PARTIAL STABILIZABILITY OF CASCADED SYSTEMS APPLICATIONS TO PARTIAL ATTITUDE CONTROL

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Keywords: Brockett's Condition, Partial stabilization, Backstepping, Partial Attitude Control.

Abstract: In this work, the problem of partial stabilization of nonlinear control cascade systems with integrators is considered. The latter systems present an anomaly, which is the non complete stabilization via continuous pure-state feedback, this is due to Brockett necessary condition. To cope with this difficulty we propose the partial stabilization. For a given motion of a dynamical system, say $x(t, x_0, t_0) = (y(t, y_0, t_0), z(t, z_0, t_0))$, the partial stabilization is the qualitative behavior of the y-component of the motion (i.e the asymptotic stabilization of the motion with respect to y) and the z-component converges, relative to the initial vector $x(t_0) = x_0 = (y_0, z_0)$. In the present work, we establish a new results for the adding integrators for partial stabilization, we show that if the control systems is partially stabilizable, then the augmented cascade system is partially stabilizable. Two applications are considered. The first one is devoted to partial attitude stabilization of rigid spacecraft. The second application is intended to the study of underactuated ship. Numerical simulations are given to illustrate our results.

1 INTRODUCTION

Control problems involving cascaded systems have attracted considerable attention in the past years. Unfortunately many controllable cascaded systems can not stabilizable by pure state feedback laws this is due to Brockett (Brockett, 1983) necessary condition. Several solutions to overcome the limitation imposed by Brockett condition have been presented in the literature knowing for example the time-varying method developed by Morin (Morin et al., 1994). The conception of time-varying feedback laws is an important method to solving the stabilization problem, nevertheless, the fact to introduce the time in these feedback laws product a oscillation of the system around his point of equilibrium see for instance Pettersen and Egeland (Pettersen and Egeland, 1996), (Morin et al., 1994), (Beji et al., 2004), Pettersen and Nijmeijer (Petersen and Nijmeijer, 2001).

In this paper, we propose the partial stabilization by smoothly state feedback laws. Partial stabilizability, is the asymptotic stability with respect to most of the system's state, and the rest converges to same position which depend to initial conditions.

The aim of the paper is to extend the well known backstepping theorem to the case of partial stabilizability of nonlinear control systems. We have shown that if the original system is partially stabilizable then the cascade systems with integrators inherits the same property, to this end we have developed the inversion Lyapunov theorem for the stability with respect to part. The theoretical result is applied to solving two problems: The first is the partial stabilization of the rigid spacecraft with two controls, where we have improve the Zuyev's (Zuyev, 2001) result that the velocity ω_3 of the third axes converges by using smooth state feedback laws. The second problem treated is the attitude of underactuated ship, we have construct two smooth feedback laws that stabilize asymptotically five components and the sixth converges.

A numerical simulations are given to valid our results.

The paper is structured as follows: The next section deals with some mathematical preliminaries. In particular, the inversion of the Lyapunov theorem of the stability with respect to part is demonstrated. The backstepping techniques and partial stabilizability is treated in section 3. In section 4 we give two applications for the backstepping result. Issues left for the future investigation are discussed in the conclusions.

Jammazi C. and Abichou A. (2006).
PARTIAL STABILIZABILITY OF CASCADED SYSTEMS APPLICATIONS TO PARTIAL ATTITUDE CONTROL.
In *Proceedings of the Third International Conference on Informatics in Control, Automation and Robotics*, pages 296-301
DOI: 10.5220/0001205802960301

2 PRELIMINARIES

In this section the concept of partial stability and partial stabilizability and some of its results will be reviewed in order to build the mathematical background for the stability proofs in the subsequent sections.
We consider the dynamical systems in finite dimension of the following form:

$$\begin{cases} \dot{x}_1 &= f_1(x_1, x_2) \\ \dot{x}_2 &= f_2(x_1, x_2). \end{cases} \tag{1}$$

here $f = (f_1, f_2)$ is supposed to be in class $C^\infty(R^n \times R^m)$, $x_1 \in R^p$, $x_2 \in R^{n-p}$ and p integer such that $0 < p \leq n$. We suppose that

$$f_1(0,\, x_2) = 0,\ \forall x_2 \in R^{n-p}\ \ f_2(0,\, 0) = 0 \tag{2}$$

Definition 1 (Partial Stability) The system (1) is said to be partially stable if the two following conditions a), b) are satisfied:
(a)

$$\begin{array}{ll} \forall & \epsilon > 0, \exists \eta > 0 \text{ s.t.} |x_1(0)| + |x_2(0)| < \eta \\ \Rightarrow & |x_1(t)| + |x_2(t)| < \epsilon,\ \forall t \geq 0. \end{array} \tag{3}$$

