A NEW METHOD FOR REJECTION OF UNCERTAINTIES IN THE TRACKING PROBLEM FOR ROBOT MANIPULATORS

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Abstract: This paper presents a new strategy for robust tracking in robot manipulators. The aim of the strategy is to reject parametric uncertainties due to model or load disturbances. The basic controller acting on the manipulator is a robust controller designed by Lyapunov's direct method. Acting on this controller there is an adaptive system responsible for the adaptation of the basic parameter of the robust feedforward term. The performance of the strategy is tested in a Puma-560 manipulator. A comparison with existing techniques is done to verify the efficiency of the presented controller.

1 INTRODUCTION

There are circumstances in which the performance of conventional controller of robot manipulator decreases. For instance when the dynamics of the robot are not precisely known or disturbances are affecting the system, the controller could perform poorly. In many of the control schemes the dynamic model is explicitly used to compute the control action. These techniques are based on a perfect knowledge of the robot model and its dynamic parameters. A perfect cancellation of the nonlinear dynamics is achieved if those two premises are satisfied, and linear controllers can then be used with satisfactory performance.

There are other techniques that do not use this exact feedback linearization approach but a local linearisation around the desired trajectory (Torres et al, 2002) or the property of linear parameterizability of n-link rigid robots to obtain a linear model of the system (Spong, 1992).

In this paper the imperfect cancellation of the nonlinear dynamics due to uncertainties is afforded. Lot of works related with adaptive control schemes (Ortega and Spong, 1989; Slotine and Li, 1987), robust control schemes (Slotine, 1985; Spong and Widyasagar, 1987; Dawson et al, 1992) and even hybrid control schemes (Su and Stepanenko, 1997) have been proposed to deal with these uncertainties. Most of robust controllers are based on the Lyapunov's direct method (LDM). These schemes add a robust term to the control input that tries to compensate the discrepancies between the estimated model and the real model of the system. This robust action presents a good performance in several circumstances, but it has to be revised at least in two cases. First, when the robot works with different payload masses, and second, when the controller is used with a robot manipulator having different dynamic parameters than the estimated model. Due to this, the robust action has to vary adequately. The present work tries to add an adaptive scheme in order to tune automatically the robust design parameter involved in this action.

2 CONTROLLER DESIGN

The control problem considered is the tracking problem of robot manipulators with uncertainties in the model. The controller has three parts (Spong, 1992; Sciavicco and Siciliano, 1996): a feedback linearisation inner loop, a stabilizing PD control law and a robust action.

The dynamics of this system are represented by:

$$u(t) = D(\theta(t))\ddot{\theta}(t) + h(\theta(t), \dot{\theta}(t)) + c(\theta(t)) \qquad (1)$$

being $D(\theta(t))$ the inertia matrix, $h(\theta(t), \dot{\theta}(t))$ the Coriolis and centrifugal force vector, $c(\theta(t))$ the gravitational force vector and $u(t)$ the applied torque to each link

The linearization is achieved considering the following input to the nonlinear model:

$$\tau_k = D(\theta_k)y_k + h(\theta_k, \dot{\theta}_k) + c(\theta_k) \qquad (2)$$

where y_k is the new input to the linear resultant model. The sub-index k indicates the instant of time, while θ_k and $\dot{\theta}_k$ refers to the measured position and velocity at the instant k. This leads to the following linear and decoupled second-order model:

$$y_k = \ddot{\theta}_k \qquad (3)$$

The following equation ensures an asymptotically stable second-order system (the time dependence is avoided in the notation for simplicity):

$$y_k = -K_p\theta_k - K_D\dot{\theta}_k + r_k \qquad (4)$$

where the components r_{ik} of the vector r_k are the reference for each joint. This can be seen taking into account equations (3) and (4), which leads to the second-order system:

$$r_k = \ddot{\theta}_k + K_D\dot{\theta}_k + K_p\theta_k \qquad (5)$$

which is asymptotically stable if K_p and K_D are positive definite matrices. Moreover, choosing a diagonal form for them, the system results decoupled. Once any desired trajectory $\theta_d(t)$ is given, the tracking problem for this trajectory is solved by choosing:

$$r_k = \ddot{\theta}_{k,d} + K_D\dot{\theta}_{k,d} + K_p\theta_{k,d} \qquad (6)$$

