum_{i=1}^{n}(p_i - \overline{p}_w)(p_i - \overline{p}_w)^T W \qquad (5)$$

where CM_w is the weighted covariance matrix, $W = \{\sqrt{w_1},...,\sqrt{w_n}\}$, are the weighs associated to each point $p_i \in N(p_j)$.

Once we have already estimated the covariance matrix, its eigen-analysis produces a robust PCA. Figure 1b, shows the correction of the problem using our PCA variant, including the inverse repartition. These variations let our algorithm detect outliers and made it robust to noise.

4.2 Features Preservation

To prevent over smoothing of the point cloud and, in consequence, the lost of sharp features, we apply a shift to the mean along of the normal direction. We obtain the new position of the mean using (6).

$$\overline{p}' = \overline{p}_w + t_m n_p \tag{6}$$

where $\overline{p}'$, is the new position of the mean, $\overline{p}_w$ is the original weighted mean estimated by (4), n_p is the normal approximation to the plane at p_i, given by the robust PCA, and t_m is the displacement calculated by (7).

$$t_m = mean \sum_{p_i \in N(p)} \left\| n_p (p_i - \overline{p}_w) \right\| \tag{7}$$

The above quantity, under the constraint $\|n_p\| = 1$, is taken as the minimal height needed for displacing the mean along of the normal direction.

4.3 Shrinkage Prevention

Linear fitting algorithms are based on the mean of the neighborhood around a point p_i, this produce data shrinkage, because these algorithms use the neighborhood centroid $\overline{p}'$ (free of noise) instead of the original point p_i. We correct this problem by applying a shift to the centroid $\overline{p}'$, in the direction of the projection of the vector $p - \overline{p}'$ onto the tangent plane to the neighborhood. The centroid $\overline{p}'$ is on the tangent plane and p_i is on the point cloud, as it is shown Figure 2.

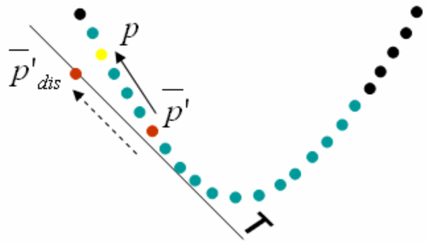


Figure 2: Data shrinkage prevention.

The new centroid $\overline{p}'_{dis}$ is calculated using (8).

$$\overline{p}'_{dis} = Ortho + \overline{p}' \tag{8}$$

$$Ortho = \langle (p - \overline{p}), T \rangle T \tag{9}$$

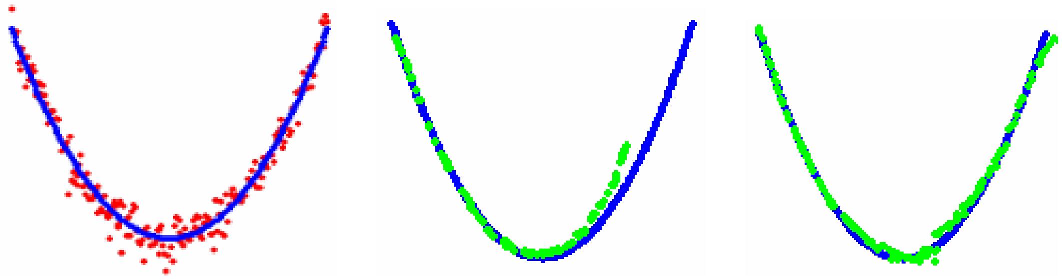
where *Ortho*, is the orthogonal projection of the vector $p - \overline{p}'$ onto the neighbourhood tangent plane T in $\overline{p}'$, and $\langle \cdot \rangle$ is the dot product operator. Figure 3a shows data points sampled from a synthetic curve (blue dots) and its corresponding noisy data (red dots). Figure 3b shows the smoothed noisy data without shrinkage prevention (green dots) and Figure 3c shows the correction of the shrinkage problem after applying (8). It is important to note that for a correct application of (8), the surface must be sufficiently sampled in regions with high curvatures.

