

A PERFORMANCE METRIC FOR MOBILE ROBOT LOCALIZATION

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Abstract: This paper focus on the problem of how to measure in a reproducible way the localization precision of a mobile robot. In particular localization algorithms that match the classic prediction-correction model are considered. We propose a performance metric based on the formalization of the error sources that affect the pose estimation error. Performance results of a localization algorithm for a real mobile robot are presented. This metric fulfils at the same time the following properties: 1) to effectively measure the estimation error of a pose estimation algorithm, 2) to be reproducible, 3) to clearly separate the contribution of the correction part from the prediction part of the algorithm, and 4) to make easy the algorithm performance analysis respect to the great number of influencing factors. The proposed metric allows the validation and evaluation of a localization algorithm in a systematic and standard way, reducing workload and design time.

1 INTRODUCTION

Experimentation in Autonomous Mobile Robots (AMR) research is not an obvious task. This type of robots are complex systems. They incorporate a great number of interrelated hardware and software subsystems. Their navigation environment must be specifically modelled and their components must operate in real time. Finally, any research contribution about autonomous behaviours in real environments requires a considerable effort in both theoretical and experimental works.

Performance evaluation for such complex systems is likewise a complex task. Experiments must be controlled and reproducible, but it is not easy to repeat the experiments of another research group because of the high number of involved variables. There exist an important need to establish general frameworks of performance evaluation, in the context of intelligent systems (Meystel et al, 2003) and more specifically about AMRs (Dillman, 2004). The work of (Hanks et al, 1993) goes ahead and remarks the need of benchmarks that not only provide performance comparisons, but that also support the scientific progress by helping to analyze

why the system behaves the way it does. Furthermore, the development of this area will be a requirement for AMR systems to reach the consumer market.

A main feature for robot autonomy is the self-localization capability. The robot must estimate by itself its pose (position and orientation) respect to a reference system, with enough precision to achieve the commended tasks. The particular problem we focus is how to measure in a reproducible way the precision of the pose estimations produced by the robot. The solution of this problem will allow the validation and performance evaluation of a localization algorithm in a systematic and standard way, reducing design time and workload.

Our hypothesis is that it is possible to perform systematic and reproducible measurements of the pose estimation error in real navigation conditions. Although there are a great number of influencing factors, we believe that they can be enumerated and modelled. In section 2 we formalize the pose estimation process, In section 3, a reproducible performance metric for robot localization is proposed. Experimental results are presented and discussed in section 4, followed by the main conclusions in section 5.

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2 THE PREDICTION-CORRECTION MODEL

Most solutions to AMR pose estimation follow the classic prediction-correction (or predict-update) closed loop state estimator model presented in Figure 1. According to (Thrun, 2002), virtually all state of the art robotic mapping algorithms are probabilistic, and the single dominating scheme for

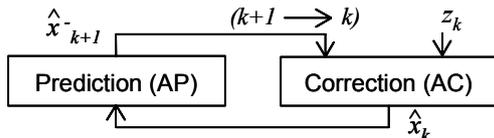


Figure 1: The generic prediction-correction model.

state estimation is the Bayes filter, a recursive estimator that matches the prediction-correction model. A particular bayesian filter of common use in AMR localization is the Kalman Filter (Welch and Bishop, 2004). But other non-probabilistic solutions also follow this model, for example the possibilistic approach of (Bloch and Saffioti, 2002). This widespread and simple scheme estimates the robot's pose in two steps: prediction and correction. The prediction is achieved by an algorithm (AP in advance) that implements the process model:

$$\hat{x}_{k+1}^- = A(\hat{x}_k^-, u_k) \quad (2-1)$$

where \hat{x}_k^- is the estimation of the robot's state at step k , u_k is the control action that was ordered at step k to reach the step $k+1$, A is the process model, and \hat{x}_{k+1}^- is the a priori estimation of the robot's state at step $k+1$. \hat{x}_k^- must contain the robot's pose, that is habitually in the form of a vector composed by two cartesian position coordinates and one orientation coordinate respect to a reference system external to the robot. In an AMR system the process model A often represents a fine tuned dead-reckoning model for the actual motor robot platform. It includes physical wheels dimensions, odometers resolution, models of the motor control algorithms, etc.

