ON ABILITY OF ORTHOGONAL GENETIC ALGORITHMS FOR THE MIXED CHINESE POSTMAN PROBLEM

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Abstract: The well known Chinese Postman Problem has many applications, and this problem has been proved to be NP-hard in graphs where directed and non-directed edges are mixed. In this paper, in order to investigate the salient feature of orthogonal design, we designed a genetic algorithm adopting an orthogonal crossover seoperation to solve this (mixed Chinese Postman) problem and evaluate the salient ability. The results indicate that for problems of small sizes, the orthogonal genetic algorithm can find near-optimal solutions within a moderate number of generations. We confirmed that the orthogonal design shows better performance, even for graph scales where simple genetic algorithms almost never find the solution. However, only the introduction of orthogonal design is not yet effective for the Chinese Postman Problem of practical size where a solution can be obtained in less than 104 generations. This paper concludes that the optimal design scale of orthogonal array to this mixed Chinese Postman Problem does not conform to the same scale as the multimedia multicast routing problem.

1 INTRODUCTION

The Chinese Postman Problem, as is well known, is to find the shortest route in a graph that uses every arc (directed or non-directed edge) and gets back to where it started. For example in the non Eulerian graph shown in Fig.1, since Postman's route traverses every arc at least once, the Postman must passes doubly through an arc of weight 6. By duplicating some arcs, the non Eulerian graph can have at leaset one Postman's route. In general, if a given graph is a non Eulerian graph, it can be said that the optimum solution of the Chinese Postman Problem is a route where the total weight of duplicated arcs is the minimum. When a given graph is an Eulerian graph, the solution is uniquely determined.

Though this problem has many applications, including robot exploration and analyzing interactive systems and web site usability (Thimbleby, 2003), this problem has been proved to be NP-hard. The multimedia multicast routing problem is also NP-hard. Paper (Zhang and Leung, 1999) proposed an orthogonal genetic algorithm for this latter problem, and concluded on the basis of solving a benchmark test problem, that for practical problem sizes the orthogonal genetic algorithm can find near-optimal solutions within a moderate number of generations. Its salient feature is to incorporate an experimental design method called orthogonal design into the crossover operation. In order to further investigate this salient feature of orthogonal design, which was applied to the sampling of genes from the parents for crossover, we will design a genetic algorithm adopting an orthogonal crossover operation to solve the mixed Chinese Postman Problem and evaluate the salient ability.

For the problem which we treat is called the Chinese postman problem on mixed networks (WCPP), heuristic solution procedures have been proposed to solve approximately (Edmond and Johnson, 1979)(Pearn and Liu, 1995)(Frederickson, 1979).

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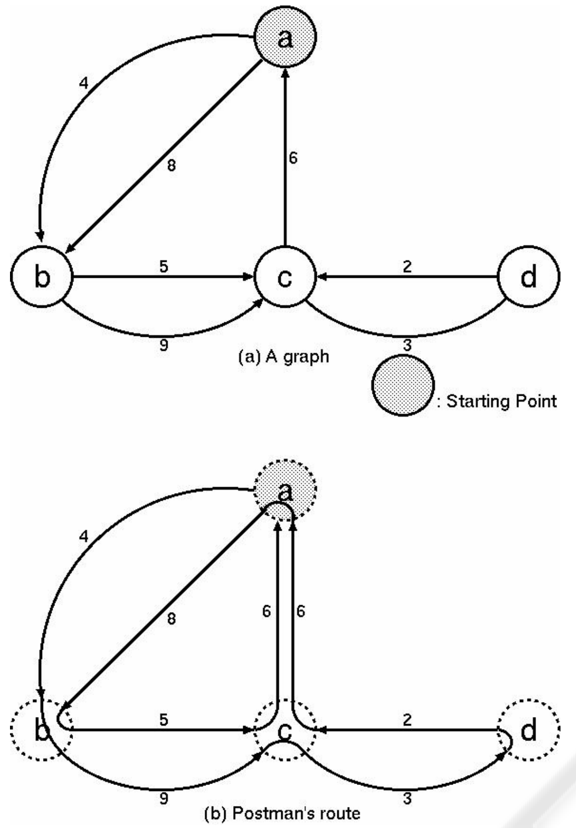


Figure 1: An example of route in Chinese postman problem.

