OPTIMAL CONTROL APPLIED TO OPTIMIZATION OF MOBILE SWITCHING SURFACES PART II : APPLICATIONS

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Abstract: To reinforce interest of a general optimization algorithm obtained in a previous paper (Jolly et al., 2005), we consider three applications : an original one about control of cycles for a thermostat with anticipative resistance, a classical one with a new resolution for a car with two gears and a last one about an obstacle-avoidance problem in robotics. For the first case, we optimize the adjustment of thermostat thresholds to control at best the room temperature. For the second case, we optimize the switching times to stop the car as near as possible of chosen points and this, in a minimum time . In the last example, we optimize parameters of the switching surfaces in order that the robot reaches a chosen target without meeting a mobile obstacle.

1 INTRODUCTION

In (Jolly et al., 2005), we have found results on the question of optimization of switching surfaces for a hybrid dynamical system (h.d.s), generalizing what was in (Wardi et al., 2004).

Here, we consider three applications that underline interest of these theorical results. The first, somewhat original, is one of a thermostat with anticipative resistance controlling a convector in a same room (Cébron, 2000), (Quémard et al., 2005). In this example, we optimize the adjustment of thermostat thresholds to control at best the room temperature. This application can be taken as a pattern for h.d.s leading to some cycle solutions.

The second application is one of a car with two gears (Gapaillard, 2003), (Hedlund and Rantzer, 2002). We optimize the switching times, firstly, to stop the car as near as possible of a first desired destination and then, after a new start-up, to stop the car as near as possible of a final destination and this, in a minimum time. Interest of this classical h.d.s problem for us is to bring a new resolution improving numerical performance.

The last application solves an obstacle avoidance problem in robotics (Boccadoro, 2004). Here, we optimize parameters of the switching surfaces in order that a robot reaches a pre-specified target without never meating a given mobile obstacle. Compared to (Boccadoro, 2004) where the considered obstacle is fixed, this example underlines interest of mobility for switching surfaces in applications.

In section 2, we briefly present the theorical algorithm found in (Jolly et al., 2005). From section 3 to section 5, we detail each application presented above. Section 6 concludes the paper.

2 OPTIMIZATION ALGORITHM REMINDER

Let t_0, $x_0 = x(t_0) \in \mathbb{R}^n$ be given initial time and state. Here, we consider a h.d.s which sustains switchings at increasing times $t_1, ..., t_N$ in $[t_0, t_{N+1}]$ (t_{N+1} is the final time) so that for $i = 1, ..., N + 1$, state $x_i = x(t_i)$ belongs to a given mobile surface parameterized by $a_i \in \mathbb{R}^{r_i}$ and of equation:

$$\Psi_i(x_i, t_i, a_i) = 0, \tag{1}$$

where Ψ_i is from C^1 class with values in $\mathbb{R}$. In $[t_0, t_{N+1}]$, state $x(t)$ is supposed to be continuous and in $[t_{i-1}, t_i]$, $i = 1, ..., N+1$, state $x(t)$ complies with dynamical system:

$$\dot{x} = f_i(x, t), \tag{2}$$

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where f_i is from C^1 class with values in $\mathbb{R}^n$. Under suitable assumptions (Jolly et al., 2005), t_i is a function of $a_1, ..., a_i$, $i = 1, ..., N+1$. For our optimization problem, the criterion we have to minimize or maximize is in the form:

$$J^0 = \sum_{i=1}^{N+1} J_i^0,$$

where $J_i^0 = \phi_i(x_i, t_i, a_i) + \int_{t_{i-1}}^{t_i} L_i(x,t)dt$ with ϕ_i and L_i from C^1 class.

Optimization problem - *Considering t_i as a function of $a_1, ..., a_i$, $i = 1, .., N+1$, we search values for $a_1, ..., a_{N+1}$ which optimize criterion J^0.*

We consider the following augmented criterion:

$$\sum_{i=1}^{N+1} J_i,\ J_i = \phi_i + \nu_i \Psi_i + \int_{t_{i-1}}^{t_i} (H_i - \lambda_i^T \dot{x})dt \quad (3)$$

where ν_i is a control parameter, λ_i is the adjoint state and $H_i = L_i + \lambda_i^T f_i$. Those variables play a key role in the following algorithm:
Let $a_1, ..., a_{N+1}$ be initialized parameters.

