EFFICIENT RENDERING OF HIGH-DETAILED OBJECTS USING A REDUCED MULTI-RESOLUTION HIERARCHY

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Abstract: In the field of view-dependant continuous level of detail of triangle-meshes it is often necessary to extract the current LOD triangle by triangle. Thus, triangle strips are only of very limited use, or only usable with a high effort. In this work a method is proposed that allows a stepwise reduction of a great, fine stepped LOD hierarchy by merging nodes. The result of this process is a reduced hierarchy, which allows the extraction of many neighboring static triangles in one step, so that triangle strips are applicable more efficiently. We show that this results in a significant decimation of processed vertices without loosing a smooth LOD transition.

1 INTRODUCTION

It is not possible to render high-detailed triangle meshes with interactive framerates even with current hardware. In many cases the triangle size in image space is very low (< 1 pixel) and it is often more necessary to achieve interactive framerates than showing fine details. Thus, level of detail techniques are an important part of many rendering techniques for large scenes and high-detailed objects. For this purpose, the original triangle mesh is simplified using global or local operations (see (Luebke et al., 2002) for an overwiew). Every operation creates a level of detail (short LOD) of the original mesh. During rendering a certain LOD is selected with respect to a high image quality versus low costs.

For doing so, there are two techniques: Discrete techniques save a certain number of offline generated LOD. Then, in every frame, an appropriate LOD is selected, e.g. depending on the viewer distance. On the other hand, continuous techniques use a data structure to extract an appropriate LOD, which contains the whole detail spectra of the original mesh. The advantage of continuous techniques is a very smooth LOD transition in contrast to discrete approaches, and they adapt the LOD better to the viewing situation, so that in most cases less triangles have to be rendered compared to using a discrete LOD.

On the other side graphics cards are specialized to accelerate the rendering of large triangles sets, e.g. using *triangle strips*. A triangle strip is a sequence of triangles in which adjacent triangles share a common edge. A strip of k triangles is described as a sequence of $k + 2$ vertices: three vertices for the first triangle and one for each additional triangle. A set of triangle strips that contains all triangles of a mesh is called a *striptification* of that mesh.

Triangle strips are very useful when using discrete LOD techniques, because the whole LOD is rendered at once. In contrast, triangle strips are less useful when using continuous techniques. This is because triangles, that belong to LOD are extracted in small sets only. Thus, the triangle strip length is limited to this set size. To overcome this limitation more sophisticated and time consuming striptification techniques have to be used. Thus, sometimes discrete LOD techniques are more efficient than continuous LOD, although discrete LOD contain more triangles (but less vertices because of longer triangle strips).

To decrease the number of processed vertices in continuous LOD by using triangle strips, a trade-off has to be found: On the one hand the calculation effort should be small (time-consuming calculations should be done offline), and triangle strips should be as long as possible on the other hand.

In this paper a continuous technique is proposed that uses triangle strips more efficiently. Thus, the rendering process is accelerated significantly. A LOD is created by small patches in our approach. Every patch contains a small amount of adjacent triangles.

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Such a patch is described by a few triangle strips. We have developed a technique to enlarge these patches offline before rendering, so that longer triangle strips are useable. In contrast to other approaches these triangle strips are static, so that the striptification is computed offline, too. This results in a fast LOD extraction during rendering time.

The remainder of this paper is structured as follows. First, in section 2, we will give a short overview on related works. Section 3 describes the basic LOD structure we use. In section 4 we demonstrate, how the patch size is increased by reducing this structure. After this, we will discuss the achieved results in section 5. We conclude with a short summary and an outlook to future work in section 6.

2 RELATED WORK

The rendering acceleration of continuous LOD meshes by using triangle strips was the issue of some previous works. The *Skip Strip* algorithm (El-Sana et al., 1999) uses a merge tree that is represented by a skip strip. For the original mesh a striptification is created, which is updated and repaired during traversal the strip. In addition, the generated triangle strips are scanned for redundant vertices before rendering.

In (Stewart, 2001) an elegant and efficient algorithm for creating a striptifaction of a static mesh is proposed using a so called *tunneling-operator*. This operator connects previously given triangle strips iteratively to reduce their number. They show how this operator can be used to update a striptification that was broken by a local vertex-split (resp. edge-collapse) operation. Thus, it can be used for continuous LOD, too.