1. **[Start]** Generate random population of *t* chromosomes (suitable solutions for the problem).
2. **[Fitness]** Evaluate the fitness *f(x)* of each chromosome *x* in the population.
3. **[New population]** Create a new population by repeating following steps until the new population is complete.
   1. **[Selection]** Select two parent chromosomes from a population according to their fitness (the better their fitness, the better their chance of being selected).
   2. **[Crossover]** Use crossover probability to cross over the parents and form a new offspring (children). If no crossover was performed, the offspring would be an exact copy of the parents.
   3. **[Mutation]** Use mutation probability to mutate new offspring at each locus (position in the chromosome).
   4. **[Accepting]** Place new offspring in a new population.
4. **[Replace]** Use newly generated population to continue the algorithm.
5. **[Test]** If the end condition is satisfied, **stop**, and return the best solution in the current population.
6. **[Loop]** Go to step **2**.

Figure 2: A basic process of genetic algorithm.

2 GENETIC ALGORITHM AND ORTHOGONAL ARRAY REPRESENTATION

A genetic algorithm (GA) is a heuristic approach used to find approximate solutions to knotty problems through application of the principles of biological evolution. Genetic algorithms make the best of biologically derived approaches such as inheritance, mutation, natural selection, and recombination (or crossover). Genetic algorithms are a particular class of evolutionary algorithms where a population of abstract representations (called chromosomes) of candidate solutions (called individuals) evolve into better solutions. That is, information treated in GA can be classified into two structures: phenotype and genotype. Phenotype represents information in the biological world, and genotype represents information in a population of chromosomes. By encoding, the information in a phenotype can be transferred into a genotype, and by decoding the opposite occurs. We will chiefly consider that in a genotype. Even though some different encodings are possible, in general the solutions are represented in binary strings of 0s and 1s.

In this mixed Chinese postman problem, we will treat a given graph G whose every arc is a directed edge or a non-directed edge. We will consider a directed graph G' where every non-directed edge in the given graph is changed into two directed edges with different directions each other. Then, we can make our chromosome type as an integer string whose element means the number of times that the postman passes the arc. The length of a string is the number of edges of G'. Therefore, in this mixed Chinese postman problem, the solutions are represented in strings of integer. The evolution starts from a population of completely random individuals and goes on in generations. In each generation, the fitness of the whole population is evaluated, and multiple individuals are stochastically selected from the current population (by judging their fitness) and modified (so called by mutation or recombination) to form a new population, which becomes current in the next iteration of the algorithm. The general process of GA is known, and is shown in Fig.2.

Orthogonal array was developed to find the smallest, yet most cost effective, and therefore best, combination by which many and consumptive combinations can be avoided (Fang and Wang, 1994). An orthogonal array is represented in Table.1-1 by $L_9(3^4)$, where 3, 4, and 9 mean the number of kinds of entries, columns, and rows, respectively. In general, we let $L_m(n^k)$ denote an orthogonal array for k factors, n levels, and m combinations of level to be tested. Orthogonal arrays can be systematically built. Label L comes originally from a "Latin" square, which is defined as a matrix where no two entries in a row (or a column) have the same value. It has been proved that the orthogonal design is optimal for use as an additive model and a quadratic model, and that the selected combinations are good representatives for all the possible combinations (Wu, 1978). The problem of building an orthogonal array is the same as the problem of finding m nodes which are at the maximum distance between any pair in the (k $\log n$)-dimensional hypercube. Table 1-2 shows 9 nodes corresponding to 9 combinations (on an 8-dimensional hypercube) in Table 1-1.

Table 1-1: A representation of orthogonal array $L_9(3^4)$.

Combination	Factor1	Factor2	Factor3	Factor4
1st	X	X	X	X
2nd	X	Y	Y	Y
3rd	X	Z	Z	Z
4th	Y	X	Y	Z
5th	Y	Y	Z	X
6th	Y	Z	X	Y
7th	Z	X	Z	Y
8th	Z	Y	X	Z
9th	Z	Z	Y	X

Table1-2: 9 nodes corresponding to 9 combinations on a (k *log* n)-dimensional hypercube.

