A Randomized Gathering Algorithm for Multiple Robots with Limited Sensing Capabilities*

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Abstract. Consider a swarm of weak, anonymous and homogeneous robots lacking memory, orientation, and communication capabilities, and having myopic sensors that tell them the directions to nearby robots, but not the distance from them. We present a simple randomized algorithm which, when performed by all members of the swarm, gathers them in a small region. We explore the interesting global phenomena that occur during the process, evident from our analysis and simulations.

1 Introduction

In this paper, we present a very simple algorithm that makes a swarm of very simple robots perform a seemingly simple task — getting together in a small region. From a practical standpoint, it can be useful for collecting multiple ant-robots after they have performed a task in the field; for enabling them to start a mission, after being initially dispersed (e.g., parachuted); or for aggregating many nano-robots in a self-assembly task. From a theoretical standpoint, it is the most basic instance of the *formation* problem, i.e., the problem of arranging multiple robots in a certain spatial configuration. While advanced intelligent robots are certainly capable of gathering, the problem is most challenging when the robots are ant-like or less — having very limited abilities, e.g., myopic, disoriented and lacking explicit communication capabilities.

Several theoretical works on this subject exist. Current approaches include agreement on a meeting point with some unique geometrical property [1–4]; using a common compass [5]; cyclic pursuit [6–8]; and others [9–11]. Sugihara et al. suggested a simple way to fill a convex shape, which is also useful for gathering [12].

These methods rely on strong assumptions about the robots (or *agents* as we shall call them henceforth): Some rely on labeling (e.g., pursuit), some on common orientation, and many on infinite-range visibility. Nearly all works rely on the agents' ability to measure their mutual distances.

In this work, we suggest a simple gathering algorithm, which relies on very few capabilities: Our agents are anonymous, homogenous, memoryless, asynchronous, myopic and are *incapable of measuring mutual distances*. We are not aware of previous works using this limited model. The inspiration and motivation for this work came from

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experiments with real robots in our lab [13], made from LEGO parts and very crude sensors, which are range-limited and do not provide usable distance measurements.

In [14], we presented a simple deterministic gathering algorithm, which is similar in idea to the aforementioned polygon-filling algorithm of Sugihara et al., yet it has the additional property of maintaining mutual visibility between the robots, in order to cope with their shortsightedness. In this work, we present a randomized variant of the algorithm. Due to space restrictions, we omitted the proofs and abridged the discussion. More details can be found in [14] and in an upcoming extended paper.

2 Model and Algorithm

We begin with a definition of the world model. Then, we discuss the conditions which guarantee that visibility is maintained between our myopic agents, and present the proposed algorithm.

2.1 Model

The *world* consists of the infinite plane $\mathbb{R}^2$ and n point *agents* living in it. We adapt Suzuki and Yamashita's convenient way of modeling a system of asynchronous agents [4]: *Time* is a discrete series of *time steps* $t = 0, 1, \ldots$. In each time step, each agent may be either *awake* or *asleep*, having no control over the random scheduling of its waking times. A waking agent senses its environment, and is able to move instantly to any point within a distance σ (the *maximum step length*). The agent is able to see other agents within distance V (the *visibility radius* or *range*). However, it cannot measure its *distance* from them. It only knows the *directions* in which the nearby agents are found, i.e., the input is a cyclic list of *angles* $\theta_1, \ldots \theta_m$ (relative to some arbitrary direction, e.g., the agent's heading). There are no collisions. Several agents may occupy the same point.[1] All agents are *memoryless*, *anonymous* (indistinguishable in their appearance) and *homogenous* (they lack any individuality or identity, and perform the same algorithm).

Regarding the agents' activity schedule, we only assume that the agents are *strongly asynchronous*: For any subset G of the agents and in each time step, the probability that G will be the set of waking agents is bounded from below by some constant $\varepsilon > 0$.

Define the *mutual visibility graph* as an undirected graph with n vertices, representing the agents, and an edge between each pair of agents, if and only if they can see each other, i.e., the distance between them is at most V.

2.2 Maintaining Visibility

We now present a sufficient condition, on any algorithm, for maintaining mutual visibility between agents. In what follows, denote a disc of radius r and center a (where a may signify the location of an agent a) by $B_r(a)$. Also, denote

$$\mu \equiv \min\left(V/2, \sigma\right) \quad .$$

[1] In this case, they have undefined relative directions and simply ignore each other.

