MONTE CARLO LOCALIZATION IN HIGHLY SYMMETRIC ENVIRONMENTS

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Keywords: Monte Carlo Localization, MCL, Symmetric Environments, Two-Stage Sampling.

Abstract: The localization problem is a central issue in mobile robotics. Monte Carlo Localization (MCL) is a popular method to solve the localization problem for mobile robots. However, usual MCL has some shortcomings in terms of computational complexity, robustness and the handling of highly symmetric environments. These three issues are adressed in this work. We present three Monte Carlo localization algorithms as a solution to these problems. The focus lies on two of these, which are especially suitable for highly symmetric environments, for which we introduce two-stage sampling as the resampling scheme.

1 INTRODUCTION

Localization is a crucial ability for a mobile robot in order to be able to navigate. It is a key competence in many successful robotic systems (see e.g. (S. Thrun, 2001), (S. Lenser, 2000)). Obviously, having a good idea of ones position is also a prerequisite for decision making and interaction with the environment. The localization problem is often divided into three subproblems. The most basic problem is position tracking, where the robot starts with a known initial position and just has to keep track of its pose. Whereas global localization requires the robot to localize itself from scratch. The most complex subproblem is the kidnapped robot problem. Here the mobile unit has to estimate its pose with a wrong initial believe. All three subproblems can be solved with a particle filter algorithm (L. Ronghua, 2004), (Fox, 2003) and (S. Lenser, 2000).

Here we also consider a fourth subproblem: Highly symmetric environments. The basic particle filter often fails to localize a robot when there are a lot of symmetries in the environment (L. Ronghua, 2004). Therefore, it is necessary to introduce a technique that allows to track multiple distinct positions on the map.

Symmetric environments need to be considered for many mobile robot applications. Examples are robots for surveillance in office buildings, where we frequently find symmetries. We observe the same situation for different kinds of buildings. Furthermore, robots operating in sewers and other tunnel like environments may have to deal with ambiguities.

In the past years several approaches to Monte Carlo localization had been proposed. Most of these fail in symmetric environments (Fox, 2003), (S. Lenser, 2000) and (S. Thrun, 2001) or use large sample sets (A. Milstein, 2002). A lot of research deals with the efficient use of the samples, e.g. (S. Lenser, 2000), in order to use small sample sets and only a few publications are concerned with variable sample set sizes (L. Ronghua, 2004) and (Fox, 2003).

In this work we present three algorithms to solve the localization problem based on Monte Carlo Methods. The first is derived from Dieter Fox's KLD-Sampling (Fox, 2003) and includes a procedure similar to Sensor Resetting Localization (S. Lenser, 2000). We call this method KLD-Sampling with Sensor Resetting (KLD-SRL). The other two algorithms use two-stage sampling for resampling in order to master highly symmetric environments.

The remainder of this paper is organized as follows. Section 2 summarizes Monte Carlo Localization. Section 3 outlines related work. Section 4 introduces an efficient and robust localization algorithm based on KLD-Sampling. Furthermore, in Section 5 we present an extension to MCL to handle symmetric environments and show how to include it into the schemes of existing MCL algorithms. Finally, In section 7 we show the experimental results followed by the conclusions.

2 MONTE CARLO LOCALIZATION

Monte Carlo Localization (MCL) is a Bayesian approach to localization. There are numerous publica-

Sehestedt S. and E. Schneider F. (2006).
MONTE CARLO LOCALIZATION IN HIGHLY SYMMETRIC ENVIRONMENTS.
In *Proceedings of the Third International Conference on Informatics in Control, Automation and Robotics*, pages 249-254
DOI: 10.5220/0001215802490254

Table 1: The three steps of the MCL algorithm.

1. Prediction: Draw $x_t^i \sim p(x_t^i \mid x_{t-1}^i, u_{t-1})$.
2. Update: Compute the importance factors $\omega_t^i = \eta p(y_t \mid x_t^i)$, with η being a normalization factor to ensure that the weights sum up to 1. Here, y_t is a sensor reading at time t.
3. Resample.

tions presenting Monte Carlo Methods in detail, examples are (A. Doucet, 2000) and (S. Arulampalam, 2002). The belief is represented by a set of weighted samples $X(t) = \langle x_t^i, \omega_t^i \mid i = 1, ..., N \rangle$, where each x_t^i is a possible position of the robot and ω_t^i the according importance factor at time step t. The calculation of the posterior probability distribution consists of three steps. Prediction, update and resampling.

