

# USING FUZZY LOGIC FOR PRICING

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Abstract: This paper deals with traditional pricing models under uncertainties. A fuzzy model is applied to the classical economical approach in order to calculate the possibilities of economical indices such as profits and losses. A realistic case study is included to illustrate a typical application of the fuzzy model to the pricing issue.

## 1 INTRODUCTION

Most of current challenges in electrical system management issues are concerned to the new world's environment i.e. competition and deregulation. The performance of a company should be measured not only by its product quality but also by the efficiency of its business in order to achieve good contracts with low risks and high profits.

One of the major fundamental tasks related to the new competitive reality is pricing a contract which can be a tough challenge.

The objective of this paper is to describe a new computational tool customized for the risk assessment. The mathematical model is based on the application of fuzzy sets to the classical economic theory and the overall solution scheme aims to provide an effective and reliable help to the Decision Maker on the new challenges of a competitive environment.

## 2 CLASSICAL ECONOMICS

In a very simplified way, the classical economic theory (Mas-Colell et al., 1995; Sher et al. 1986; Varian 1992) establishes a product price based on two main functions illustrated in Figure 1: the production cost and the consumer utility. It is important to note that every cost is associated to a desired (or sometimes regulated) quality (reliability, security) level. Therefore, the presented function must be regarded as the minimum total cost necessary to supply the load under corresponding quality constraints.

Theoretically, in ideal conditions such as perfect market, competition, etc., the equilibrium between offer and demand is achieved when the price equals production costs – the break-even point corresponds to demand  $D^*$  charged at price  $P^*$ . However, it should be noted that the future demand will not necessarily equal to the optimal  $D^*$ . A good load management scheme would therefore bring the load to the “profit” region; any commitment to supply load at the “losses” region would require

compensation in order to maintain a company economically healthy.

It is interesting to observe that the utility would achieve profits whenever the demand is lower than  $D^*$  (on the left side of the break-even point, where the user accepts a price higher than production costs), and losses if the demand is higher than  $D^*$ . A good load management scheme would therefore bring the load to the "profit" region; any commitment to supply load at the "losses" region would require compensation.

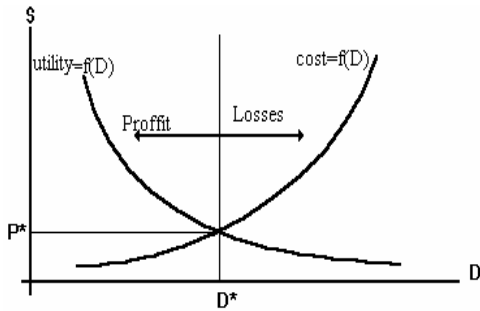


Figure 1: Production cost and Utility functions

This paper considers a model that extends the classical economic theory to accommodate uncertainties. It uses a specialized optimal expansion/ operation model to evaluate a family of possible minimum cost functions associated to each possible future scenario. One example of such functions is illustrated in Figure 2, defining the *possibility region of production costs*. As may be seen, each point of the region corresponds to the optimal operation (or, if desired, operation and expansion) cost necessary to supply a given load. The overall optimization algorithm is fully described in (Camac, 1998). Fast minimum cost flow and parametric programming algorithms were specially designed and developed in order to make it possible to construct the region within a reasonable computational effort.

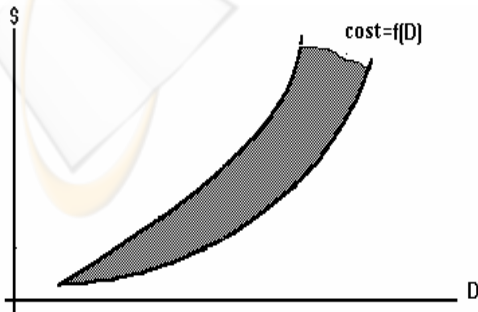


Figure 2: Possibility Region of Production Costs

The same reasoning may be used to construct the family of utility functions and therefore the *possibility region of consumer's utilities* illustrated in Figure 3.

