

# APPROXIMATE REASONING TO LEARN CLASSIFICATION RULES

Amel Borgi

Research Unit SOIE, ISG, Tunis

National Institute of Applied Science and Technology, INSAT

Centre Urbain Nord de Tunis, BP 676, 1080 Tunis, Tunisia

**Keywords:** Supervised learning, rules generation, approximate reasoning, inference engine, imprecision and uncertainty management.

**Abstract:** In this paper, we propose an original use of approximate reasoning not only as a mode of inference but also as a means to refine a learning process. This work is done within the framework of the supervised learning method SUCRAGE which is based on automatic generation of classification rules. Production rules whose conclusions are accompanied by belief degrees, are obtained by supervised learning from a training set. These rules are then exploited by a basic inference engine: it fires only the rules with which the new observation to classify matches exactly. To introduce more flexibility, this engine was extended to an approximate inference which allows to fire rules not too far from the new observation. In this paper, we propose to use approximate reasoning to generate new rules with widened premises: thus imprecision of the observations are taken into account and problems due to the discretization of continuous attributes are eased. The objective is then to exploit the new base of rules by a basic inference engine, easier to interpret. The proposed method was implemented and experimental tests were carried out.

## 1 INTRODUCTION

Facing the increase of data amount recorded daily, the detection of both structures and specific links between them, the organisation and the search of exploitable knowledge have become a strategic stake for decision making and prediction task. This complex problem of data mining has multiple aspects (Michalski et al., 1983) (Zhou, 2003). We focus on one of them: supervised learning. In (Borgi, 1999) (Borgi et al. 2001), we have proposed a learning method from examples situated at the junction of statistical methods and those based on Artificial Intelligence techniques. Our method, SUCRAGE (SUPervised Classification by Rules Automatic GEneration) is based on automatic generation of classification rules. Production rules *IF premise THEN conclusion* are a mode of knowledge representation widely used in learning systems because they ensure the transparency and the easy explanation of the classifier (Duch et al., 2004) (Haton et al., 1991). Indeed, the construction of production rules using the knowledge and the

know-how of an expert is a very difficult task. The complexity and cost of such a knowledge acquisition have led to an important development of learning methods used for an automatic knowledge extraction, and in particular for rules extraction (Haton et al. 1991) (Duch et al., 2004).

The learning method SUCRAGE is based on a correlation search among the features of the examples and on discretization of continuous attributes. Rules conclusions are of the form «belonging to a class» and are uncertain. In the classification phase, an inference engine exploits the base of rules to classify new observations and also manages rules uncertainty. This reasoning that we called *basic reasoning* allows to obtain conclusions, when the observed facts match *exactly* rules premises.

In this paper, we are interested in an other reasoning: approximate reasoning (Zadeh, 1979) (Haton et al. 1991) (El-Sayed, et al., 2003). It allows to introduce more flexibility and to overcome problems due to discretization. Such reasoning is closer to human reasoning than the basic one:

human inferences do not always require a perfect correspondence between facts or causes to conclude.

In (Borgi et al., 2001), we have proposed a context-oriented approximate reasoning. This reasoning, used as an inference mode, allows to manage imprecise knowledge as well as rules uncertainty: according to distance between observations and premises, it computes a neighborhood degree and associates a final confidence degree to rules conclusions. This model is faithful to the classical scheme of Generalized Modus Ponens (Zadeh, 1979). In this paper, we propose to see approximate reasoning under another angle. The originality of our approach lies in the use of approximate reasoning, not only as a mode of inference, but to refine the learning. This reasoning allows to generate new rules and to ease in this way problems due to discretization and imprecision of the observations. The aim is that the new base of rules will then be exploited by a *basic inference* engine more easy to interpret. In our model, approximate reasoning has then no more vocation to be a method of inference allowing to fire certain rules but joins in the process of learning itself. The software SUCRAGE was extended: new rules construction through approximate reasoning was implemented. Applications of the extended version to benchmark problems are reported.

This paper is organized as follows. In section 2, the method SUCRAGE is described. More precisely we describe the learning phase (rules generation) and the classification phase. Only the basic inference engine is presented. In section 3, we present the approximate reasoning used as an inference mode. Section 4 attempts to explain the use of approximate reasoning to generate new classification rules and its contribution to the process of learning. Tests and results obtained by computer simulations with two benchmarks are provided in section 5. Finally, section 6 concludes the study.