If the pose is estimated in open loop using only the AP, the estimation error in \hat{x}_{k+1}^- increase monotonically along the trajectory and the robot will be lost or will collide. The classic solution to this problem is using exteroceptive sensors like sonar, laser, CCD cameras, etc. to capture some actual perception z_k of the surrounding objects and comparing it against a map of the environment. Here begins the algorithm of the correction phase (in advance, AC), that fuses the a priori estimation with

the information that comes from z_k to obtain the posterior state estimation \hat{x}_k . The correction estimator function E is expressed as:

$$\hat{x}_k = E(\hat{x}_k^-, z_k, H(\hat{x}_k^-)) \quad (2-2)$$

where $H(x_i)$ models the robot's perception at a particular state x_i . It must include the sensor model and the measurement model involved in the capture of z_k . The estimator E must correct the a priori estimation by comparing the actual perception z_k with $H(\hat{x}_k^-)$, the expected perception at \hat{x}_k^- . Every variable in this loop has an associated uncertainty, that have not been represented in the previous equations for simplicity. E may use the uncertainties of \hat{x}_k^- , z_k and the uncertainty derived from the comparison of z_k with $H(\hat{x}_k^-)$ to perform the states fusion.

2.1 Estimation Error Evaluation

The AP-AC tandem conforms an algorithm for the estimation of the robot's pose (in advance, AEP). Its performance is typically evaluated by the error of the estimated pose (in advance, EEP) respect to the real pose. The particular operators that compute the EEP will be presented in the next section.

Note that the EEP may be applied to both \hat{x}_k^- and \hat{x}_k estimations. The AC is responsible of the final estimation and, apparently, is the key of the process, but, how much effective is the AC respect to the AP? Suppose a robot with good quality odometers an a fine tuned AP (easy to build nowadays), precise enough to keep \hat{x}_k^- below the application location error requirements along several meters of trajectory. In most cases the AC will be just copying \hat{x}_k^- to \hat{x}_k , but having to compute the code from H and E functions. A common situation is to find that AC (world model, sensor model, etc) complexity is much greater than the one of the AP. It is worth in such cases to spend a great deal of computation in the AC? How the computational load between the AP and the AC can be balanced without degrading the AEP performance? We propose to analyze the posterior estimation error relative to the a priori error as one of the key elements that may help to explain the performance of an AEP in terms of its internal components. This approach is one step ahead from the obtaining of a simple performance metric punctuation, in the sense of the (Hanks et al, 1993) reference in section 1, and is the main part of the performance metric presented in section 3.

Nowadays, there is no a widely accepted performance metric for measuring in a reproducible way the localization precision of a mobile robot. Furthermore, there are few published works that propose performance metrics for robot localization. In (O’Sullivan et al, 2004) an interesting performance metric for map building is presented. Although the pose estimation is intimately involved in the map building task, this metric does not separate the performance of the pose estimation from the world modelling algorithms. (Gat, 1995) emphasizes on the experiments reproducibility and propose a performance metric for AMRs that measures the traversed distance and elapsed time to reach a goal. Here again the pose estimation performance is enclosed with another robot’s subsystems. In the general robot localization literature, a very common way to evaluate the EEP is to measure quantitatively the position and orientation error, see for example (Lee and Song, 2004) and (Clarentin et al, 2005). Other works do not measure quantitatively the EEP, as (Sagüés and Guerrero, 2005) which controls the EEP by making the robot to stop periodically in a checkpoint marked on the floor and counting the times it does inside the marks. Some works, such as (Porta et al, 2005), report in detail the experiments conditions. Others, like (Castellanos et al, 2001), also report the statistical significance of the obtained EEP distributions. But only few works as (Fox et al, 1999), (Gutmann and Fox, 2002) and (Di Marco et al, 2004) present experimental results with enough quality and detail to match the requirements of a performance metric. Experiments conditions are also reported as exhaustively to be reproduced. In such cases, the pose estimation error is reported as a whole, without analysing the AC contribution to the final EEP, as exposed as follows.