In (Velho et al., 1999) an initial striptification is generated for a low detailed mesh. After this, the mesh is refined by inserting new vertices and edges, so that the resulting new triangles can be inserted in the initial striptification, without requiring more triangle strips. The position of an inserted vertex is estimated from an implicit or parametric surface description.

All these techniques do not limit the length of triangle strips. On the other side triangle strips are not static and have to be updated from LOD to LOD, so potentially in every frame. This is time consuming, especially the LOD changes a lot. In our approach the triangle strip length is limited, but they are static, and can be generated offline. Since the benefit of triangle strips decreases with their length, the overhead for processed vertices is small (as shown in section 5). In addition the whole striptification can be stored in a vertex buffer on the graphics card to further reduce the rendering time. This is not possible using dynamic triangle strips, as are used in the described approaches.

Beside these works, there are algorithms that focus on special object types, unlike our approach, which can be used for nearly all kind of objects: In (Lindstrom et al., 1996) an efficient algorithm for height fields is proposed. The single LOD of these landscapes are represented by *Quad-Trees*. The ROAM-Renderer (Duchaineau et al., 1997) was developed for landscapes, as well. It includes an iterative algorithm to get and update triangle strips with a length of four or five triangles. Both approaches also create triangle strips during or after LOD selection, unlike our approach, which uses static triangle strips.

3 BASIC LOD HIERARCHY

To render an original mesh regarding the viewing distance and other parameters, a data structure is needed that allows the access to all surface areas in different resolutions. These structures are in general hierarchies or trees, like the *merge tree* (Xia et al., 1997). We use the elegant *MT-hierarchy* developed in (Floriani et al., 1997). Using this, it is already possible to extract a small patch of two or more fixed triangles that belong to a certain LOD at once. This is an important property as shown in section 4.

The MT-hierarchy is a directed acyclic graph (abbreviated *DAG*) $G = (\mathcal{N}, \mathcal{A})$. Its nodes $\mathcal{N}$ represent local simplification operation and are labeled by a simplification error. Hierarchy arcs[1] are labeled by triangle sets. We denote a triangle set of an arc a as a patch p_a. The outgoing arcs of a node contain all triangles that are changed (or deleted) by this simplification operation, and its ingoing arcs contain these changed triangles (figure 1). There are two special nodes: The *drain node* n_D at the bottom of the hierarchy (with simplification error 0), whose ingoing arcs represents the original mesh, and the *source node* n_S on top of the hierarchy (with simplification error ∞), whose outgoing arcs contains the most simplified mesh.

3.1 Hierarchy Creation

The hierarchy is generated bottom up, as described in (Floriani et al., 1997). We use edge-collapse operations (Hoppe et al., 1993) to simplify the original mesh iteratively: All possible edge-collapse operations are ordered in a priority queue, in respect to their simplification error. This queue is processed until it

[1]Since we operate on meshes, the term ”edge” already has a meaning. For clarity the term ”arc” is used for a hierarchy edge.

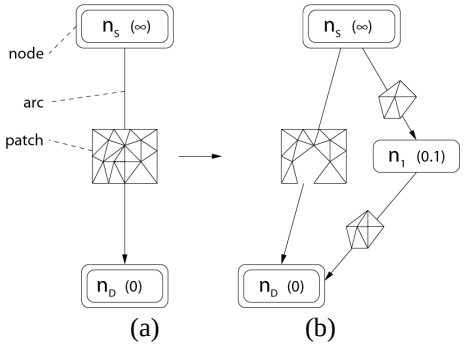


Figure 1: Hierarchy creation steps: basic mesh, the dashed edge is collapsed (a), patch hierarchy after this edge-collapse operation with simplification error 0.1 (b).

is empty. We calculate the simplification error using the widely used *quadric error metric* (short: QEM) (Garland and Heckbert, 1997). By this iterative creation and by the QEM it is ensured that the simplification error of the father node n_i of every arc (n_i, n_j) is greater than (or equal to) the simplification error of its child node n_j.

3.2 LOD Selection

To extract a LOD out of the hierarchy it is necessary to create a *cut*. A cut is an arc subset $\mathcal{C} \subseteq \mathcal{A}$ with the following two properties:

1. $\forall\, a=(n_i,n_j), a'=(n_k,n_l) \in \mathcal{C} : \not\exists\ n_j \rightarrow^* n_k$ ($\rightarrow^*$ denotes a path of any length)
2. $\mathcal{C}$ is *maximal* (no arc can be added to $\mathcal{C}$ without breaking property 1).