Prediction is done by moving the samples in the prior distribution as specified by the motion model. Then the weights are calculated, using a sensor model. Finally, in the additional resampling step, particles with low weight are replaced by those with significant weight. Doing this, the samples get concentrated in areas of high probability. The downside is, that this procedure can lead to premature convergence in ambiguous situations (A. Bienvenue, 2002). For more details on resampling see (A. Doucet, 2000). MCL is summarized in table 1.

2.1 Discussion of MCL

Beside the very appealing attributes of particle filters (S. Arulampalam, 2002), there are some issues that need to be taken into account. Firstly, the basic particle filter uses large numbers of particles, what results in high computational effort. Above this, MCL is not able to track multiple hypotheses stable, but converges to exactly one position. This can be harmful, as we loose track of possibly right positions and do not take these into account for pathplanning. Finally, the basic particle filter recovers from localization failure very slowly.

3 RELATED WORK

3.1 Sensor Resetting Localization

Sensor Resetting Localization (SRL) is an extension to Monte Carlo Localization (S. Lenser, 2000). In each time step, a number of particles is replaced with samples drawn from the sensor model. Using the equation

$$Ns = (1 - \frac{\tilde{P}}{P_t}) * N \tag{1}$$

we determine how many samples are to be replaced. With $\tilde{P} = \sum_{i=0}^{N} \omega_t^i / N$ and P_t the probability threshold, that can be adjusted to let the algorithm react more or less sensitive. The logic behind this procedure is, that if the particles have a high average weight, we can be certain that most samples are in regions of high probability. If the average weight is low, we can conclude that many particles are in unlikely positions. Then we try to replace a number of samples by sampling from the sensor model.

Therefore, we draw the new samples according to the most recent sensor reading. Hence, these particles are in areas of high probability. The problem with this procedure is, that it may be very difficult to sample from $p(y_t^i \mid x_t^i)$ (S. Thrun, 2001).

SRL is mathematically questionable as it simply removes a number of particles and replaces them with new samples, as already mentioned in (S. Thrun, 2001).

4 ADAPTIVE PARTICLE FILTERS

4.1 KLD-Sampling

Variable sample set sizes allow us to adjust the number of particles to the complexity of the problem. Clearly, we need much more particles for global localization than for tracking (Fox, 2003). Following this logic, KLD-Sampling adjusts the number of particles based on the quality of the approximation of the posterior distribution. We now give a brief description of KLD-Sampling as introduced by Dieter Fox (Fox, 2003).

It is assumed that the true posterior is given by a discrete, piecewise constant distribution. Then it can be guaranteed that with probability $1 - \delta$ the distance between the true posterior and the sample-based approximation is less than ϵ.

For this, an approximation of the Kullback-Leibler distance is used. We compute the number N of samples that is needed for the approximation based on the number k of bins with support (Fox, 2003).

$$N = \frac{k-1}{2\epsilon} \left\{ 1 - \frac{2}{9(k-1)} + \sqrt{\frac{2}{9(k-1)}} z_{1-\delta} \right\}^3 \tag{2}$$

$z_{1-\delta}$ is the upper $1-\delta$ quantile of the standard normal distribution. Then N relies on the inverse of the error bound ϵ and is in the first order linear to k. To estimate the number k of bins with support we used a fixed

Table 2: The KLD-SRL algorithm.

1. Prediction: Draw $x_t^i \sim p(x_t^i \mid x_{t-1}^i, u_{t-1})$.
2. calculate N according to (2).
   If $N > Nold$:
   draw $N - Nold$ samples from $p(y_t^i \mid x_t^i)$.
3. Update: Compute the importance factors $\omega_t^i = p(y_t \mid x_t^i)$.
4. normalize weights to sum up to 1.
5. Resample.

spatial grid, where a grid cell has support if at least one particle lies in that cell.

It is easy to integrate KLD-Sampling into the particle filter scheme. In the prediction phase we count the number of bins with support. If a sample falls into an empty bin, we increase k. With each processed sample N is updated according to equation 2. Sampling is stopped as soon as the number of samples reaches N. For a complete description of this method refer to (Fox, 2003).

4.2 KLD-Sampling with Sensor Resetting

KLD-Sampling still leaves one bottleneck. While global localization and when localization errors occur we have to use large numbers of particles. This comes from the fact that new samples are uniformly distributed over the state space. This is very inefficient in applications where it is possible to sample from the sensor model $p(y_t \mid x_t^i)$.