This is easy to view substituting (6) into (5), which leads to:

$$\ddot{\tilde{\theta}}_{k,d} + K_D\dot{\tilde{\theta}}_{k,d} + K_p\tilde{\theta}_{k,d} = 0 \qquad (7)$$

where $\tilde{\theta}_k = \theta_{k,d} - \theta_k$ (and similarly for its time derivatives). This equation gives the expression for the dynamics of the position errors. Finally,

following (4) and (6), the stabilizing control law is defined by:

$$y_k = \ddot{\theta}_{k,d} + K_D\dot{\tilde{\theta}}_k + K_p\tilde{\theta}_k \qquad (8)$$

The third part of the controller is the robust action added to correct the imperfect compensation of the nonlinear term in (1), given by the inverse dynamics control (2). In the assumption that only an estimation of the real matrices $D(\theta)$, $h(\theta, \dot{\theta})$ and $c(\theta)$ can be obtained, the equation (7) results:

$$\ddot{\tilde{\theta}}_d + K_d\dot{\tilde{\theta}}_d + K_p\tilde{\theta}_d = \eta \qquad (9)$$

where η gives the discrepancies between the real and the estimated values for the matrices (Sciavicco and Siciliano, 1996). In view of this, for this nonlinear coupled system, tracking with zero error is not ensured and PD control action is not sufficient. Following the well-known LDM, an outer feedback loop on the error can be designed in order to be robust to the uncertainty η:

$$y_{k,r} = \frac{\rho}{\left\| D^t Q \xi_k \right\|} D^t Q \xi_k \qquad (10)$$

where

$$D_{2n \times n} = \begin{bmatrix} 0_{n \times n} \\ I_{n \times n} \end{bmatrix}, \quad \xi_{2n \times 1} = \begin{bmatrix} \tilde{\theta}_{n \times 1} \\ \dot{\tilde{\theta}}_{n \times 1} \end{bmatrix}.$$

$Q_{2n \times 2n}$ is a positive definite matrix and ρ is a design parameter. The full control law is given then by:

$$y_k = \ddot{\theta}_{k,d} + K_D\dot{\tilde{\theta}}_{k,d} + K_p\tilde{\theta}_{k,d} + y_{k,r} \qquad (11)$$

To avoid the problems in (10) when the error approximates zero, the following expression is used:

$$y_{k,r} = \begin{cases} \dfrac{\rho}{\left\| D^t Q \xi_k \right\|} D^t Q \xi_k & , \; if \; \left\| D^t Q \xi_k \right\| \geq \varepsilon \\ \dfrac{\rho}{\varepsilon} D^t Q \xi_k & , \; if \; \left\| D^t Q \xi_k \right\| < \varepsilon \end{cases} \qquad (12)$$

3 IMPROVING PERFORMANCE OF EXISTING TECHNIQUES

The value of the design parameter ρ is important in order to have a good performance of the closed-loop

system. Several proposals have exist in the literature (Spong, 1992; Corless and Leitmann, 1981; Liu and Goldenberg 1993; Jaritz and Spong, 1996).

In this paper a new method to adjust this critical parameter is presented. An adaptive law based on a gradient descent method is used for the adaptation of the design parameter ρ:

$$\rho_k = \rho_{k-1} - \gamma \frac{\partial J_k}{\partial \rho_{k-1}} \qquad (14)$$

where γ is the learning rate of the adaptation. In this case, the cost function is formed by two terms. The first of them loads the error in the state of the robot. The second term loads the resultant input to the load system. The resultant cost function is given by:

$$J_k(\rho_{k-1}) = \frac{1}{2} \xi_k^{T} Q_{ad} \xi_k + \frac{1}{2} y_{k-1}^{T} R_{ad} y_{k-1} \quad (15)$$

where the 2n x 2n matrix Q_{ad} weighs the state error and the n x n matrix R_{ad} weighs the influence of the inputs to the linearised system. This choice gives the following adaptation law:

$$\rho_k = \rho_{k-1} - \gamma \left[\xi_k^{T} Q_{ad} \frac{\partial \xi_k}{\partial \rho_{k-1}} + y_{k-1}^{T} R_{ad} \frac{\partial y_{k-1}}{\partial \rho_{k-1}} \right] \quad (16)$$

To compute the derivatives in (16), a first order approximation has been applied. The error ξ_k can be approximated by:

$$\xi_k = \begin{bmatrix} \widetilde{\theta}_k \\ \dot{\widetilde{\theta}}_k \end{bmatrix} = \begin{bmatrix} \theta_{d,k} \\ \dot{\theta}_{d,k} \end{bmatrix} - \begin{bmatrix} \theta_k \\ \dfrac{\theta_k - \theta_{k-1}}{h} \end{bmatrix} \qquad (17)$$