Combination	Factor1	Factor2	Factor3	Factor4
1st	0 1	0 1	0 1	0 1
2nd	0 1	1 0	1 0	1 0
3rd	0 1	1 1	1 1	1 1
4th	1 0	0 1	1 0	1 1
5th	1 0	1 0	1 1	0 1
6th	1 0	1 1	0 1	1 0
7th	1 1	0 1	1 1	1 0
8th	1 1	1 0	0 1	1 1
9th	1 1	1 1	1 0	0 1

3 EXPERIMENTAL DESIGN METHODS

In this section, we introduce the concept of experimental design methods for our experiment mentioned later.

3.1 Phenotype

Roads are modeled as a graph G = (V,E), where V is a set of nodes and E is a set of arcs. By making every non-directed arc express as two directed arcs, change a given graph G into a directed graph G'. As mentioned before, the optimum solution is a route where the total weight of duplicated arcs is the minimum. We number the edges in G' from 1 to t. We can then represent any route R of the graph G' as an t-tuple (e_1, e_2, …, e_t), where any element (which means a gene) e_i is defined as follows:

e_i = The number of times that the postman passes arc i

From the above definition, e_i is a non negative integer and can become 0 if e_i is represented for one of two directed edges changed from non-directed edge in G. For example, assuming that edge with weight 4 is r_4, R in Fig.1(b) can be represented as (e_{r4}, e_{r8}, e_{r5}, e_{r9}, e_{r3}, e_{r3}, e_{r2}, e_{r6}) =(1, 1, 1, 1, 1, 0, 1, 2). Since R in Fig.1(b) is a Postman's route, then this phenotype (1, 1, 1, 1, 1, 0, 1, 2) is a solution.

3.2 Fitness

The Postman can return to the starting point if G is an Eulerian graph. If G is a non Eulerian graph, in order to return to the starting point the postman must traverse some arcs more than once. In general, if G is a non Eulerian graph, it can be said that the optimum solution of the mixed Chinese Postman Problem is a route where the total weight of duplicated arcs is the minimum. When G is an Eulerian graph, the solution is uniquely determined.

In order to set the fitness of the optimum solution to maximum, we will prepare the following function f of fitness.

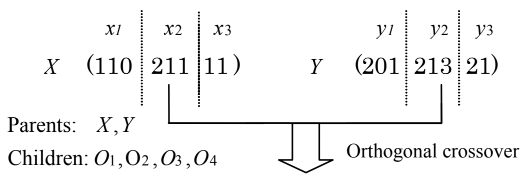
$$f = \begin{cases} 1/(\text{total weight of duplicating arcs}); \text{if G has Postman's route} \\ 0; \text{otherwise} \end{cases}$$

3.3 Orthogonal Crossover

In order to fit the mixed Chinese postman problem, we interpret orthogonal array $L_m(n^k)$ to be an orthogonal array for k factors divided from n levels (parent chromosomes), and m combinations of levels (samplings at the time of crossover). Let orthogonal array $L_4(2^3)$ be shown in Table 2. Obeying the above interpretation of $L_4(2^3)$, 2 parent chromosomes are divided into 3 factors each, and we obtain 4 new chromosomes as shown in Fig.3 In the case of $L_9(3^4)$, we obtain 9 new chromosomes from 4 parent chromosomes as shown in Fig.4.

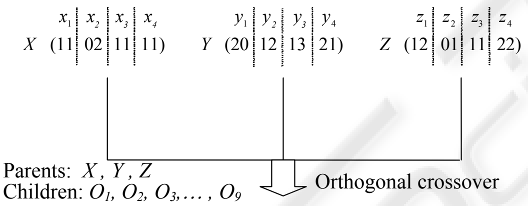
Table 2: Orthogonal array $L_4(2^3)$.