For this, we remember the number of samples we used in the last step. We then compute N according to equation (2) after the prediction phase. If N is larger than the number of samples we used in the last time step we add new samples to the sample set. The newly generated samples are not just uniformly distributed over the map but they are generated according to the most recent sensor reading.

Sampling from the sensor model can be a very difficult task depending on the sensors one wants to use. In this work we focused on laser range finders. For a defined number of positions on the map, we synthetically produce the joint probability distribution $p(y, x)$ that contains the information about the occurrence of a sensor reading y on a certain position x on the map. This distribution is represented by a piecewise constant distribution using a kd-tree.

In this way we place these particles in areas of high probability and ignore areas of low probability. This allows us to reduce the maximum size of the sample set to a few hundred particles.

5 CLUSTERED PARTICLE FILTERS

Usual MCL and KLD-Sampling often fail to correctly localize a robot in ambiguous situations. Both methods prematurely converge to possibly wrong positions on the map. As soon as the robot leaves the symmetric parts of the environment, KLD-Sampling quickly recovers from the localization failure.

Nevertheless, the pathplanner had possibly wrong data about the pose before recovery. This may have fatal consequences for the mobile unit. In the following we describe the proposed algorithms for localizing a mobile robot in symmetric environments.

5.1 Background

In this section we present the mathematical background of our approach to Monte Carlo Localization with clustering.

Using Sequential Importance Sampling (SIS), we take account of the mismatch between the proposal distribution and the target distribution (S. Thrun, 2001). In usual SIS the update equation for the importance factors can be shown to be (S. Arulampalam, 2002)

$$\omega_t^i = \frac{p(y_t \mid x_t^i) p(x_t^i \mid x_{t-1}^i) p(x_{0:t-1}^i \mid y_{0:t-1})}{q(x_t^i \mid x_{0:t-1}^i, y_{1:t}) q(x_{0:t-1}^i \mid y_{1:t-1})} \quad (3)$$

with the numerator being the target distribution and the denominator the proposal distribution. Seeing that $q(x_t^i \mid x_{0:t-1}^i, y_{1:t}) = q(x_t^i \mid x_{t-1}^i, y_t)$ we can calculate the weights according to

$$\omega_t^i = \eta \frac{p(y_t \mid x_t^i) p(x_t^i \mid x_{t-1}^i)}{q(x_t^i \mid x_{t-1}^i, y_t)} \quad (4)$$

η is a normalizing constant, which ensures that the weights sum up to 1. In that way, ω_t^i is only dependent on x_t^i, y_t, and y_{t-1}. For our clustering approach we have to take account of the existence of the clusters. Using a modified proposal distribution, we have a function $\eta(x_t^i)$ which calculates the normalizing factor for the sample x_t^i depending on the associated cluster in which x_t^i lies, in order to consider multiple hypotheses.

$$\omega_t^i = \eta(x_t^i) \frac{p(y_t \mid x_t^i) p(x_t^i \mid x_{t-1}^i)}{q(x_t^i \mid x_{t-1}^i, y_t)} \quad (5)$$

Consequently, we use the following equation to compute the weights of the samples.

$$\omega_t^i = \eta(x_t^i) p(y_t \mid x_t^i) \tag{6}$$

For C_t^j being the sum of weights of the jth cluster

$$\eta(x_t^i) = 1/C_t^j \tag{7}$$

This method allows us to use a procedure similar to Two-Stage Cluster Sampling as our resampling scheme. In Two-Stage Cluster Sampling one divides the whole sample set into clusters. In the first stage, a cluster is randomly drawn. Then, in the second stage, the actual sample is drawn (J. Hartung, 1989). Here we first draw a cluster and then apply systematic resampling on the individual particles in that cluster. Hence, we draw samples from all clusters and therefore are able to keep track of multiple hypotheses. Note, that we actually draw each of the clusters. If we only detect one cluster, the resampling procedure behaves exactly like usual (systematic) resampling. Furthermore, the association of a sample with a cluster is determined in every time step. Thus, a sample can be associated with different clusters in different time steps.

5.2 Clustering

In the following, we present an extension to Monte Carlo localization that allows for adequate localization in highly symmetric environments as frequently occurring in office like environments. Furthermore, it is shown how to incorporate this into the functionality of Sensor Resetting Localization and KLD-Sampling with Sensor Resetting.

In order to use two-stage sampling, we need to determine clusters in the sample set. Clustering is a well known technique to classify data and build groups of similar objects (W. Lioa, 2004). Here we use clustering to find groups of particles that occupy the same area in the state space. Doing this, we try to extract significant clusters that represent a possible position of our robot.