Let agent a see m agents $b_1, \ldots b_m$, and define the *allowable region* $AR(a)$ *of* a:

$$AR(a) \equiv B_\mu(a) \cap \bigcap_{i=1}^{m} B_{V/2}\left(a + \frac{V}{2} \cdot \frac{b_i - a}{\|b_i - a\|}\right) \quad . \tag{1}$$

It is easily seen (cf. Fig. 1) that $AR(a)$ is not empty if and only if all visible agents are contained within a sector or "wedge" of less than half the disc $B_V(a)$, i.e., its angle is less than π. In this case, $AR(a)$ is simply the intersection of $B_\mu(a)$ and the two discs corresponding to the agents on the wedge's bounds (e.g., b_1 and b_4 in Fig. 1(b)):

$$AR(a) = B_\mu(a) \cap B_{V/2}\left(a + \frac{V}{2} \cdot \frac{b_1 - a}{\|b_1 - a\|}\right) \cap B_{V/2}\left(a + \frac{V}{2} \cdot \frac{b_m - a}{\|b_m - a\|}\right) \quad . \tag{2}$$

Also, if $m = 0$, then $AR(a) = B_\mu(a)$. The following lemma holds.

Lemma 1. *If each agent confines its movements to the allowable region defined above, then existing visibility will be maintained.*

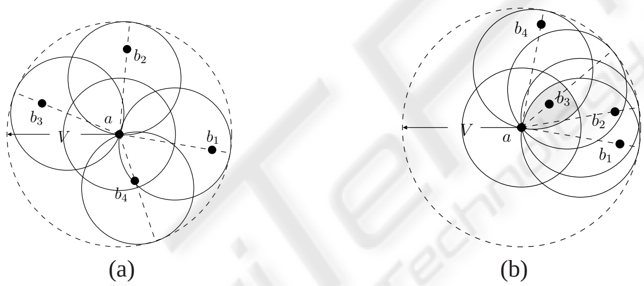


Fig. 1. Maintaining visibility. (a) a is surrounded and cannot move ($AR(a) = \varnothing$). (b) a can move only within the shaded area.

2.3 The Algorithm

The proposed algorithm is as follows:

> *Move to a uniformly-distributed random point in the allowable region* (unless it is empty, in which case do not move).

Interestingly, the algorithm doesn't seem to "care" for anything but maintaining visibility, yet its rationale is similar to that of the deterministic algorithm from [14] (where the agent moves as far as allowed along the *bisector* of the wedge): The agents inside the area occupied by the swarm do not move, while the agents at the outskirts move inside, making the swarm shrink, until all agents are gathered densely in a small cluster.

3 Results and Analysis

Despite its extreme simplicity, the algorithm effectively manages to gather all agents in a small cluster. Moreover, we observed in our simulations some very interesting global phenomena, which we discuss in this section.

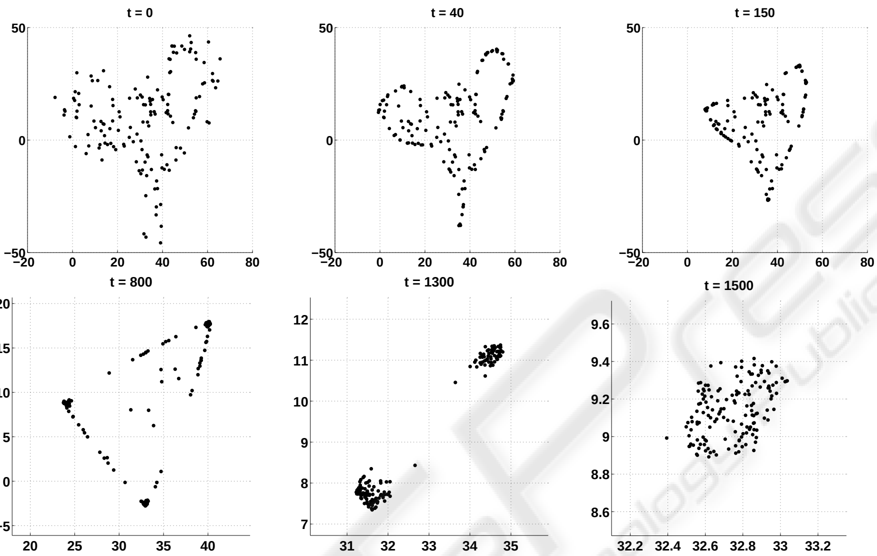


Fig. 2. A typical run. Here $n = 150, \sigma = 1, V = 10$, and each agent wakes up in each time step with probability $p = 0.6$. Note that the scale changes between frames.