Combination	Factor1	Factor2	Factor3
1st	X	X	X
2nd	X	Y	Y
3rd	Y	X	Y
4th	Y	Y	X



$$O_1 = (x_1 \quad x_2 \quad x_3) = (110 \ \ 211 \ \ 11)$$
$$O_2 = (x_1 \quad y_2 \quad y_3) = (110 \ \ 213 \ \ 21)$$
$$O_3 = (y_1 \quad x_2 \quad y_3) = (201 \ \ 211 \ \ 21)$$
$$O_4 = (y_1 \quad y_2 \quad x_3) = (201 \ \ 213 \ \ 11)$$

Figure 3: Orthogonal crossover for $L_4(2^3)$.



$$O_1 = (x_1 \ x_2 \ x_3 \ x_4) = (11 \ 02 \ 11 \ 11)$$
$$O_2 = (x_1 \ y_2 \ y_3 \ y_4) = (11 \ 12 \ 13 \ 21)$$
$$O_3 = (x_1 \ z_2 \ z_3 \ z_4) = (11 \ 01 \ 11 \ 22)$$
$$O_4 = (y_1 \ x_2 \ y_3 \ z_4) = (20 \ 02 \ 13 \ 22)$$
$$O_5 = (y_1 \ y_2 \ z_3 \ x_4) = (20 \ 12 \ 11 \ 11)$$
$$O_6 = (y_1 \ z_2 \ x_3 \ y_4) = (20 \ 01 \ 11 \ 21)$$
$$O_7 = (z_1 \ x_2 \ z_3 \ y_4) = (12 \ 02 \ 11 \ 21)$$
$$O_8 = (z_1 \ y_2 \ x_3 \ z_4) = (12 \ 12 \ 11 \ 22)$$
$$O_9 = (z_1 \ z_2 \ y_3 \ x_4) = (12 \ 01 \ 13 \ 11)$$

Figure 4: Orthogonal crossover for $L_9(3^4)$.

3.4 Creation of the Initial Population of Chromosomes using an Orthogonal Array

In order to create the initial population of chromosomes which resemble closely as possible as they can, we will present a method to use an orthogonal array in the creation of the initial population of chromosomes.

A chromosome is an t-tuple whose the i-th element is the number of times passing through the i-th arc by the Postman' route. Let divide a chromosome into the number of factors. The range of the random numbers is divided into the number of levels of orthogonal array. In addition, for each one of the same levels in orthogonal array select, let randomly correspond to a random number in the same range of the random numbers.

Let us take $L_4(2^3)$ shown in table 3 and t=12 as an example. Since a t-tuple is divided into 3 factors, then we can represent 4 chromosomes based on $L_4(2^3)$ as follows:

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	Y	Y	Y	Y	Y	Y	Y	Y
Y	Y	Y	Y	X	X	X	X	Y	Y	Y	Y
Y	Y	Y	Y	Y	Y	Y	Y	X	X	X	X

Each X and Y are corresponded to a random number in the ranges 0~6 and 7~12, respectively. Then, we obtain the following initial population of chromosomes (4 chromosomes) as an example.

1	3	5	3	5	4	2	4	5	5	4	6
5	6	4	2	7	9	8	9	10	7	9	11
8	10	9	7	2	5	1	4	11	8	9	10
7	11	8	9	10	7	8	12	2	5	1	3

4 EXPERIMENT

In this section, we will explain the experiments we performed.

4.1 Graphs

We used random graphs shown in Table 4. We applied the Genetic algorithm 30 times to each graph and evaluated the mean values.

Table 4: Experimental graphs.

|V|	|E|	Total weight
5	7	44

4.2 Orthogonal Genetic Algorithm (OGA)

We developed the following algorithm, as shown in Fig.5, where an initial group starts from a population of completely randomly generated individuals, and the probabilities of crossover and mutation are 1.0 and 0.5, respectively.

4.3 Simple Genetic Algorithm (SGA)

The simple genetic algorithm adopts a one-point crossover in Step 2.1 instead of orthogonal crossover.

4.4 Another Orthogonal Genetic Algorithm using Orthogonal Array in the Creation of the Initial Population of Chromosomes (OGA^2)

This algorithm uses an orthogonal array also in Step 1 in Fig.5 instead of random number, and the utilization method is shown in 3.4.

4.5 Results

Fig.6-1 shows the relationship between the number of obtained Eulerian graphs and the number of generations. On the other hand, Fig.7-1 shows the relationship in the case where orthogonal array is used in the creation of initial population of chromosomes. Fig.6-2 shows the relationship between the obtained minimum weight of the Postman route and the number of generations. Fig.7-2 shows the relationship in the case where orthogonal array is used in the creation of initial population of chromosomes. These results mean that our orthogonal genetic algorithm shows better performance, especially in $L9(3^4)$. SGA almost never finds the solutions for Problem 3 where the number of edges is 7.