3 PERFORMANCE METRIC

We propose a performance metric for benchmarking the correction algorithm AC based on formalizing the error sources to prevent hidden factors that could falsify the obtained EEP. The metric is composed by the following steps:

1. Experiments framework report
2. Run conditions report
3. Analysis of the absolute estimation error
4. Analysis of the estimation error relative to the a priori error

Steps 1 and 2 are a collection of requirements to describe the navigation experiments (runs) with

enough detail to be reproducible. Steps 3 and 4 are the metric itself. The term “run” is used here as a controlled experiment in which the robot travels along a monitorized trajectory. The AEP developer should decide the rooms and trajectories depending on his/her research objectives. During every run, the trajectory’s real poses x_i should be sampled with enough frequency to obtain representative statistical distributions. This sampling should be done using measurement instruments external to the robot and its precision, the ground truth of the experiment, should be at least twice the AEP’s expected precision. The robot should record the a priori and posterior estimations produced for each sampled real pose. In consequence for every run i three traces of poses should be obtained:

$$\begin{aligned} RP_i &= \{x_1, x_2 \dots x_{N_i}\} \\ PE_{post}_i &= \{\hat{x}_1, \hat{x}_2 \dots \hat{x}_{N_i}\} \\ PE_{prio}_i &= \{\hat{x}^-_1, \hat{x}^-_2 \dots \hat{x}^-_{N_i}\} \end{aligned} \quad (3-1)$$

where RP_i , PE_{post}_i , and PE_{prio}_i are the traces of real poses, posterior pose estimations and a priori pose estimations from the run i , respectively. N_i is the number of sampled real poses in the run i , and $i = 1..R$, being R the number of runs. Metric components are explained in the following sections.

3.1 Experiments Framework Report

In order to document the EEP factors with sufficient detail, the AEP developer should first describe the general experiments framework, common to every run. It should be reported the general objectives of the particular AEP development, the type of AMR (general or specific purpose, etc), the type of navigation environment (indoor, outdoor, office, domestic, industrial, etc.), and how these aspects condition the run selection.

The AC should be documented in terms of the description of the E and H functions and their uncertainty models. Additionally, it should be reported as exhaustively as possible the frequency of the AC estimations: respect to the AP estimation frequency, to the absolute time, to the robot travelled distance, etc. The procedure and frequency of real poses measurement, and its ground truth should also be reported. In the case of simulated runs, it should be described the simulator internal models.

3.2 Run Conditions Report

This report must contain the experiment features that may change between runs. For each run, the place where it has been performed should be described. At

least the walls and obstacles topology, their material properties and the ambient conditions should be presented. It should also be described any other factor that may affect the capture of the robot exteroceptive sensors. Additionally, the run trajectory should be included, at least the RP_i , $PEpost_i$, and $PEprio_i$ traces and ($i = 1..R$). A graphical 2D floor projection is a conventional way to present the trajectories. The criteria for trajectories generation should be reported.

It should be justified why each run is representative of real trajectories and how the statistical parameters derived from it are valid, in terms of sufficient number of samples, use of random or controlled trajectories, etc. The eventual environment changes during the run should be quantified, to describe the degree in which the run represents dynamic environments, for example, number of perceptible people during the run, etc.

3.3 Analysis of the Absolute Estimation Error

The objective of this analysis is the absolute AEP's performance in terms of position and orientation errors of the captured posterior estimations $PEpost_i$. Lets first define the EEP as a set of a position error EEP_{XY} and an orientation error EEP_T :

$$\begin{aligned} EEP_{XY}(\hat{x}_i) &= d_{XY}(\hat{x}_i, x_i) \\ EEP_T(\hat{x}_i) &= d_T(\hat{x}_i, x_i) \end{aligned} \quad (3-2)$$

where x_i is the real pose measured under controlled conditions when the robot produced the estimation \hat{x}_i , d_{XY} is the euclidean distance between positions over the plane of the floor, and d_T is the absolute orientations angle difference (euclidean distance over the orientation dimension). We chose euclidean distances because they are intuitive and of common use. For example, the requirements of an AEP development project can be expressed as "The 95% of the EEP should be under 20cm, 5°."

