GLOBAL OPTIMIZATION OF PERFORMANCE OF A 2PRR PARALLEL MANIPULATOR FOR COOPERATIVE TASKS *

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Keywords: Optimization of performance, parallel manipulators, cooperative manipulators, trajectory planning.

Abstract: In this paper the trajectory planning problem is solved for a 2PRR parallel manipulator which works in cooperation with a 1 degree-of-freedom (*dof*) platform. The whole kinematic chain is considered as a redundant 3-dof manipulator, and an algorithm is presented to solve the redundancy by using the joint velocities in the null space of the jacobian matrix. The internal motion of the assisted manipulator allows globally optimize the condition number of the jacobian matrix during the accomplishment of a desired task. Consequently, the accuracy of the manipulator is maximized and singular or degenerate poses are avoided. A case of study is presented to show the effectiveness of our approach.

1 INTRODUCTION

A parallel manipulator is a linkage mechanism whose end-effector is connected to a fixed frame through several independent kinematic chains. For certain applications the parallel manipulators have particularly interesting features. In fact, because of the significant global rigidity of parallel manipulators, they become more efficient for machining applications than serial manipulators. Nevertheless, the workspace of a parallel manipulator is typically too small for such applications. This handicap, however, can be overcame by incorporating a secondary cooperative manipulator to aid the main one (the parallel) to achieve the task. The secondary manipulator should continuously relocate the work-piece to the main one in such a way that the kinematic performance of the chain is enhanced.

The strategy in which a positional device is incorporated to aid a main manipulator has been applied in previous works. In one paper (Hemmerle and Prinz, 1991) the main manipulator was redundant, and the secondary was non redundant. Both manipulators cooperate to achieve a welding task: the main one (*right hand*) moves the welding torch and the positioner (*left hand*) continuously relocates the work-pieces. The redundancy should be solved for the main manipulator whereas a suitable trajectory of the positioner must be established. The solution to both problems was based on two criteria: in the former, the joint variables are moved as far as possible from its limits; in the second criterion, the joint displacements are minimized. The approach proposed in the aforementioned reference was not extended for parallel manipulators. Works were not found dealing with the synthesis of trajectories of cooperative parallel manipulators.

The trajectory planning problem of a 2PRR parallel manipulator operating in cooperation with a 1 *dof* platform is studied in this paper. A formulation is proposed to solve the problem in such a way that a manipulator's kinetostatic index of performance is optimized during the achievement of a desired task.

In our formulation the whole kinematic chain is considered as being equivalent to a redundant three degree-of-freedom manipulator. Then, the joint

* *Research completed by the IRCCyN and the ITLag in the Franco-Mexican Laboratory of Applied Automatics (LAFMAA) and supported by the CNRS and the CONACYT.*

A. Moreno H., Alfonso Pámanes J., Wenger P. and Chablat D. (2006).
GLOBAL OPTIMIZATION OF PERFORMANCE OF A 2PRR PARALLEL MANIPULATOR FOR COOPERATIVE TASKS.
In *Proceedings of the Third International Conference on Informatics in Control, Automation and Robotics*, pages 516-522
DOI: 10.5220/0001213505160522
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velocities in the null space of the Jacobian matrix are used to globally optimize both the manipulability and the condition number of the manipulator.

2 THE 2-DOF ORTHOGLIDE MANIPULATOR

A kinematic scheme of the 2PRR manipulator and the cooperative platform is presented in Figure 1. As observed, the manipulator is composed of two actuated sliding blocks which are coupled to the fixed frame. On the other hand, the blocks are connected to the links AC and BC by the revolute joints A and B, respectively. Such links are finally connected by the revolute joint C. All the revolute joints are passives. The axes of the prismatic joints are orthogonal. This mechanism, termed *2-dof Orthoglide*, has been analyzed in previous works (Wenger et. al., 2001; Majou et. al., 2002).