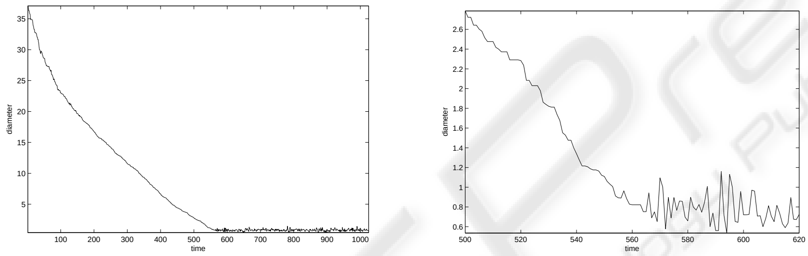
3.1 Global Behavior

The qualitative behavior of the algorithm is clearly divided into two phases (similarly to the deterministic variant). First, in the *contraction phase*, the area occupied by the agents contracts into a small dense cluster[2]. In a large swarm, the contraction process exhibits an interesting behavior, where the occupied area shrinks non-uniformly, assuming an approximate polygonal shape with a few corners and roughly straight edges between them (cf. Fig. 2). The corners are actually dense clusters of several agents. The edges are "belts", containing the agents that were swept by the contracting boundary. The density of agents along the edges is much lower than in the corners. More generally, there is a correlation between high curvature (of the boundary) and high density

[2] We use the term "area occupied by the agents" freely, as a subjective observation of the agents' distribution or the shape of the swarm. It can be defined formally as the area enclosed by laying line segments between all pairs of mutually visible agents. When all agents are mutually visible, this area equals the convex hull of the agents' locations. An alpha-shape may also be considered, however its parameter α has no clear meaning in our problem.

of agents. We believe that there is a positive-feedback relationship between density and curvature, which results in the large-scale polygonal shape, having sharp dense corners and linear edges of lower density.

The occupied area contracts until it becomes a small dense cluster with a mean diameter[3] of about μ. At this stage, the *wandering phase* begins. The dense cluster stops contracting, and begins wandering in the plane, instead. The reason to this change is clear: The agents' step sizes are not affected by the scale of the occupied area. As long as the area is large in comparison to μ, the agents on its boundary generally move inside it, making it contract. However, once the area becomes smaller, the agents' steps become relatively larger, so they leap over it, instead. As a result of these leaps, the cluster drifts along the plane indefinitely. Note that since there is full visibility between the agents at this stage, only the agents at the vertices of the convex hull are able to move.



(a) (b)

Fig. 3. The diameter in a typical run of the algorithm. Here $n = 60, \sigma = 1, V = 5$, and $p = 0.6$. (a) The phase transition is very clear. (b) Zooming in on the transition moment. Note that the mean diameter is about 0.8μ after the transition.

When $n \leq 2$, the two algorithm variants act differently. In the deterministic algorithm, a single agent will not move, and a pair will always remain on one line. In the randomized variant, the agents will roam the plane in either case.

3.2 Guaranteed Convergence

The following theorem holds (We omitted the proof for space considerations).

Theorem 2. *Given an initial configuration with a connected visibility graph, the agents will gather and forever remain in a cluster whose diameter is bounded by V, in finite expected time.*

The proof idea is as follows: We show that, in every time step, there exists an agent (specifically, one located at a vertex of the convex hull), which has a strictly positive[4]

[3] The term *mean diameter* here refers to the average diameter over time (in a given run), not probability space.

[4] i.e., positive and bounded away from zero by some constant.

chance of moving inside the convex hull *and* closer to the center of mass (average of the agents' positions) by a strictly positive amount, if it is the only agent which wakes up. By our strong asynchronicity assumption, the chance that this will happen is also strictly positive (bounded by ε). This, in turn, will make the variance (sum of squared distances from the center of mass) decrease by a strictly positive amount. Thus, with time, the variance will decrease arbitrarily with probability 1. As the variance gets smaller, the diameter must too, so at some point, the diameter will be V or less, which implies that the visibility graph is a clique. By Lemma 1, it will remain a clique, and therefore the diameter will remain bounded by V.