4.4 Bias Correction

An additional problem introduced by the linear fitting is the bias between the original point and the neighborhood mean. We correct this problem using bootstrap bias correction (Martinez, 2002). We first perform a pass using our PCA variant; then, we perform a second pass including the bootstrap bias correction according to (10). Figure 4 shows the bias problem and its correction. Figure 4a shows data points sampled from a synthetic curve (blue dots) and its corresponding noisy data (red dots). Figure 4b shows the smoothed noisy data with bias (green dots) and Figure 4c shows the bias reduction (the green dots are closer to the blue dots) after the application of the bootstrap. The bootstrap is an iterative method, but in our experimental test, a single iteration was sufficient.

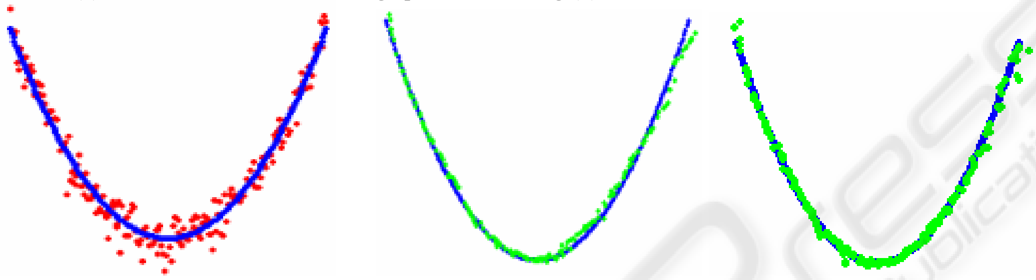
$$biasCorr = 2\overline{p} - \overline{\overline{p}} \tag{10}$$

where $\overline{\overline{p}} = \frac{1}{n}\sum_{i=1}^{n} \overline{p}_i$, is the new neighborhood mean after the second pass.



(a) (b) (c)

Figure 3: Shrinkage problem. (a) Noisy data represented by red dots (b) Smoothed data with shrinkage problem, represented by green dots (c) Smoothed data with shrinkage prevention using (8).



(a) (b) (c)

Figure 4: Bias correction (a) Noisy data represented by red dots (b) Smoothed data with bias problem, represented by green dots (c) Smoothed data with bias reduction prevention using (10).

5 ANALYSIS AND RESULTS

The proposed method has been tested in 2D and 3D data sets. The tests were run on a 1.5 GHz Athlon AMD, with 512 MB of RAM. The result shows that our algorithm has a good behaviour with different noise levels. Figure 3, used to illustrate the shrinkage problem correction, shows a synthetic data set corrupted with Gaussian noise (σ = 1% of the average spacing between the points). The points were sampled from a parabola in the interval [-1, 1].

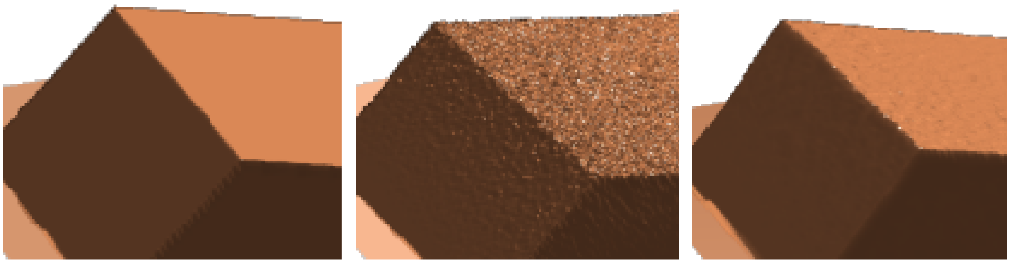
We observe how the algorithm controls the data shrinkage by applying (8), as shows Figure 3c (green points), in contrast with shown in Figure 3b, where the shrinkage prevention has not been applied (green points). The data shrinkage problem is more evident at the extremes. In our tests, the proposed method for preventing the shrinkage problem allowed to reduce the relative error, between the real and the approximated volume, from 34% to 10%.

The parabola data set (Figure 4) is also used to illustrate de bias problem. In one hand we observe the bias between the data points mean (green dots) and the original data points (blue dots). In the other hand, we observe in Figure 4c, the result of applying the bootstrap bias correction (green dots are closer to the blue dots). In our tests, after applying (10) the bias reduction reaches a 57%, i.e. the distance between the real data points and the smoothed data points after applying the bootstrap technique was reduced until a 43% of the initial distance.