## 2 THE SUPERVISED LEARNING METHOD SUCRAGE

### 2.1 Rules Generation

In this section, we describe the learning phase of the supervised learning method SUCRAGE. The training set contains examples described by numerical features denoted  $X_1, \dots, X_i, \dots, X_p$ . These examples are labelled by the class to which they

belong. The classes are denoted  $y_1, y_2, \dots, y_C$ . The generated rules are of the type:

$$A_1 \text{ and } A_2 \text{ and } \dots \text{ and } A_k \longrightarrow y, \alpha$$

where

$A_i$ : condition of the form  $X_j$  is in  $[a, b]$ ,

$X_j$ : the  $j^{\text{th}}$  vector component representing an observation,

$[a, b]$ : interval issued from the discretization of the features variation domain (here, it is the variation domain of the feature  $X_j$ ),

$y$ : a hypothesis about membership in a class,

$\alpha$ : a belief degree representing the uncertainty of the conclusion.

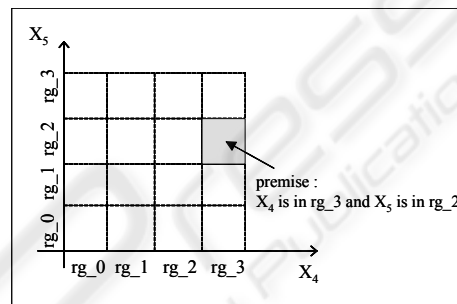


Figure 1: A partition of the correlated features space.

Our approach is multi-featured as the features that appear in rules premises are selected in one piece. This selection is realized by linear correlation search among the training set elements (Borgi, 1999) (Borgi et al., 2001). So the first step consists in computing the correlation matrix between the components of the training set vectors. Then to decide which components are correlated, this matrix is thresholded (with a threshold denoted  $\theta$ ). The idea is to detect privileged correlations between the features and to generate the rules according to these correlations. According to Vernazza's approach, we decide to group in the same premises all components that are correlated (Vernazza, 1993).

Next step in building the rules is feature discretization. Among the non supervised methods of discretization, the simplest one leads to  $M$  sub-ranges of equal width. This method called the *regular discretization* is the one we retain for this study. The  $M$  obtained sub-ranges are denoted  $rg_0, rg_1, \dots, rg_{(M-1)}$ , these values are totally ordered.

Once the discretization done, condition parts of rules are then obtained by considering for each correlated components subset, a sub-interval ( $rg_i$ ) for each component in all possible combinations. Indeed the premises of the rules form a partition of the correlated components space. Figure 1 illustrates such a partition in the case of two correlated features ( $X_4$  and  $X_5$ ) and with a subdivision size  $M=4$ .

Each premise that we construct leads to the generation of C rules (C: number of classes). The rules conclusions are of the form « belonging to a class » and are not absolutely certain, that's why each conclusion is accompanied by a belief degree  $\alpha$ . In this paper, we propose to represent the belief degrees by a classical probability estimated on the training set (Pearl 90) (Borgi et al. 01) (Borgi, 99).

### 2.2 Basic Inference Engine

The rules were generated for the purpose of a further classification use. In classification phase, the base of rules is exploited to classify new objects that don't belong to the training set. To achieve this goal, our approach consists in using a 0+ order inference engine. The inputs of this engine are the base of rules previously built and a vector representing the object to classify. The inference engine associates then a class to this vector.

We propose two reasoning models. The first one, called *basic reasoning* is presented in this section. The second one, the *approximate reasoning*, will be detailed in section 3. The *basic reasoning* allows the inference engine to fire only the rules with which the new observation components match *exactly*. The engine classifies each new observation using the classical deduction reasoning. It has to manage the rules' uncertainty and take it into account within the inference dynamic. Uncertainty management is done by computations on the belief degrees of the fired rules. Once the rules fired, we have to compute a final belief degree associated with each class. For this we propose to use a triangular co-norm (Gupta et al., 91): the final belief degree associated to each class is the result of this co-norm applied on the probabilities of the fired rules that conclude to this considered class. Experimental tests presented in this paper were realized with the Zadeh co-norm (max). Finally the winner class associated with the new observation is the class for which the final belief degree is maximum.