3.3 Evaluation of the Mean Cluster Diameter

Theorem 2 guarantees gathering to diameter V. However, the simulations clearly show further contraction to a mean diameter of about 0.8μ during the wandering phase[5], fairly indifferently to the choice of n. With the deterministic algorithm, the mean diameter typically settles at about 1.04μ.

When the agents are scattered (i.e., the diameter is much larger than μ) the diameter is much more likely to decrease, and when the agents are gathered in a small cluster, it is likelier to increase (e.g., consider a limit case of an infinitesimally small cluster). Thus, we infer that there exists a probabilistically stable equilibrium point for the diameter. This point is the expected mean diameter.

An exact calculation of the expected mean diameter seems to be difficult, yet the following rough estimate for large n provides a surprisingly good prediction of the measured results. Given a dense cluster P with diameter $D(P) \ll V$, we first approximate its convex hull shape as a disc of diameter $D(P)$. The corner agents reside on its boundary, with their wedge bisectors pointing to its center. We assume that $\sigma \ll V$ and n is large, so that the allowable region of each corner agent is approximately a narrow sector (i.e., a "pizza slice") of a disc of radius μ. Now, we approximate the expected mean diameter as that for which, for each corner agent, the probability of moving into the convex hull equals the probability of leaping over it. Geometrically, it means that the intersection of the narrow sector and the disc should contain half of the sector's area. This holds when the disc's diameter is about $\mu/\sqrt{2}$, which is quite close to the observed typical mean diameter of 0.8μ.

In the deterministic variant of the algorithm, assuming that σ is small enough, the agent simply moves a step of size μ on the bisector (i.e., along the disc's diameter in our approximation). Thus, the expected mean diameter is simply μ. Again, it agrees well with the measured typical mean diameter of 1.04μ.

3.4 Composite Random Walking

The random wandering of the cluster is composed of the movements of the individual agents in it (hence we term it a *composite random walk*). An interesting question is

[5] Although we determined the moment of phase transition subjectively, it is evident from Fig. 3 that this moment is very clear. We calculated the average diameter from about 20 time steps ahead of that moment until the end of the simulation (several hundred steps later).

whether this random walk is recurrent or not. Based on our observations, we conjecture that it is indeed. If so, then it has a very important implication: Theorem 2 guarantees that a configuration with a connected visibility graph will contract. Thus, in the general case, each connected component will contract into an independent cluster. Now, if the cluster's random walks are recurrent, then they will all meet eventually and merge into one cluster.

In order to be recurrent, a two-dimensional random walk needs to be unbiased. Obviously, when observing a single time step, the random movement is not distributed uniformly in all directions, as it depends on the exact shape of the convex hull. However, when integrating over many time steps, one may show that the cluster's displacement is distributed uniformly in all directions.

A single isolated agent performs a uniform random walk by definition. For two agents, observe that the probability distribution of the center of mass's displacement is always symmetric along the line passing through the two agents, and has a constant shape up to its orientation in the plane. Thus, the change of orientation has a constant and unbiased distribution (i.e., same for clockwise and counterclockwise changes). Therefore, over time, the orientation will be distributed uniformly, and, accordingly, the center of mass's displacement distribution will approach uniformity as well.

For $n > 2$, unfortunately, we don't have an easy proof. Even with only three agents, the state-space becomes complex and hard to analyze. Again, we conjecture that the composite random walk of three or more agents is indeed recurrent. This seems to be the case from observing long runs of the simulations.

4 Conclusions

The main contribution of our work is that we consider the gathering problem with such severe limits on the agents' sensors (being both myopic and unable to measure distances), in addition to being anonymous and memoryless. The proposed algorithm is an example of how deceptively simple individual behaviors can yield complex emergent global behaviors of the swarm — two distinct phases of contraction and wandering, where the swarm first assumes an approximate polygonal shape and then collapses into a dense cluster wandering in the plane.

If our conjecture regarding the recurrence of the composite random walk proves to be true, then the algorithm is a very powerful one, gathering all agents together into a dense cluster, regardless of their initial distribution. We find it intriguing that it does that even though it is seemingly unconnected to the gathering problem, as the agents just move randomly while trying to maintain their visibility. One may view this as an argument that gathering is the only global task that such a swarm of weak robots can do, i.e., it is a sort of an "upper bound" on the swarm behavior capabilities.

Further work should provide tighter estimates of the convergence rate, and analyze the effect of noise and error, both in sensing and movement, on the resulting global behavior.

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