For reader's information, we show the relationship between the number of generations and the computation time required in 3 algorithms (SGA, OGA, and OGA^2) in Tables 5 and 6.

In this mixed Chinese postman problem we treated, in less than 10^4 generations we can obtain a solution in graphs with nodes of less than 11. However, we can't obtain a solution in 2 or 3 days for the larger sizes.

For reference we will show the data in the case of non directed graphs G''=(V'', E''), where |V''|= 20, |E''|=30, total weight=178. The Chinese Postman problem for non directed graphs belongs in Class P. Figs.8-1 and 8-2 show the relationship between two numbers of obtained Eulerian graphs and generations, and the relationship between the obtained minimum weight of the Postman route and the number of generations, respectively.

Inputs: Graph *G=(V,E)*, edge cost *C(e)* and node degree *deg(v)*
Output: binary strings representing the Euler route

Step1) **Initialization**
Randomly create an initial generation of N binary string $P_0 = \{ X_1, X_2, \dots, X_N \}$, $X = \{ x_1, x_2, \dots, x_k \}$
and initialize the generation number *gen* to 0.
* X is a chromosome, $k = |V|$

Step 2) **Population Evolution**
WHILE (gen < MAX_GEN)
BEGIN
DO N/2 *times*
BEGIN

Step 2.1) **Orthogonal Crossover**
Randomly select n parents strings from P_{gem} and perform orthogonal crossover on them to generate m offspring $o_1, o_2, \dots, o_m$.

Step 2.2) **Mutation**
To perform mutation of offspring, flip every bit in this string with a small probability p.

Step 2.3) **Select**
Calculate the offspring fitness f, and sort them by f, and choose n for the next generation.
END

Step 3) Increment the generation number *gem* by 1.
END

Figure 5: A orthogonal genetic algorithm.

5 CONCLUSION

In order to investigate the salient feature of orthogonal design, we designed a genetic algorithm adopting an orthogonal crossover operation in the mixed Chinese Postman Problem and evaluated the salient ability. The orthogonal design shows better performance, even for graph scales where simple genetic algorithms almost never find the solution. The experimental results show that, for problems of non practical sizes, the orthogonal genetic algorithm using the orthogonal array $L_9(3^4)$ can find close-to-optimal solutions within a moderate number of generations. This optimal scale of orthogonal array was confirmed for the multimedia multicast routing problem of practical size (Zhang and Leung, 1999). However, this orthogonal design is not yet effective for the mixed Chinese Postman Problem of practical sizes. For more effective computation, our one possible extension of this research can be considered as to incorporate the orthogonal array into the

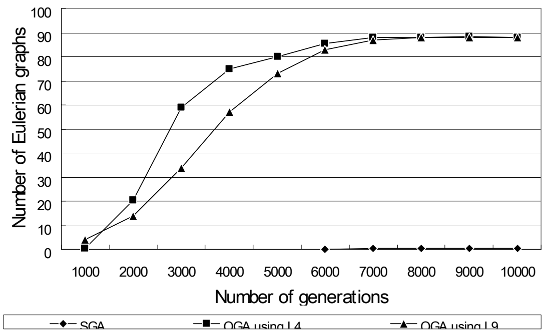


Figure 6-1: Relationship between two numbers of obtained Eulerian graphs and generations.

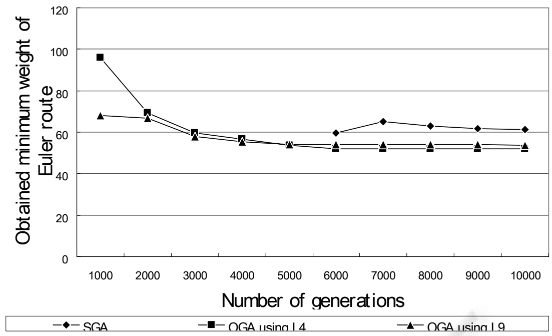


Figure 6-2: Relationship between the obtained minimum weight of the Postman route and the number of generations.