Its derivate with respect to ρ_{k-1} is:

$$\frac{\partial \xi_k}{\partial \rho_{k-1}} = -\begin{bmatrix} \dfrac{\partial \theta_k}{\partial \rho_{k-1}} \\ \dfrac{1}{h} \dfrac{\partial \theta_k}{\partial \rho_{k-1}} - \dfrac{1}{h} \dfrac{\partial \theta_{k-1}}{\partial \rho_{k-1}} \end{bmatrix} = -\begin{bmatrix} \dfrac{\partial \theta_k}{\partial \rho_{k-1}} \\ \dfrac{1}{h} \dfrac{\partial \theta_k}{\partial \rho_{k-1}} \end{bmatrix} \quad (18)$$

In order to compute this derivative, the linearised model of the system in state-space form is used:

$$\begin{aligned} x_k &= A_D x_{k-1} + B_D y_{k-1} \\ \theta_k &= C x_k \end{aligned} \qquad (19)$$

Then, the derivative of (18) is:

$$\frac{\partial \theta_k}{\partial \rho_{k-1}} = CB_D \frac{\partial y_{k-1}}{\partial \rho_{k-1}} \qquad (20)$$

If it assumed that $\left\| D^t Q \xi_k \right\| < \varepsilon$, $\forall \varepsilon$, which is true except perhaps at the beginning of the motion, expression (11) can be approximated by:

$$y_k = \ddot{\theta}_{k,d} + K_D \dot{\widetilde{\theta}}_{k,d} + K_p \widetilde{\theta}_{k,d} + \rho_k M \xi_k \qquad (21)$$

where $M = D^T Q / \varepsilon$. Expression (21) leads to the computation of the derivative in the right part of (20) as follows:

$$\frac{\partial y_{k-1}}{\partial \rho_{k-1}} = M \xi_{k-1} \qquad (22)$$

Using (22) and (20), expression (18) is written as:

$$\frac{\partial \xi_k}{\partial \rho_{k-1}} = -\begin{bmatrix} CB_D M \xi_{k-1} \\ \dfrac{1}{h} CB_D M \xi_{k-1} \end{bmatrix} \qquad (23)$$

Finally, expressions (22) and (23) can be used to evaluate the adaptation law (16).

4 RESULTS

The algorithm proposed was tested on a PUMA 560 manipulator of Unimation. The model used includes uncertainties with respect to the real model. In Figure 1, the results of a trajectory-following experiment is shown. As can be observed, the performance of the RAC strategy is considerably better than the other two. Robust controller with the Spong strategy tends to reduce the tracking error, but the new proposed strategy improves the performance of the Spong controller. In both cases the uncertainty bound parameter is bounded along the whole trajectory. However, higher values are achieved with the RAC scheme. Actually, this is the reason why the performance is better with the proposed controller.
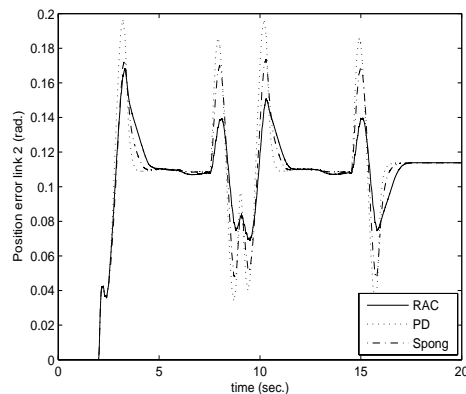
Figure 1: Tracking error comparison of the different strategies for a tracking experiment (link 2).

It is important to take into account that to setup the Spong's controller it is necessary to previously simulate the system in order to tune the parameters υ_0 and υ_1 in equation 13. However, with the RAC scheme better results are obtained and it is not necessary any previous simulation to setup the controller.

5 CONCLUSIONS

In this work an efficient self-adaptive robust controller applied in a PUMA 560 manipulator arm was presented. It is studied the case in which model uncertainties are present. The standard robust control strategy for robot manipulators is based on a robust controller with fixed design parameter or an adaptation based on the behaviour of the model in the defined reference trajectory. These schemes are inefficient: first of them requires quite trial and error proofs before reaching the appropriate value for the design parameters, and it is valid only for the current trajectory. Second of them requires an evaluation of the dynamics terms over the reference trajectory in order to get some bounds parameters to form the adaptation law. The new self-adaptive strategy designed improves the performance of the standard controllers. It was shown that the robust design parameter is very important in the closed-loop behaviour of the controller. The new strategy adds a self-tuning scheme in order to vary adequately its value. The results obtained with this new scheme show a better behavior than the standard scheme

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