The first property ensures that there are no overlapping arc patches. The second property ensures on the other hand, that the whole original mesh is covered by the arc patches of the cut.

Since the simplification error e_i, that is stored in every node n_i, decreases monotonically from top to bottom of the hierarchy, a cut is uniquely defined by:

$$C \quad = \quad \{(n_i,n_j) \in A :\ e_i \geq \varepsilon > e_j\} \qquad (1)$$

as shown in figure 2. The parameter ε describes the desired LOD accuracy. A simple algorithm scans $\mathcal{A}$ linearly for cut determination. Since the hierarchy has a logarithmic height, it is of course more efficient to traverse the hierarchy bottom up or top down to avoid unnecessary arc tests. Usually a cut does not change very much from frame to frame. Thus, it is even better to update the cut of the last frame up or down to get the cut for the current frame as described in (Floriani et al., 1998).

Since the QEM measures a *quadratic* distance in object space, ε has to be interpreted as a quadratic

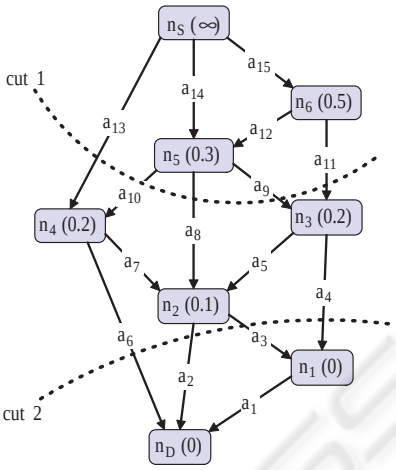


Figure 2: Demonstration of two cuts specifying different LOD. The first cut $\mathcal{C}_1 = \{a_8, a_9, a_{10}, a_{11}, a_{13}\}$ for $\varepsilon = 0.25$ and the second cut $\mathcal{C}_2 = \{a_2, a_3, a_4, a_6\}$ for $\varepsilon = 0.05$.

distance, too. However, using an image error in pixels is more intuitive. Thus, we propose to define ε by a pixel value γ in image space: Using a perspective projection the length l of a line defined in object space with a distance d to the viewer has the maximum pixel length s in image space of

$$s(l,d) \quad = \quad \frac{l}{d} \cdot \frac{h}{2\tan\left(\frac{\alpha}{2}\right)}, \qquad (2)$$

where h is the output image height (in pixels) and α is the field of view angle.

If the user defines an image error γ in pixels, then its *quadratic* length in object space is given by

$$\varepsilon = (s(l,d_O)^{-1})^2 = \left(\frac{d_O}{\gamma} \cdot \frac{2\tan\left(\frac{\alpha}{2}\right)}{h}\right)^2, \qquad (3)$$

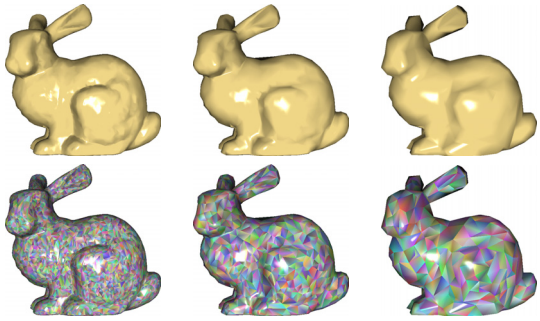
where d_O is the object distance to the viewer. Using the bounding sphere of the object with center M_O and radius r_O, the value of d_O is bounded below by

$$d_O \quad = \quad \max(|M_O - V| - r_O,\ 0), \qquad (4)$$

where V is the viewer position in object space. If γ, d_O, the image dimension h or the field of view angle α change, ε is recalculated and a new cut is determined.

In figure 3 examples for different choices of γ and their effect on image quality are shown.

This cut estimation only considers the object distance to the viewer. But it is of course extensible by using a more sophisticated cost and benefit heuristic as proposed in (Funkhouser and Sequin, 1993).



(a) $\gamma = 1$ 13k vertices (b) $\gamma = 5$ 4k vertices (c) $\gamma = 20$ 1.8k vertices

Figure 3: Showing LOD selection using the Stanford bunny: With increasing allowed pixel error the granularity of the mesh decreases significantly for the benefit of less vertices.