Distributions of the posterior EEP_{XY} and EEP_T for the run i ($i = 1..R$) are defined as:

$$\begin{aligned} EEP_{postXY}_i &= \{EEP_{XY}(\hat{x}_j) / \hat{x}_j \in PEpost_i\} \\ EEP_{postT}_i &= \{EEP_T(\hat{x}_j) / \hat{x}_j \in PEpost_i\} \end{aligned} \quad (3-3)$$

The data of interest are the R bidimensional distributions EEP_{postXY}_i vs. EEP_{postT}_i . The distributions should be analysed in terms of the following parameters:

- Ground truth limits: Every distribution point should be above the ground truth.

- Relevant percentiles: 100%, 95%, 90%, etc. They allow runs comparisons in terms of position and orientation precision.
- (If available) The theoretical limit of the optimal AEP performance. It lets to analyze what percentage of estimations are optimal.

Observed differences between run distributions should be explained in terms of the run conditions factors presented in the previous section. If the differences are well explained and the runs are representative of the robot's target navigation environment, the union of all distributions may be analyzed as a single bidimensional distribution to represent the global AEP performance metric.

3.4 Analysis of the Estimation Error Relative to the a Priori Error

This analysis focus on the AC performance. The objective is to measure the AC capability to effectively reduce the EEP, independently from the AP efficiency. The distributions of the a priori EEP_{XY} and EEP_T for the run i ($i = 1..R$) are defined as:

$$\begin{aligned} EEP_{prioXY}_i &= \{EEP_{XY}(\hat{x}_j) / \hat{x}_j \in PEprio_i\} \\ EEP_{prioT}_i &= \{EEP_T(\hat{x}_j) / \hat{x}_j \in PEprio_i\} \end{aligned} \quad (3-4)$$

The data to be analyzed are the R bidimensional distributions EEP_{postXY}_i vs. EEP_{prioXY}_i (EEP_{XY} analysis) and the R bidimensional distributions EEP_{postT}_i vs. EEP_{prioT}_i (EEP_T analysis). Both data sets will be analyzed in the following way: 1) For every run, Quantify in a factor C_{XY} the percentage of estimations \hat{x}_j that improve the EEP_{XY} :

$$EEP_{XY}(\hat{x}_j) \leq EEP_{XY}(\hat{x}^-_j) \quad (3-5)$$

2) Quantify in a factor C_T the same percentage for EEP_T :

$$EEP_T(\hat{x}_j) \leq EEP_T(\hat{x}^-_j) \quad (3-6)$$

3) Quantify in a factor C_{XYT} the same percentage that holds (3-5) and (3-6) at the same time.

C_{XY} , C_T and C_{XYT} factors represent the AC capability to really *correct* the EEP under the navigation conditions of the runs set.

In the same way as the previous analysis, the differences between run distributions should be explained in terms of the run conditions factors. If the differences are well explained and the runs are representative of the robot's target navigation environment, the union of all distributions may be analyzed as a single bidimensional distribution to represent the global AC performance metric.

The analysis may be extended by the addition of other elements, like the injection of noise to \hat{x}_k^- to increase the dynamic range of the a priori EEP. This range may be interpreted as the degradation level of the error sources that affect the AP. We can study the portion of any posterior EEP distribution in a particular interval of the a priori EEP range, and quantify how the posterior EEP distribution will be consequently affected. Additionally, the degradation of error sources that affect the another AC input, the z_k perception, may be represented by designing various run experiments with different levels of perception noise.

4 EXPERIMENTAL RESULTS

The proposed performance metric has been applied to a particular AEP developed by the authors. The four metric steps are presented in next sections.

4.1 Experiments Framework Report

The target AEP is being developed in the frame of a research project which main objective is the design robotic platforms and tools for helping the AMR research groups. The aim of this AEP development is to implement a simple self-localization module that validate the robot subsystems by showing autonomous navigation in indoor office environments.