### 3 APPROXIMATE REASONING

Approximate reasoning, in a general way, makes reference to any reasoning which treats imperfect knowledge. This imperfection has multiple facets: for instance the knowledge can be vague, imprecise, or uncertain. In spite of such imperfections, approximate reasoning allows to treat this knowledge and to end in conclusions. In (Haton et

al. 1991), approximate reasoning concerns as well the imprecision and uncertainty representation as their treatment and propagation in a knowledge based system. The term approximate reasoning has however a particular meaning of a word introduced by Zadeh in the field of Fuzzy Logic (Zadeh 1979) (Yager, 2000). In this frame, approximate reasoning corresponds to Generalized Modus Ponens who is an extension of Modus Ponens in fuzzy data. This definition of approximate reasoning is not contradictory to the first one which is more general and concerns all the forms of imperfections.

The approximate reasoning which we introduce is situated in the intersection of these two approaches. We are however more close to "fuzzifiers" as far as we remain faithful to the Generalized Modus Ponens (Zadeh 1979), but we adapt it to a symbolic frame (Borgi 1999) (Borgi et al. 01) (El-Sayed et al., 2003). We propose a model of Approximate Reasoning which allows to associate a final degree of confidence to the conclusions (classes) on the basis of an imprecise correspondance between rules and observations. This reasoning does not fire only the rules the premises of which are exactly verified by the new observation, but also those who are not too much taken away from this observation. Thus, we are in the situation described in figure 2.

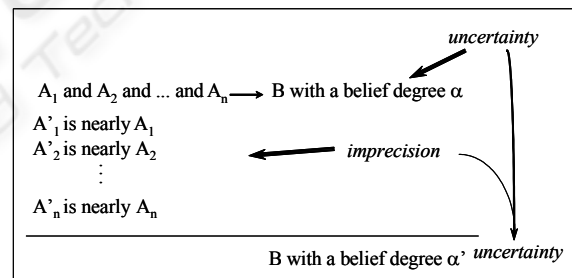


Figure 2: Particular case of Generalized Modus Ponens.

The consideration of observations close to rules premises allows to overflow around these premises. More exactly, it allows to extend beyond around the intervals stemming from the discretization and to ease so the problems of borders due to any discretization. So that our approximate reasoning can become operational, it is necessary to formalize first of all the notion of neighborhood. Then, it is necessary to model the approximate inference, that is to determine the degree of the final conclusion ( $\alpha'$ ) of the diagram shown on figure 2.

### 3.1 Proximity between Observation and Premise

In works about approximate reasoning, Zadeh (Zadeh, 1979) stresses the necessary introduction of a distance in order to define neighbouring facts. In (Ruspini, 1991), a similarity degree between two objects is introduced. In our case, to define the notion of neighbourhood we have defined two types of measure or distance (Borgi, 1999) (Borgi et al., 2001). A distance that we call local distance will measure the proximity of an observation element to a premise element. These distances will then be aggregated to obtain a global distance between the observation and the whole premise.

#### 3.1.1 A Local Distance

We consider, by concern of clearness, the following rule:

$$X_1 \text{ in } rg\_r1 \text{ and } X_2 \text{ in } rg\_r2 \text{ and... } X_n \text{ in } rg\_rn \rightarrow y_i, \alpha$$

which groups together in its premise the attributes  $X_1, X_2, \dots, X_n$ . This rule does not lose in generality: it can be obtained by renaming the attributes.

We note  $V=(v_1, v_2, \dots, v_n)$  the elements of the observation concerned by the premise. To compare  $V$  with the following premise:  $X_1 \text{ is in } rg\_r1$  and  $X_2 \text{ is in } rg\_r2$  and ... and  $X_n \text{ is in } rg\_rn$ , we begin by making local comparisons between  $v_1$  and  $X_1 \text{ is in } rg\_r1$ , between  $v_2$  and  $X_2 \text{ is in } rg\_r2$  ... So we have to define the local distances  $d_1, d_2, \dots, d_n$  of the following schema:

$A_1$  and  $A_2$  and ... and  $A_n \rightarrow B$  with a belief degree  $\alpha$   
 $A'_1$   $d_1$ -distant of  $A_1$   
 $A'_2$   $d_2$ -distant of  $A_2$   
 ...  
 $A'_n$   $d_n$ -distant of  $A_n$

---

B with a belief degree  $\alpha'$

More precisely it comes to determine the following distances  $d_i$ :  $v_1$  is  $d_1$ -distant from  $rg\_r1$ , ...,  $v_n$  is  $d_n$ -distant from  $rg\_rn$  where the distance is the formal translation of the neighboring concept.