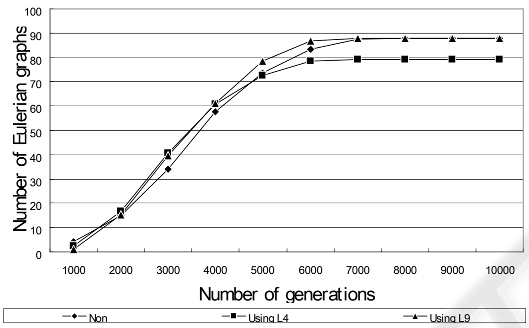


Figure 7-1: Relationship between two numbers of obtained Eulerian graphs and of generations in the case where orthogonal array is used in the creation of initial population of chromosomes.

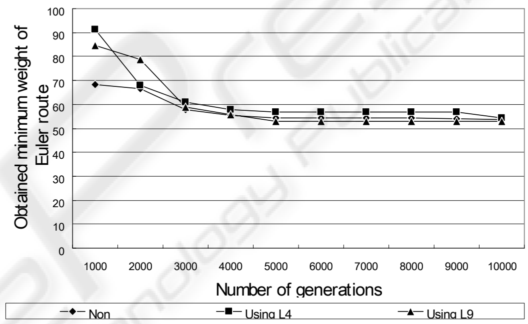


Figure 7-2: Relationship between the obtained minimum weight of the Postman route and the number of generations in the case where orthogonal array is used in the creation of initial population of chromosomes.

Table 5: Computation time required at 1000~10000 generations.

	SGA	OGA using $L_4(2^3)$	OGA using $L_9(3^4)$
generation	time (sec)	time (sec)	time (sec)
1000	247.1	279.8	437
2000	252.5	286.4	447
3000	257.8	292.6	456.6
4000	263.3	298.4	465.8
5000	268.3	304.4	475
6000	273.8	310.3	484.4
7000	279.1	316.2	493.6
8000	284.5	322.1	502.9
9000	289.9	328.1	512.5
10000	295.3	333.9	521.7

Table 6: Computation time required at 1000~10000 generations.

	OGA using $L_9(3^4)$	OGA^2 using $L_4(2^3)$	OGA^2 using $L_9(3^4)$
generation	time (sec)	time (sec)	time (sec)
1000	437	428.3	433.4
2000	447	437.7	443.2
3000	456.6	447.3	452.7
4000	465.8	456.4	462.3
5000	475	465.7	471.6
6000	484.4	474.7	480.9
7000	493.6	483.9	490.3
8000	502.9	493.1	499.5
9000	512.5	502.2	508.6
10000	521.7	511.4	517.9

experimental design methods of setting an initial group of populations. We performed this extension. The experiment results show no innovative improvement.

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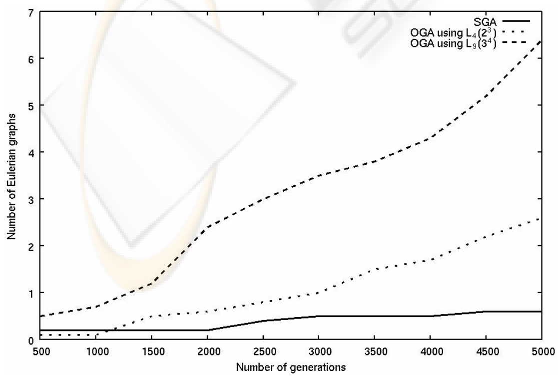

Figure 8-1: Relationship between two numbers of obtained Eulerian graphs and generations.

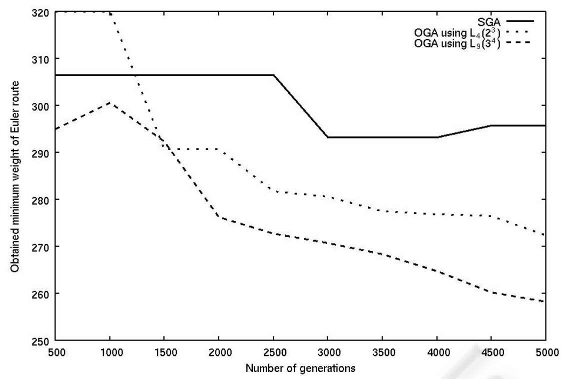

Figure 8-2: Relationship between the obtained minimum weight of the Postman route and the number of generations.