SURFACE CONSTRUCTION USING TRICOLOR MARCHING CUBES

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Abstract: This paper presents a generalized marching cubes (MC) method for 3D surface construction. Existing MC methods require the sample values at cell vertices to be different from the threshold or modify them otherwise. The modification may introduce topological changes to the constructed surface. The proposed Tricolor MC method allows cell vertices with sample values which equal the threshold to lie on the surface. It constructs the surface patches by exhausting the Eulerian circuits in the cells without changing sample values. The simulation results show that the TMC method better preserves the topology of the surfaces that pass through cell vertices and gives good results in other general cases.

1 INTRODUCTION

Three dimensional (3D) surface construction is a widely studied topic in computer graphics and computer vision. Polygonal representation of 3D surface, or mesh, is extensively used in rendering on graphics hardware, texture mapping, and shape analysis. The marching cubes (MC) method is the most popular method to generate isosurface on volumetric data at uniform grids. Proposed by Lorensen and Cline in 1987, MC method is fast in extracting high resolution isosurfaces and simple to implement (Lorensen and Cline, 1987). However, the original MC method has problems such as topology inconsistency and representation inaccuracy. Since then a variety of MC methods have been proposed for improvements.

Nielson and Hamann proposed an asymptotic decider to solve the face ambiguity which happens in a cube face with two diagonally opposite vertices marked positive and the other two marked negative (Nielson and Hamann, 1991). Chernyaev used 33 configurations to eliminate the internal ambiguity which occurs in the interior of a cube (Chernyaev, 1995). Ho et al. proposed CMS method to preserve sharp features (Ho et al., 2005).

One of the original MC assumptions is not addressed by later variations, which assumes the cell vertices lie either inside or outside the surface, not on the surface. It meets a problem when isocontours such as level set pass through cell vertices. To elide this problem, existing MC methods modify the sample values that equal threshold. This modification, however, may introduce obvious topological changes in some cases as shown in the latter part of the paper.

This paper proposes a more general MC method that does not modify the sample values. It constructs surface by finding Eulerian circuits in the cell. The paper is organized as following. In Section 2, the background for MC method is briefly introduced. In Section 3, we propose the new MC method and introduce the surface construction procedure. The experimental results are presented in Section 4. Section 5 concludes the paper.

2 BACKGROUND

The output of the marching cube algorithm is a mesh representing the 3D surface. A mesh M is a pair (K, V), where K is a simplicial complex representing the connectivity of the vertices, edges and faces, thus determining the topological type of the mesh, and $V = \{v_1, \ldots, v_m\}, v_i \in \Re^3$ is a set of vertex coordinates defining the shape of the mesh in $\Re^3$.

The input of the MC algorithm is volumetric data, which has sample value associated with each grid point. The samples at cell vertices are thresholded before isosurface extraction. Hereafter the sample

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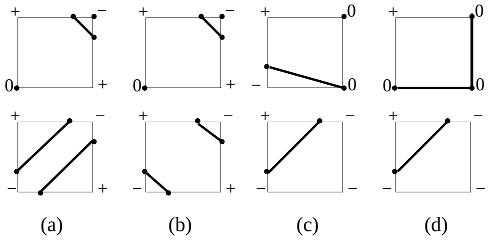


Figure 1: Topological changes caused by sample value modification. In each subfigure, the figure on the top shows the original sample values, while the figure at the bottom shows the possible topological change.

value is referred as that minus threshold. And we call the cell vertex with positive/negative/zero sample value $positive/negative/zero$ cell vertex respectively. Existing MC methods do not allow cell vertices lie on the surface, or zero cell vertices. Under this assumption, the surface is limited to intercept the cell in $2^8 = 256$ ways since each of the eight cell vertices can lie either inside or outside the surface. It is equivalent to coloring the eight cell vertices with two colors. Hence we call existing MC methods $bicolor$ methods. By exploiting the symmetries in the 256 ways, Lorensen et al. summarized the cases into 15 patterns in the original MC method (Lorensen and Cline, 1987).

To hold this assumption, existing bicolor MC methods have to modify the zero sample value in practice. For example, as level set may pass through grid points, Han et al. change the sample value at such grid points from zero to negative (Han et al., 2003). This kind of modification may introduce obvious topological changes, which is shown in Fig. 1. The thick line in the figure represents the interception of surface and cell face. Modification on sample value changes the original cell face into ambiguous faces as shown in (a) and (b). And the interceptions become completely different after modifications on (c) and (d). Secondly, the isosurface extracted should be neutral with positive and negative data samples. In other words, the isosurface should not change if we multiply the sample data with -1. Modifying the sample value, however, can lead to two different topological results for the same set of data as shown in the experiment section of this paper. Thirdly, modifying sample value may introduce nonexistent face as shown in the experiments.