1. We solve system (2) forwards for $i = 1, .., N+1$. In the same time, we compute switching times t_i, $i = 1, .., N+1$, with constraint (1).
2. Starting from t_{N+1}, $x_{N+1} = x(t_{N+1})$ just obtained, we solve system (4) backwards given by:

$$\frac{\partial H_i}{\partial x} + \dot{\lambda}_i^T = 0, i = N+1, .., 1. \quad (4)$$

   In the same time, we compute suites ν_i, λ_i, $i = N+1, .., 1$ given by:

$$\left\{ \begin{array}{l} \nu_i = -(L_i - L_{i+1} + \lambda_{i+1}^T (f_i - f_{i+1}) \\ + \frac{\partial \varphi_i}{\partial X_i} f_i + \frac{\partial \varphi_i}{\partial t_i})_{t_i} (\frac{\partial \Psi_i}{\partial X_i} f_i + \frac{\partial \Psi_i}{\partial t_i})_{t_i}^{-1}, \\ \lambda_i^T(t_i) = \lambda_{i+1}^T(t_i) + \frac{\partial \varphi_i}{\partial X_i} + \nu_i \frac{\partial \Psi_i}{\partial X_i}, \end{array} \right. \quad (5)$$

   $i = N+1, .., 1$. The notation used is that variable t_i, which follows an expression in a lower position, means that this expression is evaluated at t_i, $x(t_i)$.
   In (5), to start the backward recurrence, we define:

$$\lambda_{N+2}(t_{N+1}) = 0\, ,\, L_{N+2}|_{t_{N+1}} = 0. \quad (6)$$

3. Then, with all elements computed in the previous steps, we can deduce:

$$\frac{dJ^0}{da_i} = \frac{dJ}{da_i} = \frac{\partial \phi_i}{\partial a_i} + \nu_i \frac{\partial \Psi_i}{\partial a_i}\ , i = 1, ..., N+1. \quad (7)$$

4. Finally, with the criterion gradient, we apply a descent method to obtain optimal results.

3 OPTIMIZATION OF LIMIT CYCLES. APPLICATION TO A THERMAL DEVICE

3.1 Studied Thermal Device

Figure 1 represents a thermostat with anticipative resistance controlling a convector located in the same room. Such a thermostat is common in the industrial market (Cyssau, 1990). The principle is the following. The thermostat, which is controlled by a hysteresis phenomenon (Figure 1), heats the room through a convector (power P_c) and itself through a resistance (power P_t) until its temperature reaches its upper threshold. Then, it switches off until its temperature reaches its lower threshold.

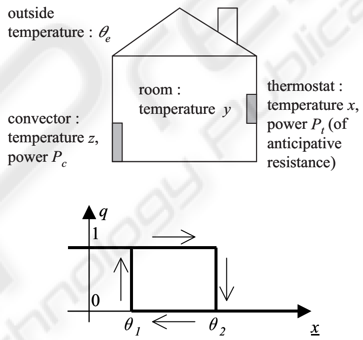

Figure 1: Thermal process and hysteresis variable

With notations of Figure 1, a power assessment and Newton law give, in the state form proposed in (Cébron, 2000), the following system:

$$\left\{ \begin{array}{l} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} -a & a & 0 \\ 0 & -(b+d) & b \\ 0 & c & -c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\ +q \begin{pmatrix} p_t \\ 0 \\ p_c \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_e \\ 0 \end{pmatrix} \\ \xi = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x \end{array} \right. \quad (8)$$

with numerical values set: $a = 0.001\ s^{-1}$, $b = 2.81\ 10^{-4}\ s^{-1}$, $c = 0.011\ s^{-1}$, $d = 0,2\ 10^{-4}\ s^{-1}$, $p_t = 0.0035\ K.s^{-1}$, $p_c = 0.1\ K.s^{-1}$, $\theta_e = 274.d\ K.s^{-1}$. Here, we consider two heating ways, say a day one and a night one, each one having its own lower (θ_1 for the day, θ_3 for the night) and upper (θ_2 for the day, θ_4 for the night) threshold. We also consider here that we change the way of heating at $t = 20000\ s$.
Discrete variable q takes the value 0 or 1. Here, we are in the same situation that the one exposed in

the second section with $t_1 < t_2 ... < t_N$ switching times in $[t_0, t_{N+1}]$, where t_0 and t_{N+1} are respectively initial and final times. A simulation with Matlab, with $q_0 = 1$ at $t_0 = 0$ and with initial values $X(t_0) = (288 \quad 288 \quad 288)^T$, $\theta_1 = 293\ K$, $\theta_2 = 294\ K$, $\theta_3 = 290\ K$, $\theta_4 = 291\ K$ gives Figure 2.