When choosing a clustering method, we mainly need to care about two things. The speed of this routine and the accuracy. Regarding the speed, grid-based algorithms are a good choice (W. Lioa, 2004). For us, a clustering algorithm is accurate, if it finds all significant particle clusters. That are those clusters with a high average weight. Clusters with low weight are not likely to represent the correct pose and can therefore be discarded. That implies, that the output of the clustering procedure only includes the significant clusters.

We used a fixed spatial grid for a first estimate of the clusters. Here a coarse grid of approximately $2\,m * 2\,m * 45\,degree$ is sufficient and since we have to do the clustering for just a few hundred samples it is fast as well. If we detect more than one cluster, it is checked if clusters can be fused. This is the case, if clusters are in close proximity of each other in the state space.

Thereby, the importance factors are computed according to equation (6). In that way, we do not loose possible positions while resampling. A cluster is considered significant if it's weight is above a manually chosen threshold. This method showed to be stable and accurate. Moreover, it allows us to simply distribute the free samples from clusters with low weight over the significant clusters. While resampling these samples are replaced with the high weighted particles of the according cluster. Alternatively, one can replace these samples by newly generated ones.

5.3 Clustered Sensor Resetting Localization

To incorporate two-stage sampling we need to regard the fact that SRL simply replaces particles in the sample set if needed. Thus, we cannot guarantee that all clusters will still exist after this replacement. In order to exclude this case, we disable the replacement of particles whenever we find more than one significant cluster. The algorithm is summarized in table 3.

The intuition behind CSRL is as follows. When the robot gets kidnapped while our filter is tracking multiple distinct hypotheses, we observe the following. The weights of the clusters become negligible and are therefore not considered significant any more. Thus, we do regard all samples as though they were in one cluster. We then can calculate a number of samples to be replaced according to Sensor Resetting Localization. The newly generated particles are distributed according to the most recent sensor reading. Then, for the new sample set, clustering is done and it is decided if we have to track multiple or a single position.

In this way, we are able to combine the ability to track multiple hypotheses stable with the advantage of using small sample sets.

5.4 Clustered KLD-Sampling with Sensor Resetting

Clustered Sensor Resetting Localization still suffers from using a sample set of constant size. Thus, we are not taking account of the different complexities of the localization problem. To overcome this issue, we developed a method to combine two-stage sampling with KLD-SRL.

Analogous to KLD-SRL the number of particles is adjusted according to equation (2). Only, if we detect more than one significant cluster, we do not allow the number of samples to be reduced. So we can guarantee that we won't loose any of the detected clusters. As soon as there is only one cluster left, the number

Table 3: The CSRL algorithm.

1. Prediction: Draw $x_t^i \sim p(x_t^i \mid x_{t-1}^i, u_{t-1})$.
2. Update: Compute the importance factors $\omega_t^i = p(y_t \mid x_t^i)$.
3. Normalize weights to sum up to 1.
4. Determine the number of significant clusters.
5. Resample using two-stage sampling.
6. If there is one cluster:
   Compute Ns according to equation (1).
   If $Ns > 0$: Replace Ns samples with samples drawn from $p(y_t^i \mid x_t^i)$.

Table 4: The Clustered KLD-SRL algorithm.

1. Prediction: Draw $x_t^i \sim p(x_t^i \mid x_{t-1}^i, u_{t-1})$.
2. calculate N according to (2).
3. If $N > Nold$:
   draw $N - Nold$ samples from $p(y_t^i \mid x_t^i)$.
4. If $N < Nold$ and number of clusters is one:
   Decrease number of samples to N.
5. Update: Compute the importance factors $\omega_t^i = p(y_t \mid x_t^i)$.
6. normalize weights to sum up to 1.
7. Resample using two-stage clustering.

of samples can be adjusted in the usual way. Note, that the number samples can still be increased while tracking multiple clusters and thus, allowing to find more clusters.

In ambiguous situations we usually have a number of significant clusters that we do not want to disappear prematurely. As soon as we detect distinct features in the environment, all but one cluster will have negligible weights and therefore be merged into the one cluster with high weight. Then, the number of samples can be decreased again.

Clustered KLD-SRL joins the advantages of using a small and variable sample set and the ability to track multiple positions over extended periods of time. Experiments show this algorithm to be accurate and robust. The algorithm is summarized in table 4.