Rule premise ( $A_i$ ) associates discrete values ( $rg\_0, rg\_1, \dots, rg\_M$ ) to observation components. But observations ( $A'_i$  or  $v_i$ ) have numerical values. In order to compare them, we introduce a numerical-symbolic interface (Borgi 99). We split each interval  $rg\_k$  into  $M$  sub-intervals of equal range, denoted  $\sigma_0, \sigma_1, \dots$ . We thus have a finer discretization, and we obtain  $M \times M$  sub-intervals ( $\sigma_0, \sigma_1, \dots, \sigma_{M \times M - 1}$ ). Figure 3 illustrates such sub-intervals obtained with

$M=3$ .

We can associate to each numerical value  $v_i$  the sub-interval  $\sigma_i$  to which it belongs. The distance  $d_i$  between  $v_i$  and  $rg\_ri$  is then defined as the number of sub-intervals of type  $\sigma$  separating  $\sigma_i$  from  $rg\_ri$ . Of course,  $d_i$  is 0 if  $v_i$  is in  $rg\_ri$ . Thus, we obtain the distance vector  $D=(d_1, d_2, \dots, d_n)$  associated to every pair (observation, premise) or (observation, rule).

#### 3.1.2 A Global Distance

In order to make approximate inferences, we want to aggregate the different local distances  $d_i$ . The result of this aggregation is a global distance that we note  $g$ -distance, and on which we wish to confer some properties (Borgi, 1999). One property that we impose to that distance is to be very sensitive to little variations of neighboring facts. This global distance that measures distance between approximately equal vectors can be insensitive when facts are very far from each other. This  $g$ -distance has to either measure the proximity between two nearby facts, or indicate by a maximal value, that they are not nearby. This is a proximity measure, and not a real distance. This distance is represented by an integer in  $[0, M-1]$ . In order to take into account the value dispersion, we do not use tools like min-max functions but we propose an aggregation based on a "dispersion" function  $S_D$ :

$$S_D: [0..M-1]^n \longrightarrow \mathbb{IN}$$

$$k \longrightarrow S_D(k) = \sum_{i=1}^n (d_i - k)^2$$

$S_D(k)$  allows, in a way similar to the variance, to measure the dispersal of the local distances  $d_i$  around  $k$ . We have then defined a global distance  $g$ -dist as follows:

$$g\text{-dist}: [0..M \times (M-1)]^n \longrightarrow [0..M-1]$$

$$(d_1, d_2, \dots, d_n) \longrightarrow \max_{k=0}^{M-1} [S_D^{-1}(\min_{k=0}^{M-1} S_D(k))]$$

The global distance is presented with more details in (Borgi 1999) and (Borgi et al. 2001). We have notably proved that the proposed aggregated distance satisfies the above mentioned property.

We can notice that it is possible to have  $g$ -distance equal to 0, even if the distance vector is not null. In other words, it is possible to have a global distance equal to 0 for an observation that does not satisfy the considered rule.

### 3.2 Approximate Inference

The use of approximate inference supposes that a meta-knowledge exists in the system and allows it to run. In our case the meta-knowledge gives the possibility to bind imprecision (observation and premise of rule) to uncertainty (conclusion degree). This meta-knowledge has two complementary aspects: the first hypothesis says that a weak difference between observation and premise induces that the conclusion part is not significantly modified. For every rule, a stability area exists around the premise of the rule. The second and stronger hypothesis says that if the distance between observation and premise increases, then uncertainty of the conclusion increases too. A maximal distance must give a maximal uncertainty (in our case, it corresponds to the minimal belief degree, i.e. a probability equal to zero) (Borgi, 1999).