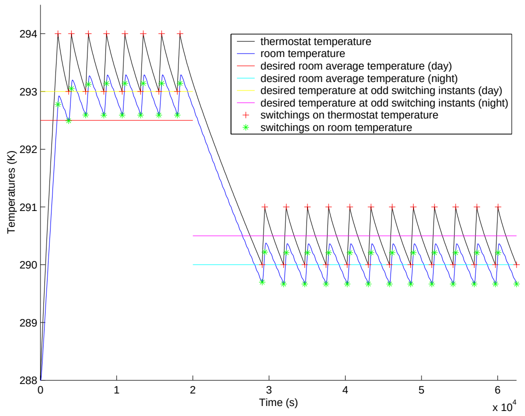

Figure 2: Temperatures before optimization

Optimization problem - *How can we choose thermostat thresholds (considered not fixed) θ_i, $i = 1, ..., 4$ to have the room temperature at odd switching times (upper stars) as near as possible of desired temperatures $\theta_{u1} = 293\ K$ (for the day), $\theta_{u2} = 290.5\ K$ (for the night)? Moreover, in the same time, how can we choose them to have the room average temperature as near as possible of desired room average temperatures $\theta_{d1} = 292.5\ K$ (for the day), $\theta_{d2} = 290\ K$ (for the night)?*

3.2 Gradient Calculus. Criterion Minimization

Results obtained in (Quémard et al., 2005) and Figure 2 let us to establish that thermostat model is a h.d.s for which a trajectory $X(t)$ can converge towards a stable limit cycle. Following notations used in the second section and particularly in equation (3), we can consider the augmented criterion $J = \sum_{i=1}^{N+1} J_i$, with:

$$J_i = q_i \frac{\alpha}{N+1}(EX_i - \theta_{uj})^2 + \nu_i \frac{1}{N+1}(DX_i - a_i)$$
$$+ \frac{1}{t_{N+1}} \int_{t_{i-1}}^{t_i} (H_i - \lambda_i^T \dot{X}_i) dt,$$

where:

- $q_i = 0$ if i is even and $q_i = 1$ if i is odd,
- $D = (1 \quad 0 \quad 0)$, $E = (0 \quad 1 \quad 0)$,
- $a_i = \theta_1$ if i is even and if $t_i < 20000$, $a_i = \theta_2$ if i is odd and if $t_i < 20000$, $a_i = \theta_3$ if i is even and if $t_i \geq 20000$, $a_i = \theta_4$ if i is odd and if $t_i \geq 20000$,
- $H_i = L_i + \lambda_i^T f_i$, where $L_i = \beta(EX_i - \theta_{dj})^2$, $f_i = AX_i + q_i B + C$,
- $\alpha + \beta = 1, \quad \alpha > 0, \beta > 0$,
- $j = 1$ if $t_i < 20000$ and $j = 2$ if $t_i \geq 20000$.

From there, we apply the algorithm we report in the second section to obtain an optimal trajectory $X(t)$ for J as a function of θ_1, θ_2, θ_3, θ_4.
Firstly, for arbitrary initial conditions, we solve direct system (8), variable with i and we compute switching times and states and the final time and state. Secondly, we solve adjoint system (4) backwards given here by:

$$\dot{\lambda}_i^T = -A^T \lambda_i^T - (0 \quad 2(EX_i - \theta_{dj}) \quad 0)^T.$$