The robotic platform used is Sancho-2 (see Figure 2), a mobile robot completely designed and built by the authors for research purposes. Its

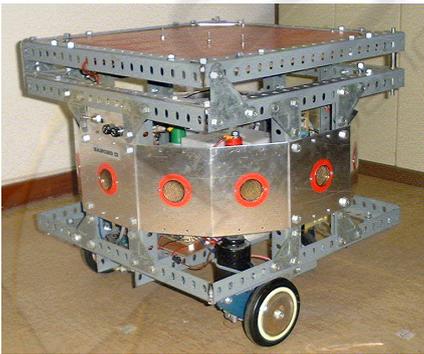


Figure 2: The mobile robot Sancho-2.

dimensions are 50cm wide, 50cm long and 50cm high, that contains the hardware of the motors and sensors control subsystems. High level software components are implemented in a laptop PC that is placed on the upper tray and connected by a serial

cable. The wheels have a tricycle structure, with two motorized wheels and one castor wheel. The resolution of the odometric sensors is 1.2cm. The environment perception are achieved by a ring of twelve ultrasonic sensors, whose resolution is 4cm.

Our AEP is an appearance-based approach and works as follows: Once the AP has produced the a priori estimation \hat{x}_k^- , a complete sonar capture is fired. It produces a 12 echoes vector (z_k) that may be interpreted as a point in a 12 dimensions vectorial space. The AC correction estimator E is expressed as:

$$\hat{x}_k = H^{-1}(f_m(z_k, H(G_{LP}))) \quad (4-1)$$

The expected perception is modelled as a points cloud in the perception space by the following way: A grid G_{LP} of local poses x_i is generated around the a priori pose estimation \hat{x}_k^- , and for each of them its expected perception $H(x_i)$ is computed. The parameters of G_{LP} are the position and orientation grid steps, $s_{XY} = 10\text{cm}$ and $s_T = 10^\circ$, and the grid radii from its centre, 40cm, 40° . These values determine the theoretical maximum EEP that our AC can correct.

The H function models the robot environment as a previously given map where the walls and obstacles are represented by a set of 2D line segments. The function returns, for each particular ultrasonic sensor pose, the distance to the nearest line segment inside a sensor cone with an aperture angle of 30° . It does not calculate any kind of sonar rebounds or outliers. This model is also used to emulate the robot's perception in the simulation tests.

The actual perception z_k is matched against the expected perception cloud using the f_m function that computes the nearest neighbour with the euclidean distance. The posterior estimation \hat{x}_k is obtained by the H^{-1} function, by the identification of the grid pose that produced the nearest perception.

As G_{LP} is a discrete regular grid, the optimal estimation respect to the EEP_{XY} or EEP_T metric is obtained when the posterior estimation \hat{x}_k is also the nearest neighbour in the positions plane or orientations axis, respectively. To formalize this concept we define the maximum reachable precisions MPA_{xy} (cm) and MPA_t ($^\circ$) parameters to be the worst EEP_{XY} or EEP_T metric values that an optimal estimation may obtain. It is easy to see that $MPA_{xy} = 0.71 s_{XY}$, and $MPA_t = 0.5 s_T$. These parameters will be needed in the section 4.3 and their values for the experiments are 7.1cm and 5° . The AP and AC estimation frequencies are equal

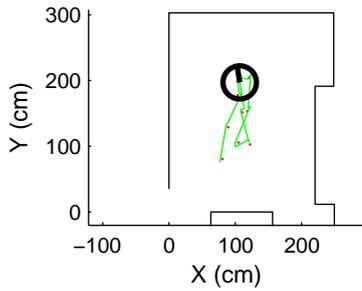


Figure 3: The run R1.

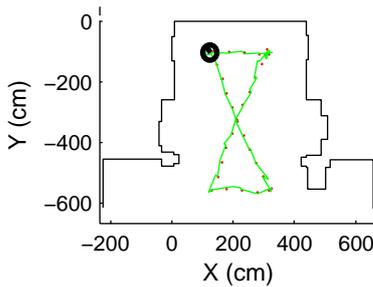


Figure 4: The run R2.

and their value is 1 estimation per each 50cm navigated. In this project's phase, the runs have been simulated. So, real pose measurement is exact and its frequency may be the same as the one for AC estimations, and the consequent ground truth is 0.0cm and 0.0°.