(b)

$$\begin{array}{ll} \exists & r > 0 : |x_1(0)| + |x_2(0)| \leq r : \\ \Rightarrow & \begin{cases} x_1(t) \to 0, t \to +\infty. \\ x_2(t) \to \alpha,\, t \to +\infty \end{cases} \end{array} \tag{4}$$

which α depends in $(x_1(0),\, x_2(0))$ only.

We consider the nonlinear control systems of the following form

$$\begin{cases} \dot{x}_1 &= f_1(x_1,\, x_2,\, u) \\ \dot{x}_2 &= f_2(x_1,\, x_2,\, u) \end{cases} \tag{5}$$

where $x = (x_1, x_2) \in R^n$ is the state, and $u(t) \in R^m$ is the control, $x_1 \in R^p, x_2 \in R^{n-p}, 0 < p \leq n$
Definition 2 (Partial Stabilizability) The system (5) is said to be partially stabilizable if there exists a continuous function $\phi : R^p \times R^{n-p} \longrightarrow R^m$, such that $\phi(0,\, x_2) = 0$ and the system in the closed-loop:

$$\begin{cases} \dot{x}_1 &= f_1(x_1,\, x_2,\, \phi(x_1, x_2)) \\ \dot{x}_2 &= f_2(x_1,\, x_2,\, \phi(x_1, x_2)) \end{cases} \tag{6}$$

is partially stable in the sense of definition 1.
Thanks to recent contribution of (Lin et al., 1995), in the dynamic given $\dot{x}_2 = f_2(x_1,\, x_2)$, we considered x_2 as a parameter, with the assumption (2) and with a result due to Lin (Lin et al., 1995), we announce the following theorem, which gives a converse Lyapunov theorem for the stabilization with respect to part of variables, this result extend the Kurzweil theorem (Rouche et al., 1977).
Theorem 1: We assume that the system (1) is partially stable with respect to x_1, then there exists a smooth function $V : R^p \times R^{n-p} \to R$ such that

(i) V is positive definite with respect to x_1

(ii) $\dot{V}(x_1,\, x_2)$ is definite negative with respect to x_1.

Proof: We suppose that the system (1) is partially stable, then by definition of partial stability, the system (1) is stable and by Persidski theorem (Rouche et al., 1977), there exist a positive definite function V_1 such that $\dot{V}_1 \leq 0$. By hypothesis we have $f_1(0,\, x_2) = 0$, then in the dynamic of $\dot{x}_1 = f_1(x_1,\, x_2)$ we can suppose that x_2 is a parameter, then by Lin (Lin et al., 1995) result, see also Rouche (Rouche et al., 1977) this system admits a smooth Lyapunov function V_2 with respect to a closed, invariant set $A = \{0\}$. Thus we have $V_2 : \mathbb{R}^p \times \mathbb{R}^{n-p} \to \mathbb{R}$ satisfying:
a) there exist two K_∞-functions α_1 and α_2 such that

$$\alpha_1(|x_1|_A) \leq V_2(x_1,\, x_2) \leq \alpha_2(|x_1|_A),$$

b) there exists a continues, positive definite function α_3 such that

$$\dot{V}_2(x_1,\, x_2) \leq -\alpha_3(|x_1|_A)$$

here $|x_1|_A = d(x_1,\, A) = d(x_1,\, 0) = |x_1|$.
We consider then the Lyapunov function defined by

$$V(x_1,\, x_2) = V_1(x_1,\, x_2) + V_2(x_1,\, x_2),$$

the candidate function V satisfies the propriety (i) and (ii).