The conclusion degree is weakened in accordance with the global distance. In our model, belief degrees ( $\alpha$ ) associated with rules are numerical, so it is hoped to conserve a numerical final degree ( $\alpha'$ ) for the whole coherence. To compute the final belief degree  $\alpha'$  of a conclusion via the approximate reasoning, given the global distance  $d$  (symbolic) between the premises and the observation and  $\alpha$  the belief degree (numerical) of the conclusion of the fired rule, we propose the following function  $F$  :

$$F : [0,1] \times [0..M-1] \longrightarrow [0,1]$$

$$(\alpha, d) \longrightarrow \alpha \cdot \left(1 - \frac{d}{M-1}\right)$$

This formula includes the two aspects of the meta-knowledge hypothesis mentioned above. It is easy to observe that little imprecisions (in cases where  $d=0$ ) do not modify uncertainty. On the other hand, a maximal distance ( $d=M-1$ ) induces a complete uncertainty ( $\alpha'=0$ ). We note that we find back the basic reasoning in the limit case  $d=0$ .

## 4 APPROXIMATE REASONING TO LEARN NEW RULES

In this part, we present the use of approximate reasoning not as a mode of inference to exploit rules in classification phase, but as a means to refine the learning. The use of approximate reasoning during the learning phase consists in generating new rules the premises of which are widened. The method consists in generating rules by using the basic approach described in section 2 then to look

"around" the rules to verify if we cannot improve them or add better rules. The objective is then to exploit this base of rules thanks to a basic inference engine by hoping to obtain results close to a basic generation of rules exploited by an approximate engine.

For reasons of legibility and simplicity, we shall call the rules generation realized by SUCRAGE in its initial version the *basic generation*. The generation of rules completed by the construction of new rules via approximate reasoning will be called *approximate generation*.

### 4.1 Method with a Constant Number of Rules

This approach can be summarized by: "from an observation situated near the rules which we generated with the basic method of SUCRAGE, we verify if we cannot widen every rule to a rule of better quality". This is made always by using the same whole learning set.

To consider that an observation  $O$  is near a rule  $R$ , we have to define a *g-threshold*, it is the maximal value authorized by *g-distance*( $O, R$ ).

For every observation  $O$  near a rule  $R$  (the mother rule) and having the same conclusion (class) as the rule  $R$ , we are going to build a new rule (the daughter rule  $R_{\text{daughter}}$ ):

- the premise of  $R_{\text{daughter}}$  is that containing *Premise*( $R$ ) and  $O$  the most restrictive possible and convex by using the ranges ( $rg\_ri$ ) and the sub-intervals of type  $\sigma$ ,
  - the class of the conclusion do not change,
  - the belief degree of  $R_{\text{daughter}}$  is recomputed on the whole training set according to the new premise.
- This new assessment of the belief degree of the daughter rule built through approximate reasoning allows integrating this reasoning into the learning process.

The sentence "Premise containing *Premise*( $R$ ) and  $O$  the most restrictive possible and convex by using the ranges ( $rg\_ri$ ) and the sub-intervals of type  $\sigma$ " means that to create the new premise, we start from the ancient premise and we add to all the conditions that  $O$  does not verify the intervals of type  $\sigma$  which would allow  $O$  to verify it.  $R_{\text{daughter}}$  contains in its premise the same attributes as the mother rule  $R$  but with wider values. For instance, as shown in figure 3, in the case of a discretization with  $M=3$ , if the given value  $O_i \in \sigma_4$  and the condition is  $X_i$  is in  $rg\_0$  then the new condition will be  $X_i$  is in  $rg\_0 \cup \sigma_3 \cup \sigma_4$  (by supposing naturally

that the condition of threshold on the global distance is verified).

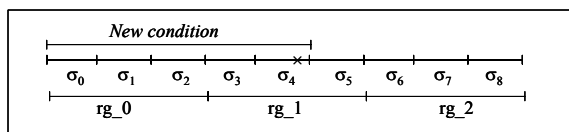


Figure 3: An example of construction of a new rule condition.

In the construction procedure of a new rule which we presented, a couple (observation, rule) verifying certain properties gives birth to a new rule. Among the mother rule and all the daughter rules we can generate, only the one who has the strongest belief degree is kept. Thus the initial number of rules does not change.