This defect is regarded as a technical problem in (Gelder and Wilhelms, 1994), and one of the several major artifacts in most existing isocontour software (Han et al., 2003). One of the reasons that existing bicolor MC methods elide this problem is too many cases are introduced without this assumption. If the sample at cell vertex is allowed to be zero, each of the eight cell vertices has three instead of two possible positions relative to the surface, i.e. inside, outside or on the surface. Thus the surface intercepts the unit cube in a total of $3^8 = 6561$ ways. It is equivalent to coloring cell vertices with three colors. We name our method $tricolor$ MC (TMC) to reflect this character. To enumerate all these ways and reduce the symmetries manually, as what has been done in (Lorensen and Cline, 1987), is extremely tedious and error-prone to human. In the following sections, we will eliminate this assumption to include zero cell vertices without modification.

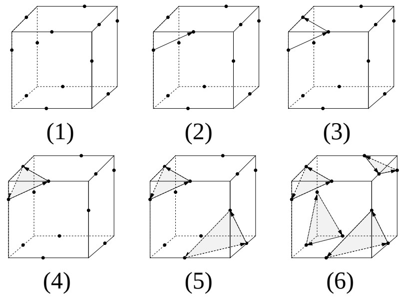


Figure 2: An example of one possible way to find 4 patches edge by edge in a cell with 12 vertices.

3 SURFACE CONSTRUCTION

For the convenience of discussion, we define $patch$ as the close interception of the isosurface and a cell, a polygon formed by connecting the surface mesh vertices. A $patch\ edge$ is the interception of the surface with a cell face, whose end points are two adjacent surface mesh vertices on the cell edge. The $degree$ of surface vertex is the number of patch edges incident on the vertex. Since we are only interested on the patch edges in one cell, hereafter the vertex degree actually refers to the degree in one cell. As mentioned in Section 2, the cell may hold the patches in 6561 ways if taking zero cell vertices into consideration. Instead of enumerating all these cases, we go back to two dimensions to examine the ways of the interception of isosurface and one cell face, or the patch edges. As the patch is enclosed by patch edges, it forms an Eulerian circuit, a graph cycle which uses each graph edge exactly once. It suggests finding the patches in the cell by finding the Eulerian circuits. We start from any of the surface vertex, find one patch edge incident on it if possible, and continue with the surface vertex that is the other endpoint of the patch edge until we return to the original vertex. After one Eulerian circuit, or a patch, is completed, repeat the process until all the patch edges in the cell have been visited. Then we find all the patches in the cell. An example of one possible way to find 4 patches edge by edge by within one cell is illustrated in Fig. 2.

As more than two surface vertices may lie on a

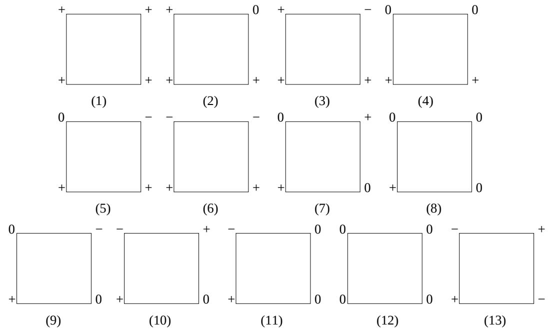


Figure 3: 13 cases of squares with zero cell vertices.

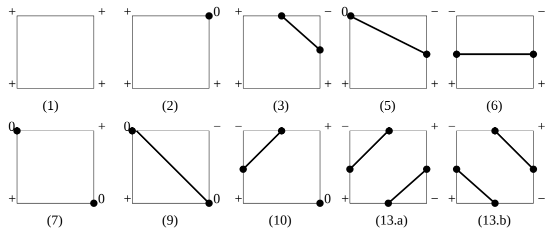


Figure 4: 10 cases out of the original 13 cases in Fig. 3 with zero cell vertices and their corresponding patch edges.