3 PARTIAL STABILIZABILITY AND BACKSTEPPING

In this section, we give an extension of the well known backstepping techniques of Coron-Praly (Coron and Praly, 1991) to partial stabilizability theory.
Theorem 2: We suppose that:

$$\begin{cases} \dot{x}_1 &= f_1(x_1,\, x_2,\, u) \\ \dot{x}_2 &= f_2(x_1,\, x_2,\, u) \end{cases} \tag{7}$$

is partially stabilizable by static state feedback of C^r, $r \geq 1$. Then the augmented cascaded systems with integrators

$$\begin{cases} \dot{x}_1 &= f_1(x_1,\, x_2,\, y) \\ \dot{x}_2 &= f_2(x_1,\, x_2,\, y) \\ \dot{y} &= u \end{cases} \tag{8}$$

(i) is Lyapunov stable.

(ii) is asymptotic stabilizable with respect to $(x_1,\, y)$ by static preliminary feedback $u_0(x_1,\, x_2,\, y)$ of C^{r-1}

(iii) there exists a scalar function $\psi \in C^0(R^{n+m})$ satisfying:

$$\psi(x,\, y) > 0,\ x = (x_1,\, x_2) \tag{9}$$

such that with the state feedback control

$$u(x,\,y) = \left\{\begin{array}{l} u_0,\, if\,|y-\phi(x)|=0, \\ u_0-(y-\phi(x))\psi(x,\,y),\, if\,|y-\phi(x)|\neq 0 \end{array}\right. \quad (10)$$

the solution $x_2(t)$ converges to a constant vector $a(x(0),\,y(0))$.

Proof: Assume that the system (7) is partially stabilizable by a state feedback of C^r, then from definition 2 there exists a C^r map $\phi\,:\,R^p\times R^{n-p}\longrightarrow R^m$ $\phi(0,\,x_2)=0,\,\forall\,x_2\in R^{n-p}$ such that the system on closed-loop

$$\left\{\begin{array}{lcl} \dot{x}_1 & = & f_1(x_1,\,x_2,\,\phi(x_1,x_2)) \\ \dot{x}_2 & = & f_2(x_1,\,x_2,\,\phi(x_1,x_2)) \end{array}\right. \quad (11)$$

is partially stable.
Theorem 1 yields the existence of a smooth Lyapunov function V for the closed-loop system (7) such that $V(x_1,\,x_2)$ is positive definite and

$$\dot{V}(x_1,\,x_2)=\frac{\partial V}{\partial x_1}f_1(x)+\frac{\partial V}{\partial x_2}f_2(x)<0,\,\forall x_1\neq 0 \quad (12)$$

Let

$$W(x_1,\,x_2,\,y):=V(x_1,\,x_2)+\frac{1}{2}|y-\phi(x_1,\,x_2)|^2.$$

We derive W along a trajectory of system (8), we obtain with the preliminary feedback

$$\begin{array}{rcl} u_0(x,\,y) & = & \frac{\partial\phi}{\partial x_1}f_1(x,\,y)+\frac{\partial\phi}{\partial x_2}f_2(x,\,y) \\ & - & G_1^T(x,\,\phi(x))\frac{\partial V}{\partial x_1}-G_2^T(x,\,\phi(x))\frac{\partial V}{\partial x_2} \\ & + & \phi(x)-y. \end{array}$$

$\forall(x,y)\in R^n\times R^m$

$$\dot{W}(x,y)=\dot{V}(x)-|y-\phi(x)|^2 \quad (13)$$

we use (12) and (13) we obtain:

$$\dot{W}(x,y)=0\Leftrightarrow(x_1,y)=(0,\phi(0,x_2))=(0,0)$$

Then W is a candidate Lyapunov function, we conclude by Risito-Rumyantsev's theorem (Vorotnikov, 1998) that $(x_1,y)=(0,0)$ is asymptotically stable, then (i) and (ii) are shown.
Convergence of x_2:
Let the functional defined by

$$T(x,\,y,\,t)=W(x,\,y)+\int_0^t|f_2(x,\,y)(s)|\,ds \quad (14)$$

We have $T(x,\,y,\,t)\geq 0$. We drive T a long a trajectory of system (8) with the new feedback law u given by (10) we obtain:

$$\begin{array}{rcl} \dot{T}(x,\,y,\,t) & = & \dot{V}(x)-|y-\phi(x)|^2 \\ & - & |y-\phi(x)|^2\,\psi(x,\,y) \\ & + & |f_2(x,\,y)| \end{array} \quad (15)$$