## 4.2 Method with Addition of Rules

We try here to widen the method of generation of a new base of rules so that the best rule is not the only one kept in the base. For that purpose, we use the "raw force" and we add in the base of rules all the rules that we can generate from each: a rule can then lead to several new rules and either as previously to a unique rule (that of stronger degree). This method allows to create a wide base close to data but this base, because of its size, becomes illegible as for interpretation by an expert. It becomes then necessary to optimize the size of the base of rules (Duch et al., 2004) (Nozaki et al. 1994).

## 5 TESTS AND RESULTS

The system SUCRAGE that we initially developed allows the generation of rules by the method presented in section 2.1 as well as their exploitation by an inference engine. This engine uses a basic reasoning or approximate one (Borgi, 1999) (Borgi et al., 2001). We completed this system by a module of rules generation via the approximate reasoning. We tested this new application on two learning bases stemming from the server of Irvine's University: those bases are Iris data and Wine data.

To compare the different results, we used the same test methods with the same parameters values for the classification system (size of subdivision  $M$ , correlation threshold  $\theta$ ). We used a ten order cross-validation (Kohavi, 1995). The obtained results are presented and analyzed in this part.

## 5.1 Results of the Method with a Constant Number of Rules

We present here the tests realized with the method of new rules construction via the approximate reasoning according to the approach with a constant number of rules. The first tests were made with  $g\text{-threshold}=0$  or  $g\text{-threshold}=1$  which seem the only reasonable values. Values superior to 2 would throw a search which we could not consider as near the rule. The analysis of the results and the emission of hypotheses to explain them can be made by examining the shape of the generated rules. We distinguish two cases in function of  $g\text{-threshold}$  (0 or 1).

The case  $g\text{-threshold}=0$  gives results (rates of good classification) almost identical to the basic generation (followed by an exact inference), so they are almost identical to results presented in column "Basic Generator, Basic Inference" of table 1. On the tested data, there are only very few changes between rules generated *basically* and *approximately*. This is mainly due to the following report: it is impossible, for a premise containing a number strictly lower than 3 attributes to have  $g\text{-dist}=0$ . All the rules containing 2 attributes in their premise can not be improved.

Let us focus now on the case  $g\text{-threshold}=1$ . Table 1 presents the rates of good classifications obtained with each of the three possible approaches: - column « *Approx. Gen.-1, Basic Inference* »: rules were generated by SUCRAGE then new rules were built via approximate reasoning, with a value of  $g\text{-threshold}=1$ . The base of rules is then exploited by a *basic* inference engine.

- column « *Basic Gen., Basic Inference* »: rules were generated by SUCRAGE in its initial version. The rules base is then exploited by a *basic* inference engine.

- column « *Basic Gen., Approx. Inference* »: rules were generated by SUCRAGE in its initial version. The base of rules is then exploited by an approximate inference engine. It is the results of this method that we hope to approach (even improve) by using approximate reasoning to build new rules.

With Iris data, we can see that the results of the approximate generation are close to those obtained with the approximate inference. Moreover, these results are very similar to those obtained with the basic generator followed by basic reasoning. Thus, it is not very interesting in view of the supplementary computations needed.

With the WINE data, the results are very interesting: approximate generation of rules allows improving the case of a basic generation followed by an approximate inference (a single case of identical results). There is also improvement with regard to a basic generation followed by a basic inference.

Table 1: Method with a constant number of rules.

Method	<i>Approx. Gen-1, Basic Inference</i>	<i>Basic Gen., Basic Inference</i>	<i>Basic Gen., Approx. Inference</i>
Parameter			
IRIS data			
M=3 θ=0.9	<b>98.00</b>	97.33	97.33
M=3 θ=0.8	<b>96.67</b>	95.33	<b>96.67</b>
M=5 θ=0.9	93.33	<b>94.00</b>	<b>94.00</b>
M=5 θ=0.8	90.67	90.67	<b>93.33</b>
WINE data			
M=3 θ=0.9	<b>88.75</b>	<b>88.75</b>	<b>88.75</b>
M=3 θ=0.8	<b>88.20</b>	87.09	87.64
M=5 θ=0.9	<b>92.68</b>	90.92	91.50
M=5 θ=0.8	<b>93.27</b>	92.05	92.05

For  $g\text{-threshold}=1$ , the observation "around" the rules is not insufficient any more here (case  $g\text{-threshold}=0$ ) but can be too much: we sometimes witness the creation of double rules. The observation near a rule can go up to another basic rule which was already generated, it is then the strongest which is going to win. We can have here a loss of information. The algorithm tends then to create an absorption of weak rules by strong rules rather than an extension of the strong rules.