cell face, there are multiple ways to define the patch edges. And the resulted patches in the cell would be completely different. So we need to count how many ways to define the edges on a cell face and make our choice. Banks et al. counted the cases to produce substitopes in the MC method (Banks and Linton, 2003). They enumerated 13 distinct cases in two dimensions with zero cell vertices as shown in Fig 3, in which symmetries between positive and negative cell vertices have been eliminated. Many of these cases, however, allow ambiguous ways to connect the patch edges. Various methods have been proposed to solve the ambiguities. The bilinear interpolation over the cell face is adopted to solve the ambiguity problem in our study,

$$\begin{aligned}\phi(u,v) = \phi_{00}(1-u)(1-v) + \phi_{01}(1-u)v \\ +\phi_{10}u(1-v) + \phi_{11}uv\end{aligned} \tag{1}$$

where $\phi(u,v)$ stands for the value at logical coordinates (u,v) on the cell face (Nielson and Hamann, 1991). This bilinear interpolation solves most cases as shown in Fig. 4. The choice of ambiguous subcases 13.a and 13.b is made according to the method proposed in (Nielson and Hamann, 1991). Note that the proposed method does not address internal ambiguity since it only allows patch edges on the cell face.

Case 1, 3, 6, 13.a and 13.b of Fig. 4 correspond to all the 15 cases in the original MC method. To show that the proposed method cover all the cases of the original MC methods, we use the 15 cases in the original MC method as input of TMC method and get the same patches as the original MC method in Fig. 5, though the triangulation is different. TMC method also generates the same results for all the possible configurations with ambiguous faces as those in (Nielson and Hamann, 1991). Due to page limit, we do not list the result here.

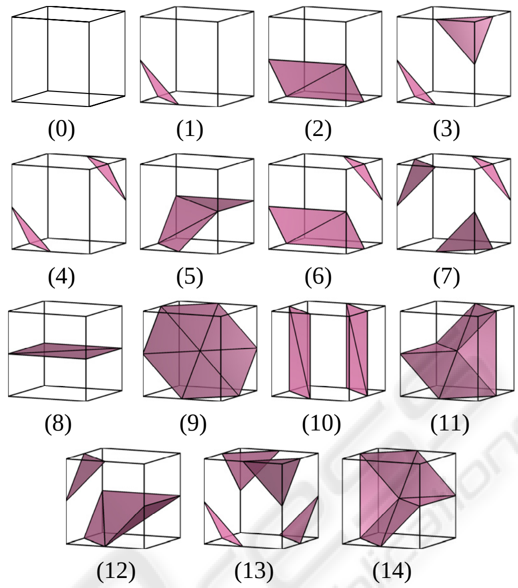


Figure 5: 15 cases in original MC method covered by TMC.

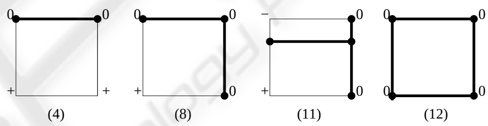


Figure 6: Bilinear Interpolation of case 4, 8, 11 and 12 in Fig. 3.

However, ambiguities in case 4, 8, 11 and 12 of Fig. 3 are not solved well by bilinear interpolation, which is shown in Fig. 6. In case 11, one patch edge intercepts with the other between its endpoints, resulting in discontinuous surface. And too little information is available to decide the patch edges at the border of a cell face, which is called *border edge*. To ensure the patch edge is on the border of two regions with different sign, both cell faces incident on the possible border edge need to be checked. The cases in Fig. 4 can be used to solve these ambiguities if the two adjacent cell faces are ”merged” into one. In the following, we enumerate the combinations of case 4, 8, 11 and 12 with the 13 cases of Fig. 3.

First we list the combinations without case 12 as shown in Fig. 7. For example, case 4 of Fig. 3 has five sub-cases from 4.a to 4.e in Fig. 7 with regard to the cell face incident on the possible border edge. And the ambiguous sub-cases 11.e.1 and 11.e.2 are solved in the same way as 13.a and 13.b of Fig. 4. As for the sub-cases 11.c.1 and 11.c.2, we choose 11.c.1 if the sum of absolute sample values at positive cell vertices is greater than that at negative cell vertices,

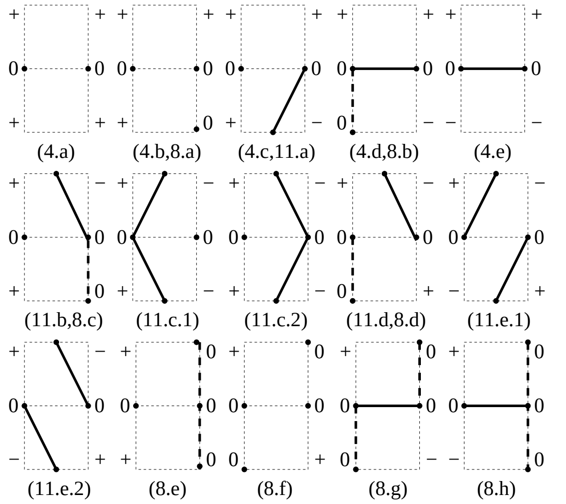


Figure 7: Solving ambiguities in case 4, 8 and 11 of Fig. 3. Solid line stands for the patch edge while dotted line for the undecided patch edge.