In the same time, we can define ν_i, $\lambda_i(t_i)$ with equations systems (5) and (6):

$$\begin{cases} \nu_i = -\frac{N+1}{Df_i}(\beta((EX_i - \theta_{dj})^2 - (EX_{i+1} - \theta_{dk})^2) \\ +\lambda_{i+1}^T(f_i - f_{i+1}) + q_i \frac{2\alpha}{N+1}(EX_i - \theta_{uj})Ef_i), \\ \lambda_i^T(t_i) = \lambda_{i+1}^T(t_i) + q_i \frac{2\alpha}{N+1}(EX_i - \theta_{uj})E \\ +\frac{\nu_i}{N+1} D, \end{cases}$$

where $k = 1$ if $t_{i+1} < 20000$, otherwise $k = 2$.
Thus, from (7), we can deduce:

$$\frac{dJ_i^0}{da_i} = -\frac{\nu_i}{N+1}, \quad i = 1, .., N+1.$$

Regrouping those terms according to values of t_i and to parity of i, we obtain the criterion gradient. Thus, we can apply a descent methode to define an optimal solution. The using of Matlab and particularly of function fmincon with initial values $\alpha = \beta = 0.5$, $\theta_1 = 293K$, $\theta_2 = 294K$, $\theta_3 = 290K$, $\theta_4 = 291K$, gives after thirteen iterations the algorithm end. We obtain Figure 3 and the following optimal values: $(\theta_1, \theta_2, \theta_3, \theta_4)$ =$(292.32, 293.744, 290.149, 291.249)$, $J^0 = 18.5046$.
This optimization leads to the following differences (indexed quantities rely on switching quantities):

- Initially (Figure 2): $|EX_i - \theta_{u1}| \simeq 0.2872$, $|EX_i - \theta_{u2}| \simeq 0.1378$. Moreover, $|\theta_{m1} - \theta_{d1}| \simeq 0.4371$ K for $t < 20000$ and $|\theta_{m2} - \theta_{d1}| \simeq 0.0132\ K$ for $t \geq 20000$ whith θ_{m1} and θ_{m2} corresponding respectively to the obtained room average temperature for $t < 20000$ and for $t \geq 20000$.
- After optimization (Figure 3): $|EX_i - \theta_{u1}| \simeq 0.0222$, $|EX_i - \theta_{u2}| \simeq 0.0865$. Moreover, $|\theta_{m1} - \theta_{d1}| \simeq 0.0362\ K$ for $t < 20000$ and $|\theta_{m2} - \theta_{d1}| \simeq 0.2058\ K$ for $t \geq 20000$. So, just this last result is not improved.

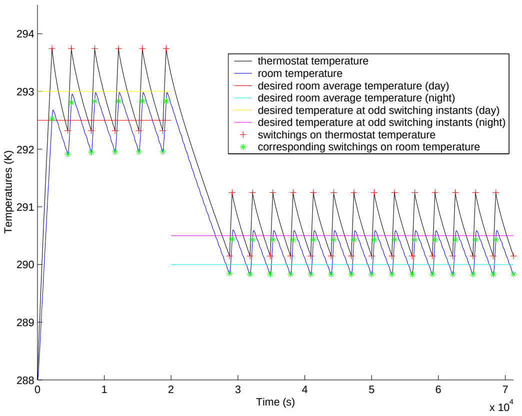

Figure 3: Temperatures after optimization

4 OPTIMIZATION OF SWITCHING TIMES. APPLICATION TO A CAR WITH TWO GEARS

4.1 Studied Car Model

Following (Hedlund and Rantzer, 2002), we consider the following system:

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = \frac{1}{m}(-cx_2 + kx_3) \\ \dot{x_3} = -x_2 + \frac{g_q(x_2)}{k}u \end{cases} \tag{9}$$

where $q = 1,2$. In (Hedlund and Rantzer, 2002), the authors find that optimal input throttle $u \in [-0.1, 1.1]$ is essentially a bang-bang pattern what we take in assumption. So, here, we choose $u \in \{-0.1, 1.1\}$.

The three continuous states of the system represent respectively the car position (x_1), the car velocity (x_2) and the rotational displacement of its transmission shaft (x_3). Function g_q, plotted in Figure 4, represents the efficiency of gear number q. Constants m (mass of the car), c (frictional damping) and k (constant of transmission shaft) are set to 1 without loss of generality.