to have $\dot{T}\leq 0$, it's sufficient to have

$$\begin{array}{l} \dot{V}(x)-|y-\phi(x)|^2+|f_2(x,\,y)| \\ \leq|y-\phi(x)|^2\,\psi(x,\,y) \end{array} \quad (16)$$

two cases are presented.
Case 1: $|y-\phi(x)|=0$
In this case all $\psi(x,\,y)>0$ is appropriate. We have

$$y=\phi(x)$$

the sub-manifold $\{\dot{W}=0\}$ is reduced to $\{(0,\,x_2,\,0)\}$ and the system (8) is asymptotically stabilizable with respect to $(x_1,\,y)$.
The component x_2 satisfies the ordinary differential equation

$$\dot{x}_2=f_2(x_1,\,x_2,\,\phi(x))$$

x_2 converges by hypothesis (because $(x_1,\,x_2)$ is solution of the system (7)).
Case 2: $|y-\phi(x)|\neq 0$.
Because $\psi(x,\,y)>0$, the inequality (16) becomes

$$\frac{\dot{V}(x)-|y-\phi(x)|^2+|f_2(x,\,y)|}{|y-\phi(x)|^2}\leq\psi(x,\,y) \quad (17)$$

with (17), we can choose ($e^x\geq x,\,\forall x\in R$)

$$\psi(x,\,y)=exp(\frac{\dot{V}(x)-|y-\phi(x)|^2+|f_2(x,\,y)|}{|y-\phi(x)|^2}) \quad (18)$$

since with (14), (16) and (18) we have $\dot{T}\leq 0$, then T is a positive decreasing function with respect to time t, we conclude that has a finite limit

$$\lim_{t\to+\infty}T(x,\,y,\,t)=T_\infty.$$

This implies that the integral

$$\int_0^{+\infty}|f_2(x_1,\,x_2,\,y)(s)|\,ds<+\infty$$

4 APPLICATIONS

4.1 Partial Stabilization of Rigid Spacecraft with Two Controls

The problem of attitude stabilization of a rotating rigid body with two controls has already been studied extensively in the literature.
A means importing to get round the obstruction of Brockett is to conceive instationnary feedback laws. Nevertheless, the fact to introduce the time in these laws can produce oscillations of the system around its point of equilibrium (see for instance, Morin et al (Morin et al., 1994)). To surmount these difficulties, we present a partial stabilizability method to solve the partial attitude stabilization with smooth controls with

respect to the state only.
In this work we will improve Zuyev's (Zuyev, 2001) result, and we prove that the velocity ω_3 converges.
Equation of motion
We consider the Euler-Poisson parameterization see Tsiotras (Tsiotras, 1996), or Zuyev (Zuyev, 2001) which describe the motion of the rigid-body, it is written in the following form:

$$\left\{ \begin{array}{lcl} \dot{\omega}_1 & = & u_1 \\ \dot{\omega}_2 & = & u_2 \\ \dot{\omega}_3 & = & \omega_1\omega_2 \\ \dot{\nu}_1 & = & \omega_3\nu_2 - \omega_2\,\nu_3 \\ \dot{\nu}_2 & = & \omega_1\,\nu_3 - \omega_3\,\nu_1 \\ \dot{\nu}_3 & = & \omega_2\,\nu_1 - \omega_1\,\nu_2. \end{array} \right. \qquad (19)$$

We will be interested to stabilize partially the equilibrium $\omega_1 = \omega_2 = \omega_3 = 0$, $\nu_1 = \nu_2 = 0$, $\nu_3 = 1$. We notice that $\dot{\nu}_1\,\nu_1 + \dot{\nu}_2\,\nu_2 + \dot{\nu}_3\,\nu_3 = 0$, then $\nu_1^2 + \nu_2^2 + \nu_3^2$=constant. Then we can suppose that:

$$\nu_1^2 + \nu_2^2 + \nu_3^2 = 1$$

We choose, on the hemisphere $\nu_3 > 0$, the equality $\nu_1^2 + \nu_2^2 + \nu_3^2 = 1$, which implies:

$$\nu_3 = \sqrt{1 - (\nu_1^2 + \nu_2^2)}.$$

To simplify our task we use the theorem 2. It's easy to show that the reduced system of (19) is locally equivalent to the system given by:

$$\left\{ \begin{array}{lcl} \dot{\omega}_3 & = & u_1u_2 \\ \dot{\nu}_1 & = & -u_2 - u_2\,g(\nu_1,\nu_2) + \omega_3\nu_2 \\ \dot{\nu}_2 & = & u_1 + u_1\,g(\nu_1,\nu_2) - \omega_3\,\nu_1 \\ \dot{\nu}_3 & = & u_2\,\nu_1 - u_1\,\nu_2 \end{array} \right. \qquad (20)$$

where g is smooth fonction satisfies $g(0,0) = g'(\nu_1,\nu_2)(0,0) = 0$.
Proposition 1: Let $\alpha > 0$, we choose the feedbacks u_1 and u_2 in this manner:

$$u_1 = -\alpha\,\nu_2 + \nu_2\,\omega_3,\ u_2 = \alpha\,\nu_1 - \nu_1\,\omega_3.$$

Then
i) The system (20) is stable with respect to $(\nu_1,\nu_2,\nu_3,\omega_3)$.
ii) The system (20) is exponentially stable with respect to (ν_1,ν_2).
iii) The angular velocity ω_3 converges.
iv) The point ν_3 converges to 1.
Proof: In closed loop the system (20) can be writhen in Lyapunov-Malkin form (Zenkov et al., 2002). We have:

$$\begin{pmatrix} \dot{\omega}_3 \\ \dot{\nu}_3 \end{pmatrix} = S(\nu_1,\,\nu_2,\,\nu_3,\,\omega_3)$$

$$\begin{pmatrix} \dot{\nu}_1 \\ \dot{\nu}_2 \end{pmatrix} = \begin{pmatrix} -\alpha & 0 \\ 0 & -\alpha \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} + R(\nu_1,\,\nu_2,\,\nu_3,\,\omega_3)$$

The matrix

$$\begin{pmatrix} -\alpha & 0 \\ 0 & -\alpha \end{pmatrix}$$

has $-\alpha < 0$ as eigenvalues. Besides the functions $R(\nu_1,\nu_2,\nu_3,\omega_3)$ and $S(\nu_1,\nu_2,\nu_3,\omega_3)$ have a nonlinear terms and vanishing together at $(0,0,0,\omega_3)$ and at $(0,0,0,0)$.
The Lyapunov-Malkin theorem and the center manifold theory allow us to conclude (i), (ii) and (iii).
By using the fact that $\nu_3 > 0$ and the relation $\nu_1^2 + \nu_2^2 + \nu_3^2 = 1$ to conclude $\lim_{t\to+\infty}\nu_3 = 1$.

In this proposition we give the feedback controller that achieve the partial stabilization of the system (19).
Proposition 2: The feedback controller that ensure the partial stabilisability of the system (19) are given by:

$$\begin{array}{l} \phi_1(x) = -k(\omega_1 - u_1(x)) \\ \phi_2(x) = -k(\omega_2 - u_2(x)) \end{array} \qquad (21)$$

$u_1(x)$ and $u_2(x)$ are given in the proposition 1; with k is large enough and $x = (\omega_i,\nu_i)$, $i = 1,2,3$.
Proof: We note that the system (20) its homogeneous of degree 0 with respect to dilation $\delta_\lambda(x) = (\lambda\nu_1,\ \lambda\nu_2,\ \lambda^2\nu_3)$, then we use the result due to Morin et al (Morin and Samson, 1996) to conclude the asymptotic stability of the system (19) with respect to $(\omega_1,\ \omega_2,\ \nu_1,\ \nu_2,\ \nu_3)$. By using the proposition 1 (ii), we conclude that there exists $k_1,\ k_2,\ C > 0$ such that

$$|u_1(x)| \le C\,e^{-k_2t},\ |u_2(x)| \le C\,e^{-k_1t} \qquad (22)$$

then it's easy to conclude that

$$\omega_1 \in L^2[0,+\infty),\ \omega_2 \in L^2[0,+\infty) \qquad (23)$$

Thus with $\dot{\omega}_3 = \omega_1\,\omega_2$ and the Cauchy-Schwarz inequality to conclude that

$$\dot{\omega}_3 \in L^1[0,+\infty)$$

which prove that ω_3 converges.