We consider the lengths of bars AC and BC as being $L=1$ *m*. The values of the joint variables are constrained as follows: $-1 \le \rho_1 \le 1$ m, $0 \le \rho_2 \le 1.5$ m and $0 \le \rho_3 \le 1.5$ m. These intervals define the domain of admissible configurations in the joint space of the manipulator.

We assume that the tasks to be carried out by the 2-*dof* Orthoglide are in machining operations. So, in order to enlarge its workspace, we integrate a cooperative manipulator which should move the work-pieces. This secondary manipulator consists of a 1-*dof* table having translational motion on an axis whose direction is defined by θ_1 in Figure 1.

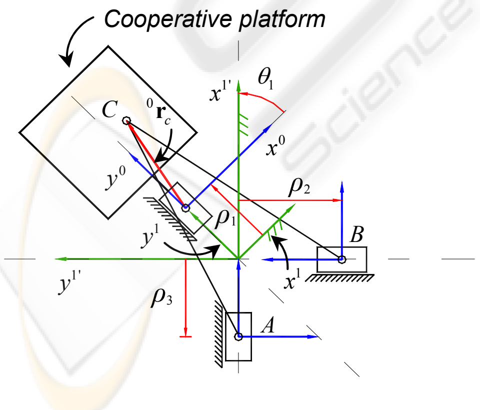


Figure 1: Kinematic scheme of the 2 dof Orthoglide manipulator.

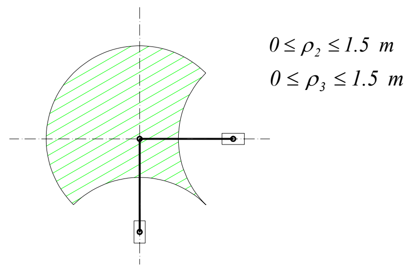
3 KINEMATICS AND SINGULARTIES OF THE MANIPULATOR

It can be observed in Figure 2 that the manipulator's workspace is increased by incorporating a cooperative platform. By doing this, however, the trajectory planning of the manipulator in coordination with the platform becomes a complex issue. Indeed, from the point of view of the relative motion manipulator-platform, the manipulator becomes kinematically redundant; i.e. that infinity of relative motions between the manipulator and the platform allow the achievement of the task. Thus, an optimal solution should be found in order to solve the redundancy. In this paper, the parallel manipulator with the cooperative platform will be termed *assisted manipulator*.

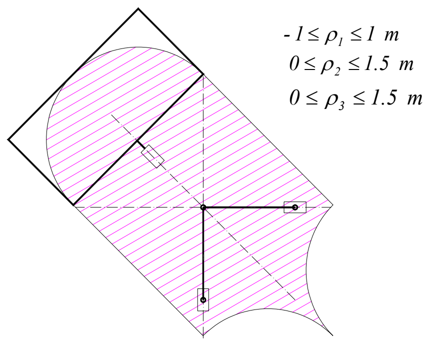
3.1 Kinematics of the Assisted Manipulator

The joint and operational velocities vectors, $\dot{\boldsymbol{\rho}}$ and $\mathbf{t}$, respectively, of a parallel manipulator are related as follows:

$$\mathbf{A}\mathbf{t} = \mathbf{B}\dot{\boldsymbol{\rho}} \tag{1}$$



(a) Without cooperative platform.



(b) With a cooperative platform.

Figure 2: Workspace of the 2 dof Orthoglide Manipulator.

In Equation 1, **A** and **B** are, respectively, the parallel and serial Jacobian matrices, (Gosselin and Angeles, 1990). The entries of these matrices depend on both the design variables and manipulator pose.