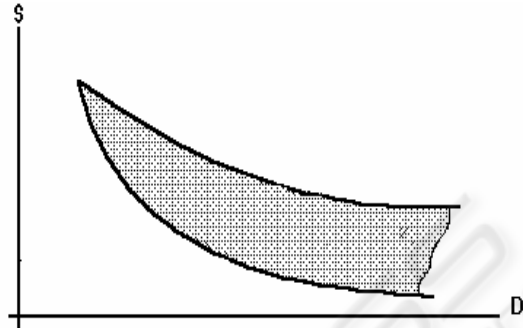


Figure 3: Possibility Region of Consumer Utilities

The resulting region is illustrated in Figure 4.

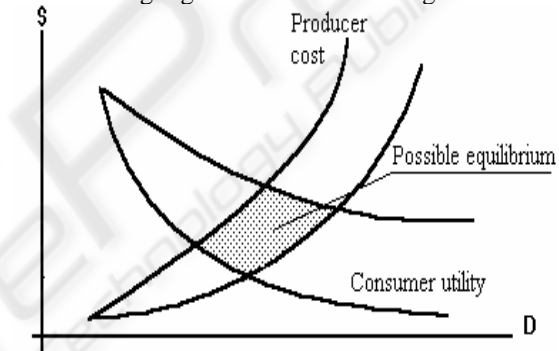


Figure 4: Possibility Region of Equilibrium

It may be seen that the equilibrium point will lie within a region delimited by the possible production/ consumption scenarios.

### 3 FUZZY MODELING

This paper uses the fuzzy set theory as a basis to model the possibility regions of costs and utilities and construct a risk assessment framework. Each scenario of production cost and consumer utilities is assigned a corresponding *possibility*  $\mu_c$  or  $\mu_u$ . (membership functions in fuzzy set theory). Figure 5 illustrates the possibility cost function for demand  $D^*$ .

It may be shown that the possibility of a future scenario  $s$  where cost  $c^*$  and utility  $d^*$  jointly occur is given by

$$\mu_s = \mu_{c^*} + \mu_{d^*} \tag{1}$$

Using the same reasoning one could accommodate uncertainties in load values, growth, fuel costs, etc. The total scenario possibilities will be the union of each individual possibility.

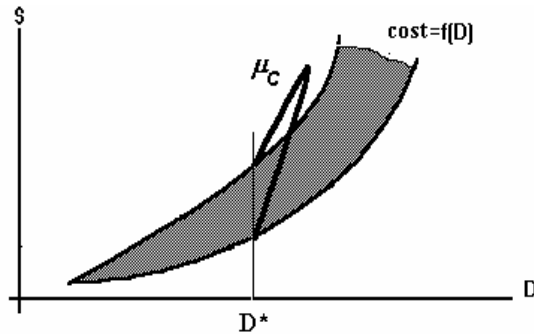


Figure 5: Possibility Function for a Scenarios

Energy pricing is a multi-disciplinary task and involves efforts of many different people in a company. A realistic price structure is generally based on complex philosophies, objectives and goals, and cannot be briefly described, as commercial interests and contractual constraints prohibit a comprehensive and detailed report. Nevertheless, an intuitive reasoning would state that the price must cover costs and the consumer must be able to pay the price.

The quality of a business (for instance, a sales contract) will be measured by indices, such as incomes, profits, etc. A realistic quality index should reflect the company’s philosophies and goals, and may combine more than one component (for instance, incomes, profits and risks) in order to suitably represent the company’s aims and objectives.

In general, it is better to loose a business than to do a bad business. Our first analysis will focus on the first need of a company to recovering expenses. Suppose, for instance, that the decision maker must price a supply of  $D^*$ , whose associated costs are presented in Figure 6, ranging from lower and upper bounds  $\underline{C}$  and  $\bar{C}$ .