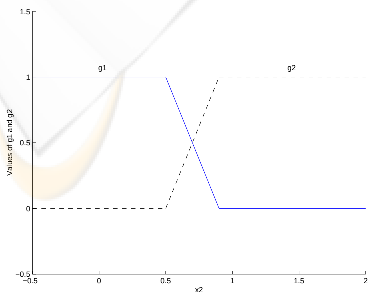

Figure 4: g_1 and g_2 behaviors

Optimization problem - *Firstly, contrary to (Hedlund and Rantzer, 2002), we impose rules rather natural for the car evolution which are listed below:*

	$[t_0, t_1[$ $[t_4, t_5[$	$[t_1, t_2[$ $[t_5, t_6[$	$[t_2, t_3[$ $[t_6, t_7[$	$[t_3, t_4[$ $[t_7, t_8[$
Action	accelerate	accelerate	brake	brake
Gear	1^{st}	2^{nd}	2^{nd}	1^{st}

Here, we optimize switching and final times t_i, $i = 1,..,8$ to stop the car as near as possible of a first chosen destination ($x_1 = 0$), then, after a new start-up, to stop it as near as possible of a second chosen destination ($x_1 = 5$) and this, in a minimum time.

4.2 Gradient Calculus. Criterion Minimization

Following notations used in the second section and particularly in equation (3), we can consider the augmented criterion $J = \sum_{i=1}^{N+1} J_i$, $N = 7$, with:

$$J_i = \frac{\alpha}{N+1}(\frac{x_2}{\delta} - v_{g_i})^2 + \nu_i \frac{t_i - a_i}{N+1} + \int_{t_{i-1}}^{t_i} (H_i - \lambda_i^T \dot{X}_i)dt,$$

where:

- $\delta = 50$: tolerable changing amplitude,
- $v_{g_i} = 0.8$, $i = 1,5$, $v_{g_i} = 1.2$, $i = 2,6$, $v_{g_i} = 0.2$, $i = 3,7$, $v_{g_i} = 0$, $i = 4,8$: recommended changing velocity,
- $a_i = t_i$, $i = 1,..,8$,
- $H_i = L_i + \lambda_i^T f_i$ where $L_i = \beta(x_1^2(t) + x_2^2(t))$, $i = 1,..,4$, $L_i = \beta((x_1(t)-5)^2 + x_2^2(t))$, $i = 5,..,8$ $f_i = AX_i + B$, with:
  * $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 0 \end{pmatrix}$,
  * $B = \begin{pmatrix} 0 \\ 0 \\ ug_q \end{pmatrix}$ with $u = 1.1$ for $i = 1,2$, $u = -0.1$ for $i = 3,4$, $g_q = g_1$ for $i = 1,4$, $g_q = g_2$ for $i = 2,3$.

Then, like for the thermostat problem, we apply the optimization algorithm related in section 2. Firstly, we solve numerically direct system (9) to define switching and final times and states. Then, we solve adjoint system backwards given by (4) which is given here by:

$$\dot{\lambda}_i^T = -A^T\lambda_i^T - (2\beta(x_1(t) - d) \quad 2\beta x_2(t) \quad 0)^T.$$

with $t \in [t_{i+1}, t_i[$, $d = 0$ for $i = 1, .., 4$, $d = 5$ for $i = 5, .., 8$.
In the same time, we obtain suites ν_i, $\lambda_i(t_i)$ given by (5) which, applied to the car problem for $i = 8, .., 1$ and considering (6), gives system:

$$\begin{cases} \nu_i = -8(\beta((x_1(t_i) - d)^2 + x_2^2(t_i) \\ -(x_1(t_{i+1}) - d_2)^2 - x_2^2(t_{i+1})) \\ +\lambda_{i+1}^T(f_i - f_{i+1}) + (0 \quad \frac{\alpha}{4}(\frac{x_2(t_i)}{50} - v_{g_i}) \quad 0)f_i), \\ \lambda_i^T(t_i) = \lambda_{i+1}^T(t_i) + (0 \quad \frac{\alpha}{4}(\frac{x_2(t_i)}{50} - v_{g_i}) \quad 0), \end{cases}$$

where $d = 0$ for $i = 1, .., 4$, $d = 5$ for $i = 5, .., 8$, $d_2 = 0$ for $i = 1, .., 3$ and $d_2 = 5$ for $i = 4, .., 7$.
Then, we deduce from (7):