4 PATCH ENLARGEMENT

Our purpose is to accelerate rendering by using triangle strips, because the number of processed vertices is a frame rate limiting factor. Since we construct a certain LOD out of several patches, triangle strips can only be defined within these patches. But an arc patch of the hierarchy contains only ≈ 2 triangles in average, so triangle strips are not very efficient. Thus, it is useful to enlarge these patches.

4.1 Arc-Collapse Operator

To achieve a larger average patch size, a coarser local simplification operations than the common used edge-collapse operator could be used, e.g. the removal of more than one vertex at once, with a following retriangulation of the created mesh hole. But it is hard to archive a specified average patch size using this approach. In addition other operators have to be used (and implemented), if a different patch size is desired. Thus, we propose another way: To increase the average patch size we introduce an *arc-collapse operator*, which merges two adjacent hierarchy nodes. This has the effect that the ingoing arcs (resp. outgoing arcs) of both merged nodes with the same father node (resp. child node) are merged to one arc each (figure 4). Thus, larger patches are created. This arc collapsing can be interpreted as the application of two simplification operations at once instead of one after the other.

To guarantee that the used hierarchy is still a DAG after an arc-collapse operation, it has to be ensured that there is no other path between the merged nodes than this arc, otherwise a cycle is created (figure 5). This would cause a conflict situation during cut estimation.

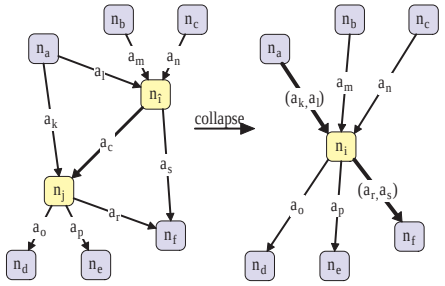


Figure 4: Arc collapse operation: If arc $a_c = (n_i, n_j)$ is collapsed to a new node other arcs a_k, a_l and a_r, a_s are merged to new arcs, that contain larger patches.

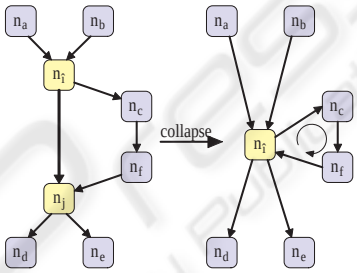


Figure 5: Error situation: If there is another path between two nodes of an arc, this arc can not be collapsed, because a cycle would be created.

A collapse of randomly selected arcs is not purposeful, because several criteria should be considered: On the one hand a desired average patch size should be achieved by collapsing as few arcs as possible. On the other hand an efficient striptifaction of the resulting patches and the keeping of a smooth LOD transition are important.

For achieving an average patch size two factors are relevant.

1. The number of triangles in the patch p_a of the collapsed arc a, denoted as $|p_a|$: If $|p_a|$ is small the average patch size is bigger after collapsing this arc than if $|p_a|$ would be greater. Hence, a small $|p_a|$ should be preferred.
2. The patch sizes of merged arcs: $\mathcal{M}_a$ denotes the set of the merged arc pairs (in figure 4: $\mathcal{M}_a = \{(a_k, a_l), (a_r, a_s)\}$). Using this, the sum ps of the patch sizes is estimated by:

$$ps(a) = \sum_{(a',a'') \in \mathcal{M}_a} |p_{a'}| + |p_{a''}| \qquad (5)$$

If ps is big, few large, or many small patches are created, which yields a bigger average patch number as wanted. On the other side, it is better to merge as many small patches as possible in order to get a nearly constant patch size over the whole

(a) original 7512 patches | (b) av. patch size 5 3575 patches | (c) av. patch size 10 1941 patches | (d) av. patch size 20 1018 patches

Figure 6: Rendering of the same LOD of the Armadillo model with different average patch sizes (each patch is differently colored).

hierarchy, because patches can only be enlarged. Thus, it is useful to use ps in relation to the number of merged arcs:

$$ps_{rel}(a) \quad = \quad \frac{ps(a)}{|\mathcal{M}_a|}. \tag{6}$$

It sometimes happens that merging two arcs in $\mathcal{M}_a$ does not form a contiguous patch. Thus, for these patches at least 2 triangle strips have to be used, so that there would be no benefit in the number of processed vertices. Denotes $\mathcal{U}_a \subseteq \mathcal{M}_a$ the set of arc-pairs with neighboring patches:

$$\mathcal{U}_a = \{(a', a'') \in \mathcal{M}_a : p_{a'}, p_{a''} \text{ neighboring}\}, \tag{7}$$

then a relation adj that considers this is given by:

$$adj(a) \quad = \quad \frac{|\mathcal{U}_a|}{|\mathcal{M}_a|} \tag{8}$$

and should be considered for selecting an arc to collapse, whereas adj should be as large as possible.