The data sets used to illustrate the result of our method for point cloud denoising were corrupted with Gaussian noise (zero mean and variance σ) along of the normal direction. We specified the noise magnitude as a percentage of the average z coordinate or the diagonal of the bounding box of the point cloud.

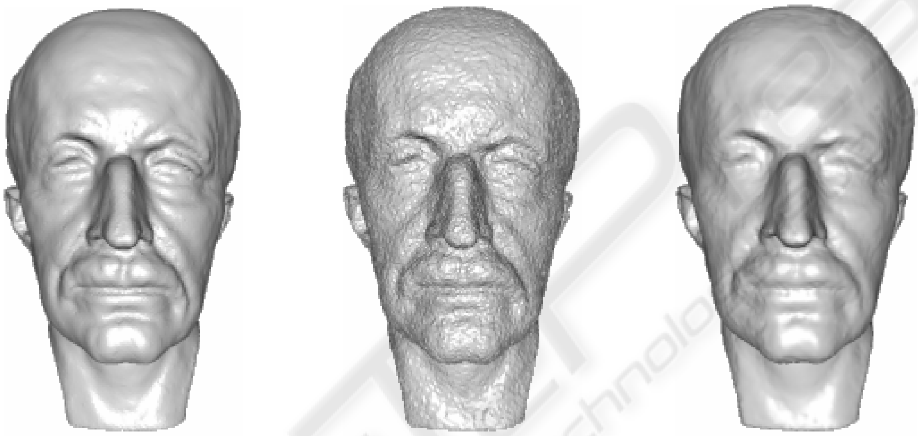
Figure 5 shows a cube model corrupted with Gaussian noise (σ = 5% of z coordinate average). We observe how our algorithm preserves sharp features (corners and edges) while smoothes the point cloud.

In Figures 6 the Max Planck model (50k points) corrupted with Gaussian noise (σ = 0.08% of diagonal of the bounding box) is shown. The model was smoothed in 13 seconds. Figures 7 and 8 show the Venus model (70k points) corrupted with Gaussian noise (σ = 0.05% and σ = 0.025% of the diagonal of the bounding box respectively). The model was smoothed in 19 seconds. In Figure 9 a bird model was corrupted with Gaussian noise (σ = 0.1% of z coordinate average) to illustrate the model detail before and after applying our algorithm. It is observed the effectiveness of the method eliminating the noise and preserving the features.



(a) (b) (c)

Figure 5: Sharp features preservation. (a) Original cube model (b) Cube model corrupted model with Gaussian noise (c) Corrupted cube model after filtering.



(a) (b) (c)

Figure 6: Max Planck model. (a) Original Max Planck model. (b) Max Planck model corrupted with Gaussian noise (c) Corrupted Max Planck model after filtering.

(a) (b) (c)

Figure 7: Venus model left view. (a) Original Venus model. (b) Venus model corrupted with Gaussian noise (c) Corrupted Venus model after filtering.

(a) (b) (c)

Figure 8: Venus model right view. (a) Original Venus model. (b) Venus model corrupted with Gaussian noise (c) Corrupted Venus model after filtering.

(a) (b)

Figure 9: Bird model. (a) Bird model corrupted with Gaussian noise (b) Corrupted bird model after filtering.

6 CONCLUSION AND FUTURE WORK

In this paper we have presented a new and robust method for point clouds denoising. The method is a PCA variant that preserves sharp features of the original surface. In contrast with previous work, our method does not require high order local fitting algorithms (like MLS), or global approximation to surface (like triangular meshes or graphs). The proposed method operates directly on the points and does not require neither nonlinear optimization algorithms nor parameters provided by users. The method is computationally efficient and easy to implement.

There is a way to improve our method using an adaptive neighborhood size instead fixed size. The neighborhood size should depend on the local characteristics like curvature and density, this will allow a better normal estimation and, in consequence, would improve the application of the operator $p' = p + tn_p$. Taking into account the local curvature, we can reduce the bias between the neighborhood mean and the data point, in this way, we are closer to the point of the original surface.

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