6 EXPERIMENTAL RESULTS

In this section we present the results of our experiments. Due to the fact that it is very difficult to obtain quantitative results in real world experiments, we focus on simulations. Different maps have been used to test accuracy and tracking multiple hypotheses. Simulations were conducted on a 1.7 GHz Pentium 4 computer with 512 MB of memory.

Firstly, we ran simulations with KLD-SRL. Since this algorithm is not able to cope with symmetric environments properly, accuracy, robustness and speed are the measures of interest. In situations like the one illustrated in figures 1(a) and 1(b) KLD-SRL usually fails. The average localization failure was approximately $3\,cm$, with a mean update time of 0.03 seconds. Kidnapping experiments revealed the time for recovery from localization failure to be approximately 2 seconds in the average. Further experiments showed that the according values are in the same magnitude for CSRL and Clustered KLD-SRL.

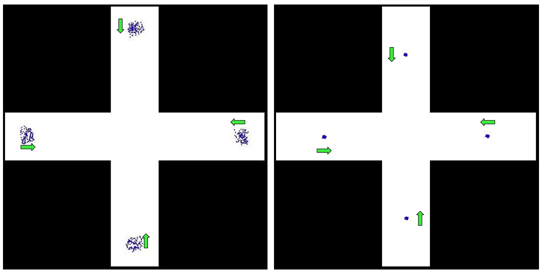
To test the ability of tracking multiple hypotheses stable over extended periods of time, we generated a completely symmetric map consisting of a cross shaped area of approximately $15\,m * 15\,m$ (figures 1 and 2). The green arrows indicate the estimated heading of the robot according the position estimate. This looks the same for Clustered Sensor Resetting Localization and Clustered KLD-Sampling with Sensor Resetting. Methods that do not facilitate techniques to handle symmetries converge to one single position and therefore this estimate is wrong in most cases.

Figure 1(a) shows the initialization of the filter, where all particles are distributed according to the most recent sensor reading. Thus, the samples cover four possible positions on the map. Since there is no way to figure out which of these poses is the right one, the filter keeps track of all of them (figures 1(a) to 2(b)). Even as the estimates cross in the middle, we do not loose any of them.

Furthermore, we conducted experiments with maps of office buildings, where we observe a lot of symmetries and asymmetries. Global localization was performed at random starting points and was successfully done in all cases. Above this, we frequently kidnapped the robot and let it relocalize itself. Most of the time we placed it on symmetric parts of the map and kidnapping was also done while the filter tracked multiple hypotheses.

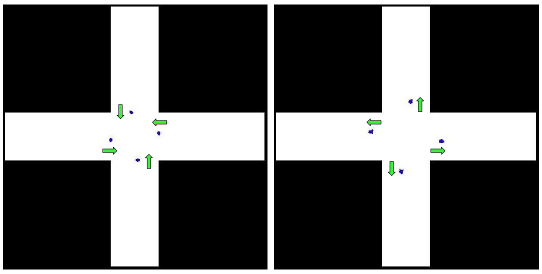
In these experiments, relocalization was usually performed within 2 seconds and as soon as the robot left the symmetric part of the environment, the filter converged to the correct position. Thus, both, CSRL and Clustered KLD-SRL, are able to localize a robot in highly symmetric environments and thereby showed to be very robust and accurate.

Regarding computational complexity, CSRL is in-



(a) (b)

Figure 1: CSRL: a) After initialization on a cross shaped symmetric map. The green arrows denote the actual heading of the robot, according to the four possible positions. b) The robot moves. Simulated data.



(a) (b)

Figure 2: CSRL: a) The four distinct position estimates come to close proximity in the middle of the cross. b) The robot moves on. Simulated data.

ferior, since it uses a sample set of constant size. The drawback of using variable sample set sizes is, that sometimes we need significantly more computation time. Thus, we always have to regard the worst case or design the robotic system in a way, that some processes can sleep for a while without affecting the usual operation too much.

7 CONCLUSIONS

The focus of this work is on handling highly symmetric environments with a particle filter algorithm. Nevertheless, computational complexity and robustness are still important issues, that need to be regarded. We introduced a resampling scheme that is easy to integrate into a particle filter.

Furthermore, we presented two algorithms that are able to localize a mobile unit correctly in symmetric environments at all times. CSRL uses small but constant sample sets. Therefore, CKLD-SRL is superior to this method, for its use of variable sample sets. Experimental results show the good quality of the position estimate and high robustness produced by these methods.

Further research is concerned with the viability of such methods for 3D/6D localization. Furthermore, we are interested in using such filters for multi target tracking (A. Kraeussling, 2005).

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