$$\frac{dJ_i^0}{da_i} = -\frac{\nu_i}{8}, i = 1, .., 8,$$

which is the criterion gradient.
Thus, we apply a descent method to define an optimal solution. We use again Matlab and function fmincon with initial values: $\alpha = \beta = 0.5$, $t_1 = 1.7$, $t_2 = 5.1$, $t_3 = 6.9$, $t_4 = 8$, $t_5 = 10.1$, $t_6 = 12.8$, $t_7 = 14.1$, $t_8 = 15.4$. The algorithm stops after thirteen iterations and gives the following optimal results: $(t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8) = (2.2597, 5.4952, 6.9851, 7.8253, 10.2369, 12.7535, 13.9783, 15.3068)$, $J^0 = 203.1983$. Figure 5 shows car trajectory in the phase portrait of x_1 and x_2.

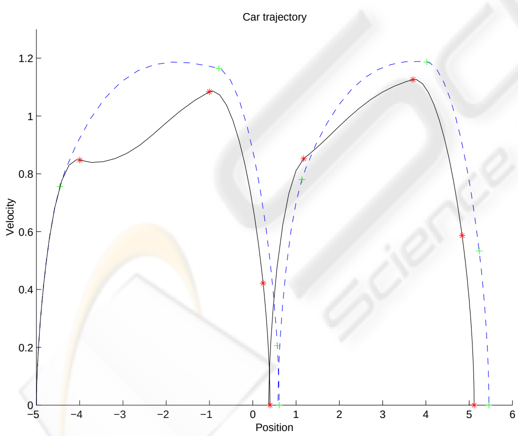

Figure 5: Trajectories before (dotted line) and after (solid line) optimization

Figure 5 confirms that our optimization algorithm enables the car to approach desired destinations. Beyond the simplifying assumption about bang-bang control u we have made, this algorithm is numerically less expensive than the one based on dynamic programming used in (Hedlund and Rantzer, 2002).

5 OPTIMIZATION OF SWITCHING RULES. APPLICATION TO ROBOTICS

5.1 Studied Robot Model

Following (Boccadoro, 2004), we consider system (10):

$$\begin{cases} \dot{x} = vcos(\phi) \\ \dot{y} = vsin(\phi) \\ \dot{\phi} = w \end{cases} \qquad (10)$$

where (x, y) is the robot position, ϕ is its orientation, v and w are the controlled translational and angular velocities. Moreover, the robot can move using two modes, an approach-goal one and an avoid-obstacle one, which are respectively given by:

$$\text{Mode 1} \begin{cases} v = 1 \\ w = C_1(\phi_g - \phi) \text{ with } \phi_g = arctan(\frac{y_g - y}{x_g - x}) \end{cases}$$

$$\text{Mode 2} \begin{cases} v = 1 \\ w = C_1(\phi - \phi_o) \text{ with } \phi_o = arctan(\frac{y_o - y}{x_o - x}) \end{cases}$$

Point (x_g, y_g) defines the position of the target that the robot has to reach and (x_o, y_o) defines the position of the obstacle that the robot has to avoid. Here, contrary to (Wardi et al., 2004), we choose a mobile obstacle which follows a circle of equation $(x_o - 1)^2 + (y_o - 1)^2 - (0.3)^2 = 0$.
The crossover between the two modes can be described as follows. We define for each obstacle position two switching surfaces of equation:

$$\Psi(x, y, a_i) = (x - x_o)^2 + (y - y_o)^2 - a_i^2, i = 1, 2.$$

Firstly, the robot operates in mode 1 until it crosses a switching surface of radius a_1 and then, it switches to mode 2. It remains in mode 2 until it crosses a switching surface of radius a_2 and then, it goes back to mode 1.