To preserve a smooth LOD transition inside the hierarchy as far as possible, the relation e_{diff} of the simplification errors of a collapsed arc's father node and child node is considered. If it is *small*, a smooth LOD transition is preserved after collapsing this arc. The value e_{diff} for every $a = (n_i, n_j)$ is given by:

$$e_{diff}(a) \quad = \quad \frac{\max(e_i, \mu)}{\max(e_j, \mu)}, \tag{9}$$

where μ is a very small constant. This constant is necessary because e_j can be 0 (e.g. if n_j is the drain node n_D). Thus, $e_{diff} = \infty$, which would have the effect that such edges would not be collapsed, although a smooth LOD transition could be given. Our choice for μ is the smallest simplification error $\neq 0$ of all nodes in $\mathcal{N}$ divided by 10, which seems to be a practicable value.

To use all these factors for creating an order of arc collapse operations, it is useful to combine them to a single weight $w(a)$ for every arc a. We found that it is very effective to multiply these factors (or their reciprocal) equally weighted, so that a small weight means to collapse an arc before higher weighted arcs.

$$w(a) = |p_a| \cdot e_{diff}(a) \cdot ps_{rel}(a) \cdot \frac{1}{adj(a)}. \tag{10}$$

To achieve a specific average patch size our procedure is as follows: All arcs are ordered in a priority queue, starting with the smallest weight. Then, iteratively, the arc at the head of this queue is collapsed (if it does not produce a cycle, of course). During this, weights of the in- and outgoing arcs of the merged nodes are updated. We stop collapsing arcs if the desired average patch size is reached (or if there is only one arc left in the hierarchy).

Results of such a hierarchy reduction by using arc-collapse operations are shown in figure 6: The number of patches gets significantly smaller with bigger patch sizes.

4.2 Node Error Adjustment

As illustrated in figure 7(a) the simplification error of an arc's father node can get smaller than its child node after an arc-collapse operation. Thus, the LOD selection algorithm (3.2) is non-deterministic. To solve this problem the hierarchy is repaired by an additional breadth-first traversal after reduction. This traversal detects these problematic arcs. If such an arc is found the simplification error of its father node is set to its child's simplification error (figure 7(b)). This slightly decreases the LOD selection precision. Thus, a higher LOD is selected than necessary, which reduces exiguously the efficiency of the reduced hierarchy.

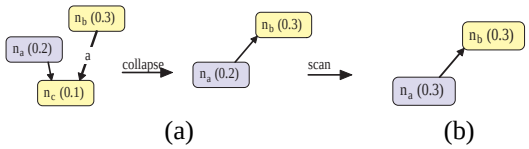


Figure 7: After collapsing arc a the monotonic bottom-up increase of simplification errors is broken.

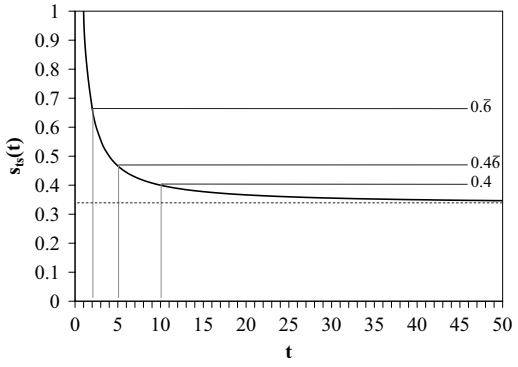


Figure 8: Efficiency of triangle strips in comparison to render each triangle of the strip separately.

5 DISCUSSION AND RESULTS

After introducing a possible way to achieve a certain average patch size easily, in this section we will discuss, what patch size is appropriate for what purpose. In addition the efficiency of our approach is shown, especially in comparison to discrete LOD.