For the assisted manipulator, the joint velocity vector is given by $\dot{\boldsymbol{\rho}} = \begin{bmatrix} \dot{\rho}_1 & \dot{\rho}_2 & \dot{\rho}_3 \end{bmatrix}^T$ and the operational velocity vector is $\mathbf{t} = [\dot{r}_{cx}\ , \dot{r}_{cy}\ ]^T$. The scalars r_{cx} and r_{cy} are the orthogonal components of the position vector ${}^0\mathbf{r}_c$ (corresponding to point C) referred to a frame x^0-y^0 attached to the translational platform (Figure 1). For the assisted manipulator we have:

$$A = \begin{bmatrix} {}^0\hat{\mathrm{r}}_4^{\ T} \\ {}^0\hat{\mathrm{r}}_5^{\ T} \end{bmatrix} \tag{2}$$

$$B = \begin{bmatrix} \hat{\mathrm{r}}_1^{\ T}\ \hat{\mathrm{r}}_4 & \hat{\mathrm{r}}_2^{\ T}\ \hat{\mathrm{r}}_4 & 0 \\ \hat{\mathrm{r}}_1^{\ T}\ \hat{\mathrm{r}}_5 & 0 & \hat{\mathrm{r}}_3^{\ T}\ \hat{\mathrm{r}}_5 \end{bmatrix} \tag{3}$$

The symbol ^ on a vectorial term of these equations designates unit vector. In matrices (2) and (3) the unit vectors correspond to the position vectors associated with manipulator's links, as shown in Figure 3.

The velocity of point C (end-effector) of the manipulator relative to the table is obtained from Equation 1 as

$$\mathbf{t} = \mathbf{A}^{-1}\mathbf{B}\dot{\boldsymbol{\rho}} \tag{4}$$

On the other hand, since the assisted manipulator can be considered as a redundant one, the general solution of the inverse kinematic problem is written as follows:

$$\dot{\boldsymbol{\rho}} = \mathbf{J}^+\mathbf{t}\ + (\mathbf{I} - \mathbf{J}^+\mathbf{J})z \quad , \tag{5}$$

where:

$$\mathbf{J} = \mathbf{A}^{-1}\mathbf{B} \tag{6}$$

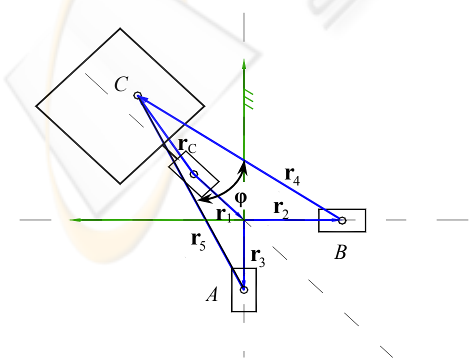


Figure 3: Vectors associated to the manipulator's links.

In Equation (5), $\mathbf{J}^+$ is the pseudo inverse matrix of $\mathbf{J}$, z is an arbitrary vector $\in \mathbf{R}^3$, and $\mathbf{I}$ is the unit matrix $\in \mathrm{R}^{3x3}$. The term $\mathbf{J}^+\mathbf{t}$ is the least norm solution of the inverse kinematic problem which provides the least norm vector $\dot{\boldsymbol{\rho}}$ that allows the end-effector to satisfy the specified velocity **t**. The expression $(\mathbf{I} - \mathbf{J}^+\mathbf{J})\ z$, is the homogenous solution, which represents the projection of z on the null space of **J**; it is a joint velocity vector which produces only internal motion of the manipulator in direction of z. This internal motion could be used to improve the kinetostatic performances as much as possible. To do that, the vector z may be defined as follows:

$$z = k\nabla H(\boldsymbol{\rho}), \tag{7}$$

where:

- $\nabla H(\boldsymbol{\rho})$ is the gradient of the kinetostatic index of performance to be optimized.
- k is a scale factor of the vector $\nabla H(\boldsymbol{\rho})$; it is defined positive if the index $H(\rho)$ must be maximized, or negative if this one will be minimized.