4.2 Run Conditions Report

Figures 3, 4 and 5 show the walls topology and trajectories of the three runs, respectively. We have selected real places from our Faculty buildings with different areas and walls topologies to represent the indoor navigation environments in an typical office building. The navigation areas are small (2m², R1), medium (13m², R2), and big (81m², R3), and the walls topologies are square (R1), open square (R2), and corridor (R3). Every wall segment has been modelled as having the same acoustic properties. Regarding to the ambient conditions, we have considered the room air temperature as a factor that may change the sound speed and influence the ultrasonic sensor precision. In our experiments this temperature was 25°C. The runs do not include neither moving objects nor furniture changes.

To determine the trajectories, we have adopted the criteria of travelling over most of the navigation area and preventing trajectories coincidences. In the corridor R3 the trajectory has been planned to diagonally traverse only one time the place. In the

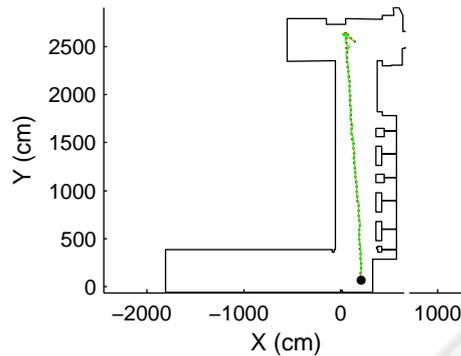


Figure 5: The run R3.

places R1 and R2 the trajectory draws an "8" over the floor, passing by the four extremes of an imaginary navigation rectangle. The runs produced 18 (R1), 62 (R2) and 61 (R3) estimations, resulting in a total of 141 estimations. We did not increment the R1 estimations to prevent the distribution slant because this place is small and its punctuations are better than the other ones.

Our AP a priori EEP distributions are upper-bounded by the limits of 10cm and 5°. To full characterize the AC response, we have injected to the a priori estimation \hat{x}^-_k an uniform noise distribution of which ranges are the theoretical limits of our AC, the grid radii (40cm, 40°).

4.3 Analysis of the Absolute Estimation Error

Figure 6 shows the three run distributions, under the described experimentation conditions, including the uniform noise injection to \hat{x}^-_k . Each run distribution is represented with a different icon (see also Figure

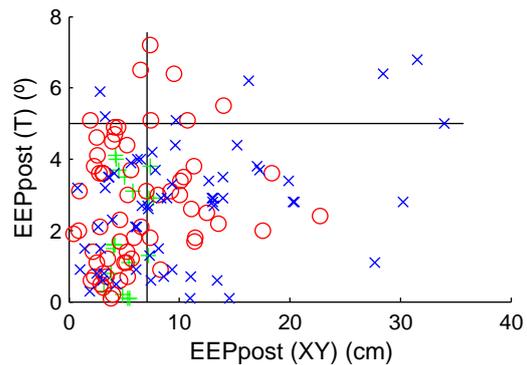


Figure 6: Absolute estimation error distributions.

7). Vertical and horizontal lines show the *MPAt* and *MPAXy* limits, respectively (see section 4.1).

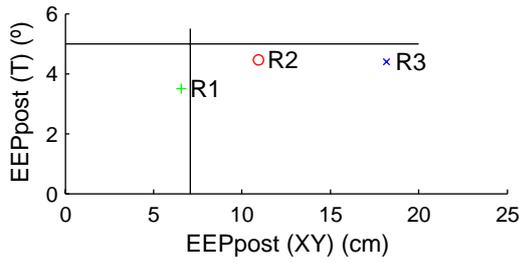


Figure 7: Absolute runs merit factor.

In order to explain the runs differences, Figure 7 reduces each run distribution to one point whose coordinates are the the merit factors mfx_{y_i} and mft_i :

$$\begin{aligned} mfx_{y_i} &= \mu\alpha_{y_i} + \sigma\alpha_{y_i} \\ mft_i &= \mu t_i + \sigma t_i \end{aligned} \quad (4-2)$$

where $\mu\alpha_{y_i}$ and μt_i are the means of EEP_{postXY_i} and EEP_{postT_i} , respectively, and $\sigma\alpha_{y_i}$ and σt_i are the standard deviations of EEP_{postXY_i} and EEP_{postT_i} , respectively. We explain the differences between the 3 runs by the size and walls disposition of the place R1, the best result, is a small room and R3, the worst, is a corridor.