### 5.2 Results of the Method with Addition of Rules

Table 2 presents the results obtained with the second method of new rules generation via the approximate reasoning: this time every new generated rule is added to the initial base of rules. The column "*Approx. Gen. Add., Basic Inference*" of this table contains the results obtained with this approach, the

title of the last two columns is unchanged in comparison with table 1. In addition, every cell contains the rate of good classifications followed by the number of rules between brackets (for this method the number of rules takes importance).

Table 2: Method with addition of rules.

Method	<i>Approx. Gen. Add., Basic Inference</i>	<i>Basic Gen., Basic Inference</i>	<i>Basic Gen., Approx. Inference</i>
Parameter			
IRIS data			
M=3 θ=0.9	<b>97.33</b> (61.4)	<b>97.33</b> (23.5)	<b>97.33</b> (23.5)
M=3 θ=0.8	<b>96.67</b> (123.6)	95.33 (21.5)	<b>96.67</b> (21.5)
M=5 θ=0.9	<b>95.33</b> (119.1)	94.00 (37.7)	94.00 (37.7)
M=5 θ=0.8	<b>94.67</b> (303.9)	90.67 (39.7)	93.33 (39.7)
WINE data			
M=3 θ=0.9	<b>90.45</b> (245.2)	88.75 (96.7)	88.75 (96.7)
M=3 θ=0.8	<b>89.93</b> (214)	87.09 (97.9)	87.64 (97.9)
M=5 θ=0.9	89.35 (388.2)	90.92 (152.4)	<b>91.50</b> (152.4)
M=5 θ=0.8	91.57 (343.8)	<b>92.05</b> (152.2)	<b>92.05</b> (152.2)

The analysis of these results shows that they are very correct at the level of good classifications rate: with the *approximate generator with addition* the rates of good classifications are generally improved or maintained in comparison with the basic generator followed by a basic inference as well as the basic generator followed by an approximate inference. With the WINE data, two cases of light depreciation are to be noted.

On the other hand, the number of generated rules increases very widely. Moreover, it is evident that we generate many useless rules, even harmful rules entailing a decline of the results. A work to reduce the number of rules becomes here indispensable as well to eliminate the harmful rules that for reasons of legibility of the base of rules (Nozaki et al. 1994) (Duch et al., 2004). A work was realized in this sense: we used Genetic Algorithms to reduce the size of the base of rules without losing too much performance. This approach tested in the case of basic generation of rules led to very interesting experimental results (Borgi, 2005).

## 6 CONCLUSION

The supervised learning method SUCRAGE allows to generate classification rules then to exploit them by an inference engine which implements a basic reasoning or an approximate reasoning. The originality of our approach lies in the use of approximate reasoning to refine the learning: this reasoning is not only considered any more as a second running mode of the inference engine but is considered as a continuation of the learning phase. Approximate reasoning allows to generate new wider and more general rules. Thus imprecision of the observations are taken into account and problems due to the discretization of continuous attributes are eased. This process of learning refinement allows to adapt and to improve the discretization. The initial discretization is regular, it is not supervised. It becomes, via the approximate reasoning, supervised, as far as the observations are taken into account to estimate their adequacy to rules and as far as the belief degrees of these new rules are then computed on the whole training set. Moreover the interest of this approximate generation is that the new base of rules is then exploited by a basic inference engine, easier to interpret. Thus approximate reasoning complexity is moved from the classification phase (a step that has to be repeated) to the learning phase (a step which is done once). The realized tests lead to satisfactory results as far as they are close to those obtained with a basic generation of rules exploited by an approximate inference engine.

The continuation of the work will focus on the first method of new rules generation (with constant number of rules) to make it closer to what takes place during approximate inference. The search for other forms of g-distance can turn out useful notably to be able to obtain results of generation between the g-threshold value 0 (where we remain too close to the observation) and the g-threshold value 1 (where we go away too many "surroundings" of the observation). The second method, which enriches the base of rules with all the new rules, is penalized by the final size of the obtained base. An interesting perspective is to bend over the manners to reduce the number of rules without losing too much classification performance.

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