To improve the accuracy of the manipulator as much as possible the term $H(\rho)$ will be defined as the condition number of the Jacobian matrix **J**. In addition, the minimization of such index allows preserving the manipulator's configurations as far as possible from singularities of **A** and **B**. If such singularities arise then the velocity of the end-effector cannot be controlled. *Parallel* or *serial singularities* of the manipulator may occur if **A** or **B**, respectively, are not full-rank matrices. More details on the condition number are given in Section 4.

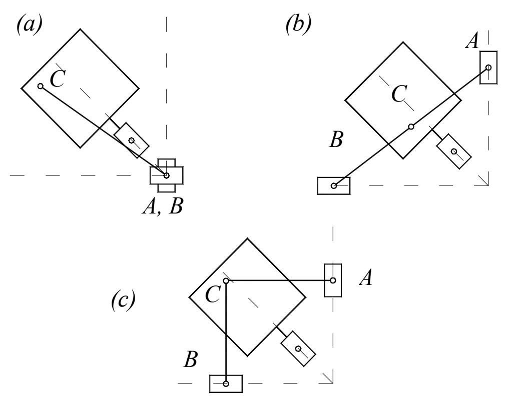


Figure 4: Parallel (*a, b*) and serial (*c*) singularities of the manipulator.

3.2 Singular Configurations

It can be shown that matrix **A** is singular when vectors $\mathbf{r}_4$ and $\mathbf{r}_5$ (i.e. links AC and BC) are parallels; in such case the manipulator is on a parallel singularity. On the other hand, in Equation 3 it can be observed that matrix **B** losses its full rank when $\mathbf{r}_2$ and $\mathbf{r}_4$, or $\mathbf{r}_3$ and $\mathbf{r}_5$, become orthogonal and the manipulator verifies a serial singularity. These conditions are satisfied by the singular configurations shown in Figure 4. In cases *(a)* and *(b)* of this Figure, which correspond to parallel singularities, we observe that not any component of force being perpendicular to the links AC and BC can be supported by the point C of the manipulator; consequently such point could move, even if the actuators are braked. On the other hand, in case *(c)*, when the manipulator is on a serial singularity, the point C cannot be displaced by the manipulator. Clearly, the aforementioned difficulties to control the manipulator, associated with both categories of singularities, are not desirables during the completion of a task and they should be avoided.

It can also be noted on the one hand that $\mathbf{r}_1$ has no influence on matrix **A**; and on the other hand, such a vector cannot produce a reduction of the rank of **B**.

4 KINETOSTATIC INDEX OF PERFORMANCE

The behavior of the Condition Number of the Jacobian matrix for the assisted manipulator is examined in this Section. This index has been largely applied in both parallel and serial manipulators (i.e. Angeles and Lopez-Cajun, 1992; Majou *et. al.*, 2002); it bounds the error propagation from the joint velocities to the operational velocities. Consequently, it should be minimized in order to preserve a suitable accuracy of velocity and applied force of the end effector.

For the assisted manipulator, we analyze the condition number of $\mathbf{J}=\mathbf{A}^{-1}\mathbf{B}$, which can be obtained from:

$$\kappa(\mathbf{J}) = \frac{\lambda_M}{\lambda_m} \quad , \tag{8}$$

where λ_M and λ_m are, respectively, the maximum and the minimum singular values of **J**. When $\kappa(\mathbf{J})$ becomes one, then the Jacobian matrix is qualified as isotropic, and the manipulator achieves an ideal configuration since its accuracy is the best. On the other hand, the condition number becomes infinity on singular configurations. The ideal value of $\kappa(\mathbf{J})$ can be obtained by the optimal configuration only if the design of the manipulator is isotropic (Angeles and Lopez-Cajun, 1992).