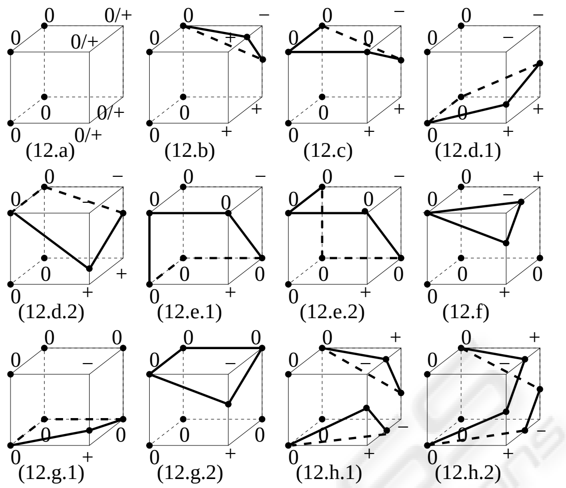
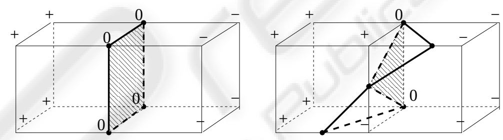


Figure 8: Solving ambiguities in case 12 of Fig. 3.



(a) Case 12 of Fig. 3 (b) Case 11 of Fig. 3

Figure 9: Insert one patch between two adjacent cells represented by the filled region.

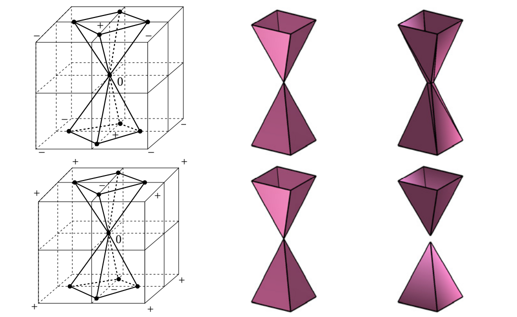
and 11.c.2 otherwise. As each of the sub-cases of case 8 in Fig. 7 has two possible border edges, we need to check the two adjacent cell faces incident on the two possible border edges.

As for the combinations with case 12 of Fig. 3, the other eight cell vertices in the cell have only 13 distinct configurations of Fig. 3 due to symmetry. So we can enumerate all the 13 configurations and generate patch edges accordingly. To use the results in Fig. 7, we "shrink" the cell face of case 12 into a border edge. Fig. 8 shows the patch edges for the twelve sub-cases of case 12 of Fig. 3 in 3D. Note sub-case 12.a represents the configurations in which the other eight vertices in the cell are the cases of 1, 2, 4, 7, 8 or 12 of Fig. 3. As for the ambiguous sub-cases 12.d.1 and 12.d.2, 12.e.1 and 12.e.2 or 12.g.1 and 12.g.2, we choose 12.d.1, 12.e.1 or 12.g.1 if the sum of absolute sample value at positive cell vertices is greater than that at negative cell vertices, and 12.d.2, 12.e.2 or 12.g.2 otherwise. And the ambiguous sub-cases 12.h.1 and 12.h.2 are solved in the same way as 13.a and 13.b in Fig. 4.

By solving the 13 tricolor cases of Fig. 3 according to Fig. 4, Fig. 7 and Fig. 8, we can decide patch edges for all the 6561 cases in 3D. But we still need to ensure that TMC method can find all the patches by searching the Eulerian circuits in the cell. The condition of Eulerian circuits is no surface vertex with odd degree. We show briefly in the following that the degree of surface vertex in one cell is either zero or two. And the surface vertex with zero degree is isolated and simply ignored.