4.2 Partial Stabilization of the Ship

This subsection is devoted to the study the underactuated ship, it was shown by Pettersen and Egeland (Pettersen and Egeland, 1996) that no continues or discontinues static-state feedback law exist which make the origin of the ship system asymptotically stable. Our treatment enable us to overcome the difficulties imposed by the Brockett condition. The stabilization problem for the under-actuated ship in treated in the sense partial stabilization.
One of the most difficult operations of the captain of the ship, it is to put the boat on the quay. In this work we develop a smoothly feedback controls, that assure

the locally convergence of the ship on the quay.

Equation of Motion: The ship see Pettersen-Nijmeijer (Petersen and Nijmeijer, 2001) can be model by the simplified one

$$\left\{\begin{array}{lcl}\dot{x}_1 & = & u_1\\ \dot{x}_2 & = & -c\,x_1\,x_3 - x_2\\ \dot{x}_3 & = & u_2\\ \dot{\theta} & = & x_1 cos\psi - x_2 sin\psi\\ \dot{\phi} & = & x_1 sin\psi + x_2 cos\psi\\ \dot{\psi} & = & x_3\end{array}\right. \tag{24}$$

x_1, x_2, x_3 are the velocities in surge, sway and yaw respectively and θ, ϕ, ψ denote the position and orientation of the ship in the earth frame. u_1 and u_2 are the controls. The reel $c > 0$. The system (24) is presented in cascaded form, to study the partial stabilizability of (24), we applied the theorem 2. The reduced system of (24) is in the following form:

$$\left\{\begin{array}{lcl}\dot{x}_2 & = & -c\,u_1\,u_2 - x_2\\ \dot{\theta} & = & u_1 cos\psi - x_2 sin\psi\\ \dot{\phi} & = & u_1 sin\psi + x_2 cos\psi\\ \dot{\psi} & = & u_2\end{array}\right. \tag{25}$$

Theorem 3: With the feedback control given by

$$\begin{array}{lcl} v_1 & = & -\mu_1(x_1 - u_1(x))\\ v_2 & = & -\mu_2(x_3 - u_2(x))\end{array} \tag{26}$$

where $\mu_i > 0$ is large enough, $u_1(x)$ and $u_2(x)$ are given by

$$\begin{array}{ll} u_1 : & = -k_1\theta + x_2\psi\\ u_2 : & = -k_2\psi.\end{array} \tag{27}$$

where k_1, k_2 are large strictly positively. The system (24) is partially stabilizable in the sense that $(x_1, x_2, x_3, \theta, \psi) = (0, 0, 0, 0)$ is asymptotically stable and ϕ converges.

5 NUMERICAL SIMULATIONS

5.1 Simulations of Rigid Spacecraft

In this subsection we present a numerical simulations to valid our results with the feedback controls $\phi_1(x) = -10(\omega_1 - u_1(x))$, $\phi_2(x) = -10(\omega_2 - u_2(x))$, $x = (\omega_i, \nu_i)$ where $u_1 = 10\nu_2 + \nu_2\omega_3$, $u_2 = 10\nu_1 - \nu_1\omega_3$ The results are shown in Fig. 1-3. These simulations show that the proposed controls laws partially asymptotically stabilizable the system given by equations (19).

5.2 Simulations of Underactuated Ship

In this subsection, we take the feedback controls $v_1 = -\mu_1(x_1 - u_1(x))$, $v_2 = -\mu_2(x_3 - u_2(x))$ with $\mu_1 = \mu_2 = 10$ and $u_1 = -5\theta + x_2\psi$, $u_2 = -5\psi$

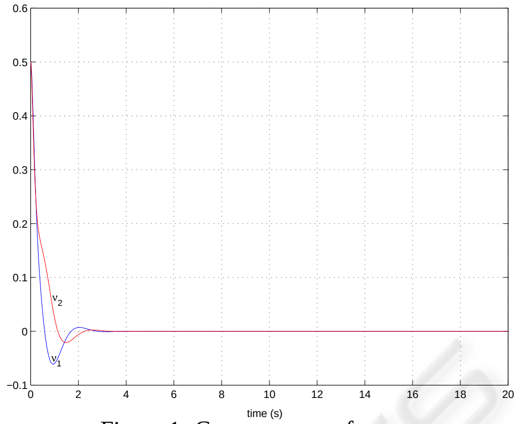

Figure 1: Comportment of ν_1, ν_2.