5 TRAJECTORY PLANNING

The trajectory planning problem as considered here is established as follows: *given a main task for the assisted manipulator specified by a desired trajectory of the end-effector referred to the mobile platform, to obtain the joint trajectories in such a way that the manipulator's configurations define values of the condition number of* **J** *as small as possible during the execution of the task.*

It is assumed that the task is specified in a discrete way by a sufficiently large sample of points of the desired trajectory. Thus, the process of solution can be carried out in the following steps for each path-point:

i) For the point p_i of the desired path, propose an arbitrary value of the first joint variable ρ_{1i} which is a member of the admissible domain of configurations.

ii) For the current point p_i of the path, solve the inverse problem of position to find the values of ρ_{2i} and ρ_{3i} corresponding to the proposed value of ρ_{1i}. Note that the pose obtained in this step is completely arbitrary. This one, however, will be successively improved as described in the next step.

iii) Find the optimum pose $\boldsymbol{\rho}_i$ which satisfies the current path point p_i and minimizes the condition number of the jacobian matrix. In order to do that, in an internal process of this step, successively optimal vectors of joint velocities $\boldsymbol{z'}_i$ in the null space of the jacobian matrix must be obtained which allows sequentially improve the manipulator's configuration (starting with that obtained in step *ii*) satisfying the current point p_i until obtaining of the optimal pose $\boldsymbol{\rho}_{i\ \mathrm{opt}}$. An optimal vector $\boldsymbol{z'}_i$ is such that the decreasing rate of $\kappa_i(\mathbf{J}_i)$ is as large as possible. For each optimal vector $\boldsymbol{z'}_i$ found in the internal process, an improved pose is computed by

$$\boldsymbol{\rho}'_i = \boldsymbol{\rho}_i + \dot{\boldsymbol{\rho}}_{hi}\,\Delta \boldsymbol{t} \ , \tag{9}$$

where $\Delta\, \boldsymbol{t}$ is an arbitrary small time interval, and $\dot{\boldsymbol{\rho}}_{hi}$ is computed by applying the projection of the optimal vector z'_i on the null space of $\mathbf{J}$ as follows:

$$\dot{\boldsymbol{\rho}}_{hi} = (\mathbf{I} - \mathbf{J}_i^{+}\mathbf{J}_i)\ \ z'_i \qquad (10)$$

Note that vectors obtained in the internal process are equivalent to the gradient of the condition number $\kappa(\mathbf{J})$.

When the optimal configuration has been determined for the actual point p_i, then the user changes to the next path-point until arriving at the last one.

6 CASES OF STUDY

We apply the proposed procedure to the trajectory planning of the assisted manipulator for a circular path to be followed by the end-effector for a period of 10 seconds. The objective of the trajectory planning consists in the global minimization of the condition number of the Jacobian matrix during the execution of the desired task. In order to evaluate the usefulness of our method, for the same task we study first the behavior of the condition number in two additional cases: (*a*) without motion of the cooperative platform; and (*b*) using only the least norm solution of the inverse kinematic problem.

The diameter of the circular path is 0.8 m and the coordinates of the center are x=0.1 m and y=0.55 m relative to the frame attached to the platform. The software for computations to solve the problem was developed in *Matlab*©. The algorithm proposed in Section 5 for optimization of the index of performance was applied only for the case (*c*).

Figures 5, 6 and 7 show the initial poses used by the assisted manipulator for each case. The path followed by the point C on both the mobile platform and the fixed frame when the manipulator carries out its task are also illustrated in these figures. The obtained joint trajectories are shown in figures 8, 9 and 10, and the behaviors of the condition number for the three cases are shown in figures 11, 12 and 13.

The expected benefits concerning the kinetostatic performance of the 2 dof Orthoglide with the cooperative platform were demonstrated. In the case of the fixed platform, it can be observed that poor values of $\kappa(\mathbf{J})$ were obtained from 0.5 to 4 sec. The manipulator is near to singularities when time is around of 2.2 sec. An improvement of the index was observed after 4 sec.

Interesting improvements of the manipulator's index of performance were observed by incorporating the cooperative table. Better results were obtained in the case of the global optimization.