5.1 Appropriate Patch Sizes

To answer the question what average patch size provides most benefits, we first look at the saving factor that triangle strips provide. This is the number of vertices of a strip in relation to the number of triangle vertices:

$$s_{ts}(t) = \frac{2+t}{3t}, \qquad (11)$$

where t is the number of triangles. As you can see in figure 8, the benefit of triangle strips converges to $1/3$ very fast with increasing triangle number. Using the original hierarchy with an average patch size of 2 you can save $\approx 33\%$ vertices by using triangle strips (assuming that every patch is represented by only one strip). If the patch size is increased to 10, an additional 27% is saved, which is only $6.\bar{6}\%$ above the theoretical minimum. That this benefit is nearly reached can be seen in figure 9(a), which shows the number of processed vertices for our Armadillo model at different LOD for some different average patch sizes, and

Table 1: Number of arcs and needed vertex-buffer memory using hierarchies for the Armadillo model with different patch sizes.

patch size	#arcs	vb size	saving
original ($\approx$2)	722076	147.8MB	–
5	258504	99.1MB	32.9%
10	118403	84.5MB	42.8%
20	55612	76.5MB	48.2%

in figure 9(b), which shows the vertex number in relation to the original hierarchy. In some cases the results are not exactly as good as expected. This is because for some large patches more than one triangle strip is needed.

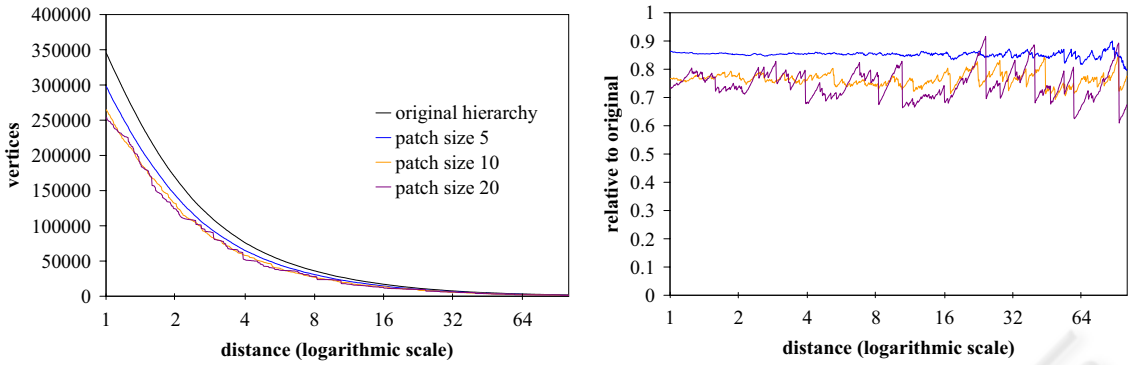
It can also be seen in figure 9(b) that the results for using bigger patch sizes are always better than using the original hierarchy, which shows that the patch size is very uniform over the whole reduced hierarchy. It is also easy to see in figure 9(b) that the relation varies significantly on low LOD using bigger patch sizes. This is because the LOD transitions get coarser with higher distances too, and the LOD do not adapt to user distance as well as when using the original hierarchy.

Another positive effect of reducing the hierarchy is that by collapsing arcs, the patches of these arcs are deleted, too. Thus, the number of vertices that are held in memory is reduced additionally. As shown in table 1 the number of arcs in a hierarchy with an average patch size of 20 is 13 times less than the number in the original hierarchy. If the triangle strip vertices of all patches are stored in one vertex buffer, using three attributes each (position, normal and color), for the Armadillo model you need 147.8MB using the original hierarchy. If the hierarchy of patch size 20 is used, this is reduced by over 48%. In addition, the cut estimation (as shown in section 3.2) is even faster, because less arcs have to be checked, whether they belong to cut or not.

We can conclude that if a fine LOD transition is important, an average patch size of $5-10$ is a good choice. If the memory requirements of the vertex buffer is more important, e.g. it should fit in the very limited graphics card memory, a value over 10 should be used instead. Hence, an average patch size of 10 is a good compromise between a fine LOD transition, vertex number and memory requirements.

5.2 Patch Striptification

The used triangle strip algorithm has a big influence on our approach. We use the tunneling-algorithm (Stewart, 2001), because it always reaches better results than the classical *SGI* algorithm (Akeley et al., 1990) or the *STRIPE* algorithm (Evans et al., 1996). Using this we reach an average triangle strip number



(a) (b)

Figure 9: Number of triangle strip vertices of different LOD at different distances using different patch sizes (a) and these numbers of vertices in relation to to the original hierarchy (b).

per patch of less than 2 even when using a patch size of 50.