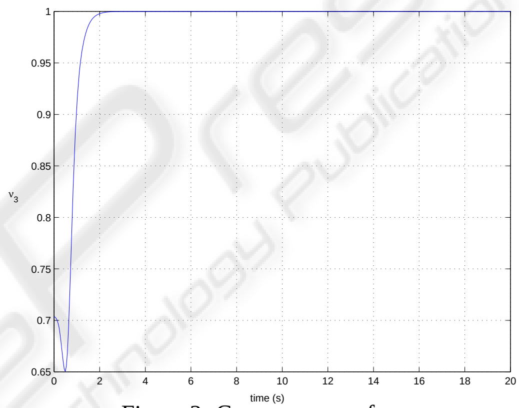

Figure 2: Comportment of ν_3.

6 CONCLUSION

The problem of partial stabilization by means of smoothly time-invariant feedback laws has been considered in the paper. Our treatment enables us to overcome the difficulties imposed by Brockett's condition. The main result shown that the backstepping techniques can be extended to partial asymptotic stability for nonlinear control systems, and that this theorem can be used for solving the partial stabilization of many control systems. The first problem treated in this paper is the attitude control of rigid spacecraft, in this sense we have improve the Zuyev's result and we have shown that the velocity ω_3 of the 3^{th} axes converges.

The second problem treated is the partial stabilization of under-actuated ship, by using the backstepping techniques we synthesized a smooth feedback controls to make the axes ϕ of the ship in the earth-fixed frame converges. This theoretical is desirable in many practical situation, indeed, the feedback control developed here make easy (for the captain) to put the ship on the quay.

The future work is to extend the backstepping tech-

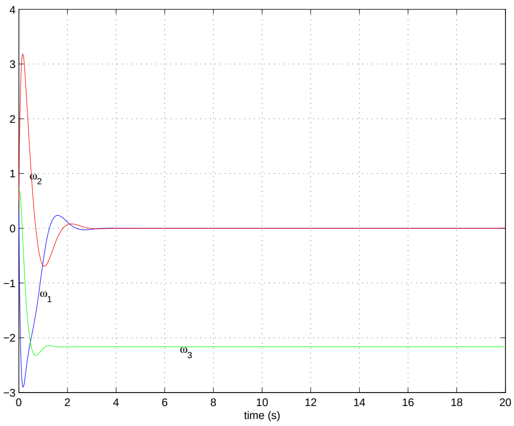

Figure 3: Comportment of the angular velocity of ω_1, ω_2, ω_3.

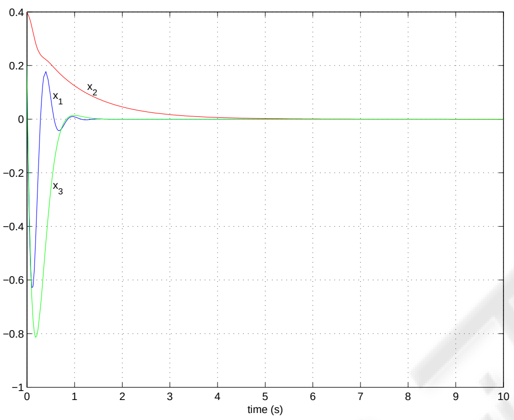

Figure 4: Comportment of the velocity x_1, x_2, x_3.

niques for the partial stabilizability by bounded feedback laws, and to applied it to construct a bounded feedback laws to assure the partial stabilization of the satellite (respectively of the under-actuated ship).

ACKNOWLEDGEMENTS

The first author would like to thank Professor Jean-Michel Coron for several interesting discussions and helpful comments.

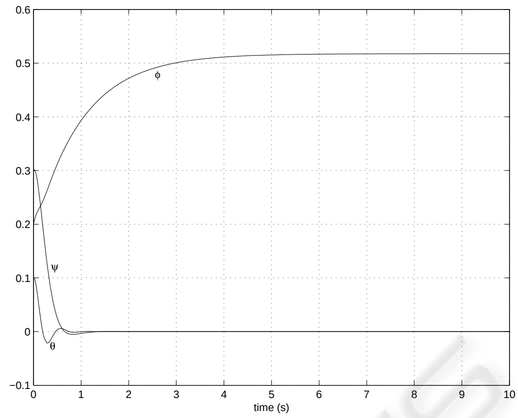

Figure 5: Positions of the axes θ, ϕ, ψ.

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