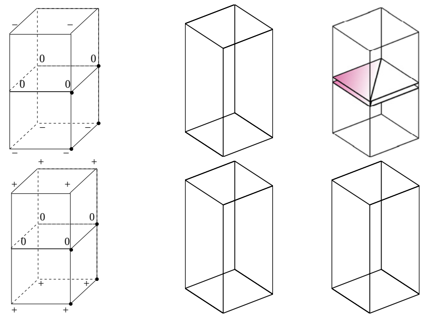
Firstly, it is easy to verify the degree of surface vertex, which is not zero cell vertex, is two. And the degree of surface vertex in Fig. 8 is either zero or two. Secondly, we show that the degree of surface vertex is at most two for the cases with zero cell vertices in Fig. 4 and Fig. 7. In Fig. 4, if one zero cell vertex connects to three patch edges like case 5 or 9, it will cause a contradiction, in which the cell vertices at the two sides of one patch edge are of the same sign. In Fig. 7, we only need to check the degree of zero cell vertex that possibly connect more than one patch edge. Since only two cell vertices are not known in Fig. 7, we get the degree is two by enumerating the nine combinations of the two unknown cell vertices in sub-cases 4.d, 11.b, 11.c.1, 11.c.2, 8.e, 8.g and 8.h. Finally we show that the degree of zero cell vertex is not one. We only need to check the zero cell vertex that connects only one patch edge in Fig. 4 and Fig. 7. From case 5 and 9 of Fig. 4, the zero cell vertex connects to only one patch edge on one cell face if its two adjacent cell vertices are of different sign. Similarly it connects to at least one patch edge in its other adjacent cell faces, if its third adjacent cell vertex (not displayed) is positive or negative. If its third adjacent cell vertex is zero, only two other unknown cell vertices (not displayed) affect its degree. We get the degree is two by enumerating the nine combinations of the two unknown cell vertices. In Fig. 7, the degree of zero cell vertex in sub-cases 4.c, 4.e, 11.e.1, 11.e.2 is two since its two adjacent cell vertices are of different sign. And we get the degree is two by enumerating sub-case 4.d, 11.b, 11.d, 8.e, 8.g and 8.h.

As the cell face of case 12 of Fig. 3 is shared by



(a) Original Topology (b) TMC (c) Bicolor MC

Figure 10: Topologies comparison between TMC and the Lewiner's MC implementation on cases 2 in Fig. 4.



(a) Original Topology (b) TMC (c) Bicolor MC

Figure 11: Topologies comparison between TMC and the Lewiner's MC implementation on case 12 in Fig. 3.

two adjacent cells, if it separates two regions of different sign, we need to insert a patch between the two adjacent cells as shown in Fig. 9(a). Moreover, as the cell face of case 11 may result in different patch edges in two adjacent cells, a new patch is also needed as shown in Fig. 9(b). Note these patches inserted between adjacent cells do not affect the previous conclusion about the vertex degree in the cell.

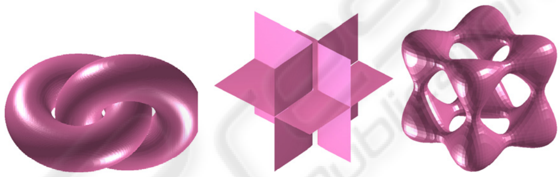
The orientation of patch vertices is adjusted to keep the patch normal pointing outward the surface consistently. Once all the patches are found, we complete the whole front surface. To get the triangulate mesh, we triangulate each patch by using one vertex as the common triangle vertex. To avoid self intersection between patches within one cell (Gelder and Wilhelms, 1994), we add a new vertex, which is the average of other vertices, as the common triangle vertex for polygons with more than five edges.

Banks and Linton also counted the tricolor cases in three dimensions in MC method by numerical algorithms (Banks and Linton, 2003). They found 147 cases out of the original 6561 cases. However, it is unrealistic to verify this result by hand. Enumerating all the ambiguous cases for these 147 cases is also a tedious task.

Another advantage of the TMC method is that 2D contours have already been generated by patch edges within each patch. By collecting the patch edges on every plane, we can get the 2D contour at specified position. Ho et al. also constructed surface by finding 2D segments in their CMS method (Ho et al., 2005). But the CMS method does not consider the zero cell vertex, hence it inherits the topological artifacts of the bicolor algorithms.