5.3 Comparison to discrete LOD

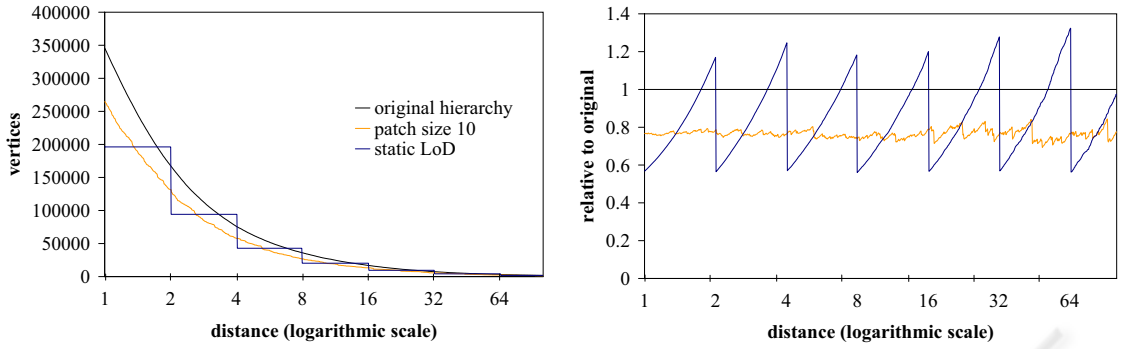
A discrete LOD transition has several benefits compared to a continuous LOD, because the whole LOD mesh is given explicitly. Thus, no LOD extraction is necessary. In addition the whole LOD mesh can be striptified. Thus, the triangle strip length is not limited, which generally results in less triangle strips, unlike in our approach. Hence, also fewer vertices are used to render the LOD. As shown in section 5.1 and figure 8, the theoretical maximum of this vertex decimation is over 33% compared to the original hierarchy with a patch size of 2. But if an average patch size of 10 is used, this decimation is only about 7%. In addition a discrete LOD algorithm does not produce a smooth LOD transition.

We compared our approach with a discrete LOD mechanism. To do so, we created a cut using the original hierarchy for every distance with a positive integer power of 2, and used these cuts to create discrete LOD. Every discrete LOD got a validity up to the next lower LOD. In figure 10(a) the number of vertices used for rendering is shown for these discrete LOD in comparison to the original hierarchy and a hierarchy with patch size 10. As you can see, the discrete LOD is better at the beginning of each distance interval. But the reduced hierarchy adapts the number of vertices better to distance, so that it is more efficient towards the end of each interval. In figure 10(b) the number of vertices in relation to the original hierarchy is shown. Especially here you can see the range of distance where our approach delivers better results (compared to discrete LOD) exceeds the number of distance intervals where the results are poorer, while it also provides smooth LOD transitions, that discrete LOD does not.

6 CONCLUSION AND FUTURE WORK

In this paper we described a technique that allows to use the benefits of continuous LOD (smooth LOD transition) and discrete LOD (good striptification, fast LOD selection). Using a fine-granular LOD-hierarchy as a base, we reduce this by iteratively using an introduced $arc - collapse$ operator. This reduced hierarchy allows the access to larger surface areas (patches) of a specific resolution than before. Thus, using triangle strips for these patches, a certain LOD is described by less vertices, which yields a significant reduction in vertices that have to be processed by the graphics card. Since the number of vertices adapts well to the viewing situation, our approach mostly uses even less vertices than a discrete LOD. This is shown by results.

Our technique works well for all kind of objects that can be described by previous multi-resolution techniques, too. For objects of a high complexity, like plants, this is not the case because these objects can not be simplified by local triangle based simplification operations very efficiently. Thus, even the lowest LOD contains many triangles. For such objects, point based approaches or combined approaches using hybrid hierarchies in the sense of (Cohen et al., 2001) should be preferred, because points are not defined by edges as triangles are, but by isolated vertices. One could imagine to use our approach for point hierarchies as well. Since point hierarchies mostly store points in nodes and not in arcs, a *node-collapse operator* is imaginable to reduce such point hierarchies. Thus, larger amounts of points could be rendered at once. This may accelerate point based LOD rendering using such reduced point hierarchies.



(a) (b)

Figure 10: Processed vertices using the original hierarchy and a hierarchy of patch size 10 in comparison to discrete LOD (a) and in relation to the original hierarchy resp. (b).

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