7 CONCLUSION

The performance of a parallel 2 dof manipulator assisted by a mobile platform was studied in this paper. The mobile platform was incorporated in the workstation of the parallel manipulator in order to increase the manipulator's workspace and to aid in improving the performances of the manipulator. Then an approach was proposed for the global optimization of the performance of the assisted parallel manipulator. In fact, our approach allows the coordination of motion of both the parallel manipulator and the mobile platform in such a way that an efficient control was obtained on the condition number of the jacobian matrix. Thus, a high level of accuracy of the manipulator can be preserved during the achievement of the desired task. The proposed approach is an extension of a previous method applied to non-cooperative parallel robots (Alba et al., 2004).

The efficiency of our approach was shown by applying it to a case of study. The global optimization of the condition number for the assisted manipulator was obtained during the execution of a circular path: only isotropic poses were determined to achieve the task.

The level of performance obtained by using our method was compared to that attained in two supplementary cases of the manipulator: without mobile platform (Case *a*), and with platform by using only the least norm solution of the assisted manipulator (Case *b*). The cases *b* and *c* have shown the advantages that can be obtained by using a collaborative device with a parallel manipulator. A poor behavior of the condition number was obtained in Case (*a*). On the other hand, in Case (*b*) an adequate behavior of the index was established: values near to one were obtained for the complete path. Nevertheless the more effective control on the behavior on the index was obtained by applying our approach.

Even if the least norm solution gave satisfactory results for the considered path, some difficulties could be found in paths having other geometries and the risk of configuration degeneration could arise.

Our approach was developed for the 2 dof Orthoglide and a 1P cooperative platform; however

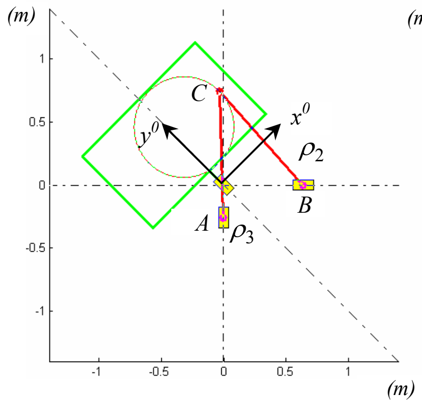


Figure 5: Initial configuration and path followed without cooperative platform.

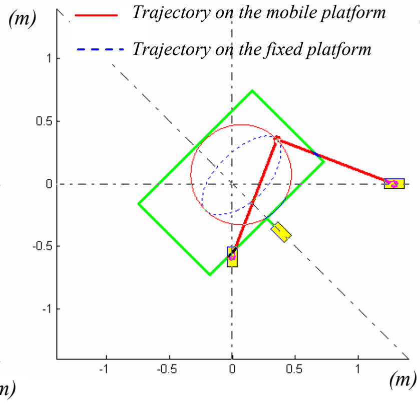


Figure 6: Initial (optimal) configuration and path followed by using the Least Norm Solution.

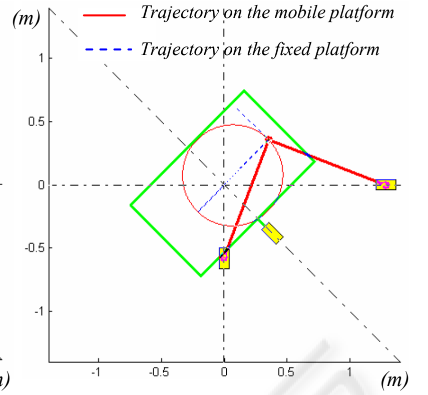


Figure 7: Initial (optimal) configuration, and path followed by applying Global Minimization of κ(J).

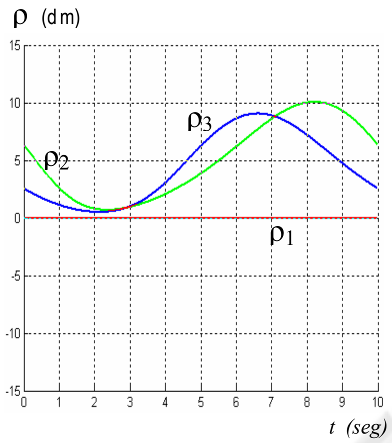


Figure 8: Joint trajectories without cooperative platform.