Every punctuation is over the ground truth (0.0cm, 0.0°). The maximum errors are 34.0cm and 7.2° that fall to 20.4cm, 5.7° at the 95% percentiles. The percentage of optimal estimations, i.e. inside both MPA_{xy} and MPA_t limits, are 53.2%.

The AEP show better performance in orientation than in position estimation. If the final robot application may accept a small orientation error degradation, we could increase the s_T grid parameter to reduce the poses number of G_{LP} , and consequently reducing the computational cost of our AC.

4.4 Analysis of the Estimation Error Relative to the a Priori Error

Figure 8 compares the posterior EEP_{XY} vs. a priori EEP_{XY} distributions. Figure 9 shows the same comparison in terms of the orientation error EEP_T . The line in the figures shows the limit where the posterior EEP is equal to the a priori EEP. The correction factors of our AEP are $C_{XY} = 88\%$, $C_T = 95\%$ and $C_{XYT} = 84\%$. In future experiments we plan to perform runs for producing a controlled perception degradation.

4.5 Discussion

Performance metrics should measure the parameter of interest, the measurement precision should be enough and the metric process should be

reproducible. The proposed procedure may effectively measure the interest parameter, the AC estimation error, independently from the particular

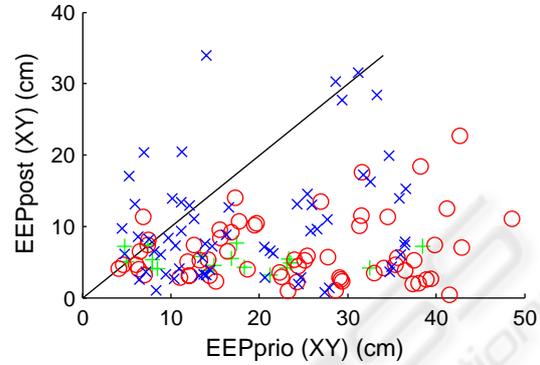


Figure 8: Relative position error distributions.

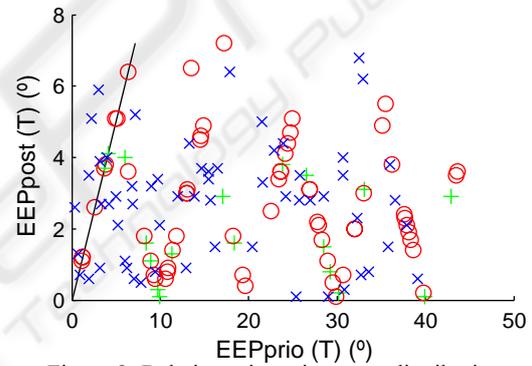


Figure 9: Relative orientation error distributions.

AP to be used and other components performance, through the noise injection to the a priori estimation \hat{x}_k^- . It has been shown that it is adequate to measure it in two ways: absolute and relative to the a priori error. It also has been shown that it is adequate to make this measurement in the form of a set of distributions instead of a single distribution or numeric value, because each run distribution may be affected by different factors from each others.

The sufficient metric precision is justified by the requirement to the AEP developer to measure and report the ground truth of the real pose measurement process.

The metric reproducibility is guaranteed by the high detail of the run conditions report and the runs separation requirement. A run may be easily reproduced using the report's information. This is also a tool for experiment validation.

This metric allows a great number of useful performance comparisons. Different run

distributions may represent different run places, robot platforms (APs), AEPs, temperature or illumination conditions, trajectories in the same room, etc.

5 CONCLUSION

The proposed performance metric offers a contribution to the area of the mobile robotics performance measurement, in particular in the robot localization field. This metric differs from the works found in the literature in the fact that it fulfils at the same time the useful properties of 1) to effectively measure the estimation error of a pose estimation algorithm, 2) to be reproducible, 3) to clearly separate the contribution of the correction algorithm, and 4) to make easy the analysis of the algorithm performance respect to the great number of influencing factors.

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