Considering a given price  $P^*$ , it may be seen that, for scenarios corresponding to costs lower than  $P^*$ , there is a positive profit  $\alpha$  given by

$$\alpha = P^* - C^* \tag{2}$$

Conversely, the company will experience losses, or a negative profit, if costs are higher than sales price. The possibility of losses may be evaluated by

$$\phi_l = \int_{P^*}^{\bar{C}} \mu_c \tag{3}$$

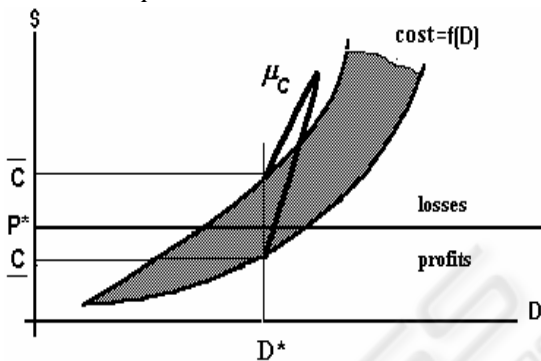


Figure 6: Pricing based on Production Costs

where the integral operation is performed under the  $D^*$  constraint and represents the accumulated possibility from  $P^*$  to  $\bar{C}$ .

#### 4 CASE STUDY

The described model was applied to a realistic Peruvian system. In order to protect confidential information, only part of the system was modeled and some key parameters were slightly changed. Therefore, the obtained results cannot be interpreted as real and do not reflect company data, targets, costs or prices. The represented system is composed by 5 hydrological plants (total installed capacity of 1500 MW) and one thermal plant (total installed capacity of 250 MW). In this simple case, no investment costs will be considered. However, practical applications of the proposed model may include operation and investment costs.

The nine possible future load scenarios are presented in Table 1.

Table 1: Possible Demand Scenarios

Possible Scenario	Demand (GWh)
1	510
2	612
3	734
4	807
5	888
6	933
7	979
8	1028
9	1080

Table 2: Production Costs for each possible scenario (Millions of US\$)

	HYDROLOGICAL SCENARIOS									
	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	367
3	0	0	0	0	0	0	0	0	6	2396
4	0	0	0	0	0	0	0	2	22	2724
5	0	0	0	0	5	401	740	1149	1759	5281
6	0	0	0	218	697	1138	1502	2053	2916	6944
7	0	3	241	914	1393	1865	2243	2928	3887	8063
8	1	250	972	1653	2169	2732	3229	4009	5062	9239
9	228	696	1759	2470	3165	3885	4463	5243	6296	10472

Table 2 presents corresponding production costs for ten possible hydrological inflows.

The Decision-Maker must choose an offer price for a contract of 1000 GWh supply along a two-year horizon. According to company, pricing it should follow a Cost-recovering philosophy, and a 5% maximum risk of losses. Figure 7 presents the fuzzy region of production costs as a function of the supplied load. For simplicity reasons, a cumulative scenario possibility function is represented. It may be seen that the price associated to a 5% risk of losses is slightly above US\$ 4000 million.

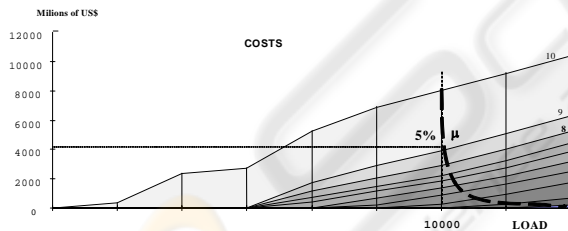


Figure 7: Fuzzy Cost Production Region

### 5 CONCLUSIONS

This paper presented a model for the risk assessment of an electrical system under a competitive environment. The proposed approach extensively used efficient optimization algorithms to build the regions of possible future scenarios. A fuzzy reasoning framework then treated these possibility regions in order to obtain the strategies corresponding to the company's objectives. It is interesting to notice the difference between the

proposed and the classical tools. While the classical approach requires the user to adopt a given objective (for example minimum operation costs, minimum variance, minimum regret, etc.), the new model adapts to the user targets and philosophies, producing the adequate results to the new company needs. The presented model aims to be an effective and useful tool to risk analysis and management.

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