Optimization problem - *How can we choose radii a_1 and a_2 in order that the robot reaches the prespecified target without never meating the mobile obstacle ?*

5.2 Gradient Calculus. Criterion Minimization

Following notations used in the algorithm reminder and particularly in equation (3), we can consider the augmented criterion $J = \sum_{i=1}^{N+1} J_i$, $N = 2$, with:

$$J_i = \nu_i[(x - x_o)^2 + (y - y_o)^2 - a_i^2] + \int_{t_{i-1}}^{t_i} (H_i - \lambda_i^T \dot{X}_i)dt,$$

where $H_i = L_i + \lambda_i^T f_i$, with $L_i = (x - x_g)^2 + (y - y_g)^2$, $f_i = (v cos(\phi(t_i)) \quad v sin(\phi(t_i)) \quad w)^T$.
From there, we apply algorithm of section 2. Firstly, we solve direct system (10) forwards. Secondly, we solve adjoint system (4) backwards which is given here by:

$$\dot{\lambda}_i^T = -A^T \lambda_i^T - (-2(x_g - x) \quad -2(y_g - y) \quad 0)^T,$$

where, if the robot operates in mode 1:

$$A = \begin{pmatrix} 0 & 0 & C_1 \frac{y_g - y}{(x_g - x)^2 + (y_g - y)^2} \\ 0 & 0 & -C_1 \frac{x_g - x}{(x_g - x)^2 + (y_g - y)^2} \\ -v sin(\phi) & v cos(\phi) & -C_1 \end{pmatrix}$$

and if the robot uses mode 2:

$$A = \begin{pmatrix} 0 & 0 & -C_2 \frac{y_o - y}{(x_o - x)^2 + (y_o - y)^2} \\ 0 & 0 & C_2 \frac{x_o - x}{(x_o - x)^2 + (y_o - y)^2} \\ -v sin(\phi) & v cos(\phi) & C_2 \end{pmatrix}.$$

In the same time, we compute ν_i, $\lambda_i(t_i)$, $i = 3, .., 1$ given by (5). Considering (6), we obtain:

$$\begin{cases} \nu_i = -((x(t_i) - x_g)^2 + (y(t_i) - y_g)^2 \\ -(x(t_{i+1}) - x_g)^2 - (y(t_{i+1}) - y_g)^2 \\ +\lambda_{i+1}^T (f_i - f_{i+1}))(2v(cos(\phi(t_i))(x(t_i) - x_g) \\ +sin(\phi(t_i))(y(t_i) - y_g)))^{-1} \\ \lambda_i^T(t_i) = \lambda_{i+1}^T \\ -2\nu_3((x_o - x(t_3)) \quad (y_o - y(t_3)) \quad 0). \end{cases}$$

Then, equation (7) yields:

$$\frac{dJ_i^0}{da_i} = -2\nu_i a_i,$$

which is the criterion gradient.
Thus, we apply a descent method to define an optimal solution. We use again Matlab and we choose the following initial values: $C_1 = 1.2$, $C_2 = 0.5$, $x_g = 3$, $y_g = 2.5$, $a_1 = 0.85$, $a_2 = 1.05$. The algorithm stops after nine iterations and gives the following optimal results: $(a_1, a_2) = (1.0724, 1.2724)$, $J^0 = 39.8023$. Figures 6 and 7 show respectively Matlab simulations before and after optimization. The robot is nearer of the target after optimization than before. Crosses and stars represent respectively switching times for the robot trajectory and for the obstacle trajectory. Lengths of the solid circle archs measure trajectory durations. They also illustrate the interest of our study compared to (Boccadoro, 2004).

6 CONCLUSION

These three applications reinforce theorical results obtained in (Jolly et al., 2005) and show all the diversity of applications areas in which our optimization algorithm can be useful.

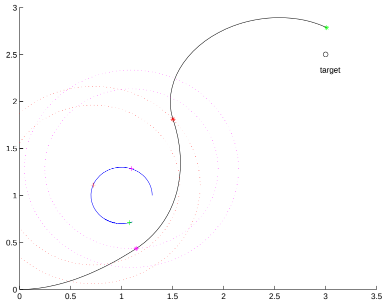

Figure 6: Trajectory of the robot before optimization

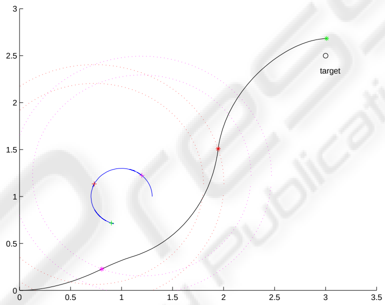

Figure 7: Trajectory of the robot after optimization

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