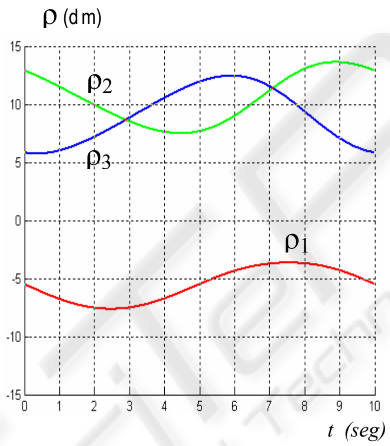


Figure 9: Joint trajectories by using Least Norm Solution.

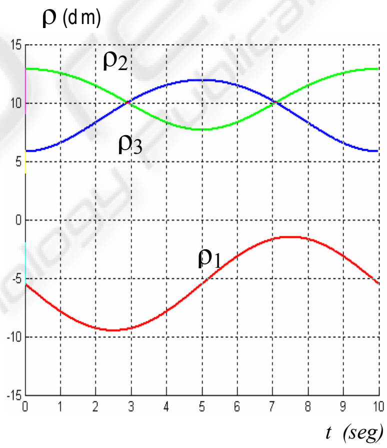


Figure 10: Joint trajectories by using Global Minimization of κ(J).

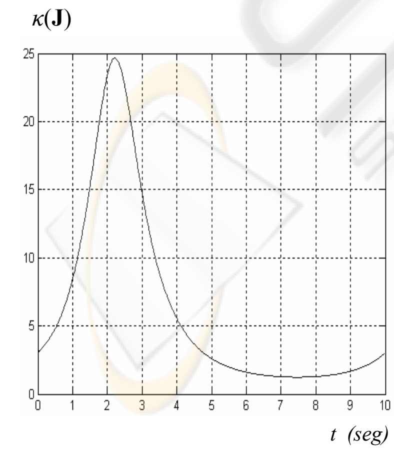


Figure 11: Behavior of κ(J) without cooperative platform.

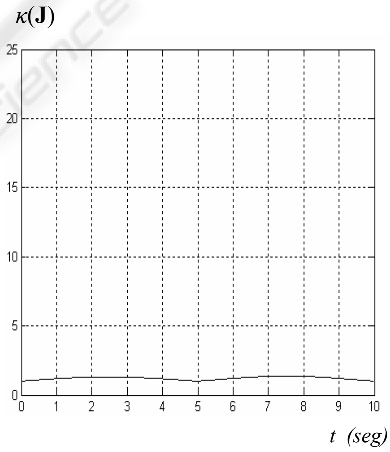


Figure 12: Behavior of κ(J) by using Least Norm Solution.

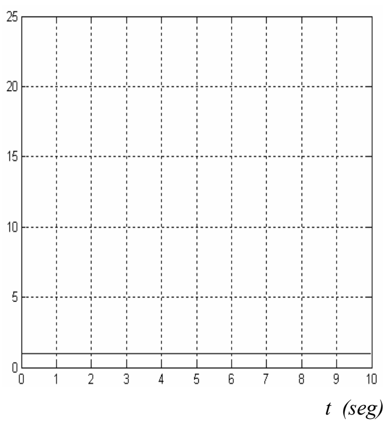


Figure 13: Behavior of κ(J) by using Global Minimization of κ(J).

it could be extended for assisted parallel manipulators having different architectures. Notice that, depending on the architecture, unit mismatch could arise in the Jacobian matrix. In such a case, this matrix must be homogenized by using a characteristic length (Daniali *et. al*., 1995, Alba-Gomez *et al*, 2005).

ACKNOWLEDGEMENTS

This work is a part of a project of LAFMAA, which is supported by CNRS and CONACYT.

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