4 EXPERIMENTAL RESULTS AND ANALYSIS

To show the advantage of the proposed TMC method, we test it over the cases shown in Fig. 10 and compare it with an existing bicolor MC method implemented by Lewiner et al. (Lewiner et al., 2003). Lewiner's implementation is based on Chernyaev's technique, to solve the ambiguity problem on the cell face (Chernyaev, 1995). It modifies the zero sample value at cell vertex to a small positive value. For the case shown in top row in Fig. 10, Lewiner's method reports eight times of case 2 of Fig. 5 and generates 16 triangles. When the data are multiplied by -1 as shown in the bottom case in Fig. 10, the proposed TMC method consistently preserves the original topology, while Lewiner's method reports eight times of case 1 of Fig. 5 and generates 8 triangles. To illustrate, we enlarge its small positive value to 0.05 and get the result in Fig. 10. Note this enlargement does not change its topology or the number of triangles generated.

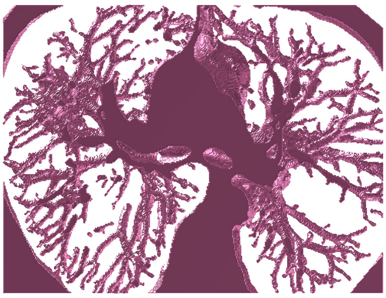


(a) Two torus (b) Case (c) Blooby

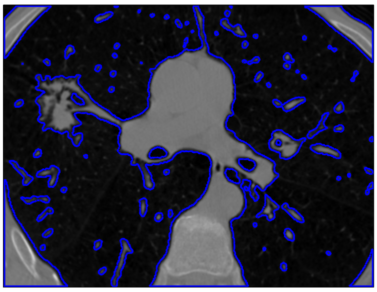
Figure 12: Examples constructed by TMC.

On case 12 of Fig. 3, Lewiner's MC implementation outputs two planes for the top case in Fig. 11. But it generates no plane when the data samples are multiplied by -1 as the bottom case in Fig. 11. In comparison, TMC gives no plane consistently since these zero cell vertices are not the boundary of two regions with different signs.

To illustrate TMC can extract the isosurface correctly, we synthesize volume data with known underlying functions and use them as the input (Lewiner et al., 2003). The results are accordant with the underlying functions as shown in Fig. 12. We also test the proposed algorithm on volumetric data with unknown underlying surface function. A series of CT lung scans have been downloaded from the Na-



(a) 3D surface



(b) 2D contour

Figure 13: Internal lung structures constructed from segmentation results.

tional Cancer Institute [LIDC]. First the 3D image in the region of interest (ROI) is segmented by level set method. Then TMC is used to construct surface from the level set results. Fig. 13(a) shows the results on the ROI with $290 \times 380 \times 62$ voxels. As mentioned, the 2D contours can be conveniently collected during extracting isosurface by TMC method as shown in Fig. 13(b), while tremendous efforts are needed for generating 2D contours in most existing MC methods. In the image, the thin structures like lung bronchia and fine surface details are well captured. And the results have been validated by experienced physicians.

We also compare the running time of TMC method with Lewiner's MC implementation as shown in Table 1. TMC method is a little bit slower than Lewiner's method because of the added cases. It is still a fast algorithm, which can generate 4 million triangles in 8.5 seconds in one experiment. The table shows that the number of zero cell vertices are about 1% to 4% of that of the resulted triangles. All the experiments are carried out on a Pentium IV 3.2GHZ processor with 1G RAM.

5 CONCLUSION

We propose a tricolor marching cubes (MC) method for 3D surface construction. Existing bicolor MC methods do not allow cell vertices whose sample values equal threshold, or zero cell vertices. Modifying sample value at such vertices may change the topology of original surface. The proposed TMC method allows the third "color", i.e. zero value prevail cell vertices after thresholding. It constructs the isosurface by finding Eulerian circuits in the cells. Comparing with the existing MC methods, the proposed method has the following advantages. First it best preserves the original topology since it does not modify sample values at cell vertices. Secondly it avoids enumerating a large number of the cases introduced by zero cell vertices. Thirdly it generates 2D contours of the surface automatically. Simulation results show that TMC method preserves topology better than existing MC methods on contacting surfaces and is an efficient method in constructing 3D surfaces.

Table 1: Running time of TMC and MC method.

	Dimension	Zero Cell Vertices	Triangles	Time (s)
MC	$200 \times 360 \times 28$	7407	260545	0.469
TMC			267167	0.718
MC	$290 \times 380 \times 62$	22081	548321	1.375
TMC			546660	1.75
MC	$512 \times 512 \times 28$	16054	1118504	2.063
TMC			1137072	2.578
MC	$512 \times 512 \times 59$	49227	4130056	5.0
TMC			4180029	8.469

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