MODELLING AND LQ-BACKSTEPPING CONTROL FOR A QUADROTOR

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Keywords: UAV, VTOL, quadrotor, LQ, Backstepping.

Abstract: Thanks to significant advances during the last decades in the miniaturized robotic area, many Unmanned Aerial Vehicle (UAV) projects were launched. Among them, the QuadriXflyer is an UAV quadrotor designed to evolve autonomously between waypoints given by an operator before flight. In this paper, we propose a modelling and a new hybrid control approach for the QuadriXflyer; a controller integrating the advantages of a Linear Quadratic (LQ) and those of a backstepping approach allowing to compensate the nonlinearities of the system. With this new approach, the gravity will be compensated directly without time delay. Robustness of the controller is then studied to ensure the stability of the quadrotor to exogenous (wind for example) and internal (noise on measurements, uncertainties on the inertia for example) perturbations.

1 INTRODUCTION

With reduction in cost and progress in miniature technology, many studies have been launched in order to build autonomous miniaturized flying robots (Bouabdallah, Murrieri, Siegwart, 2004, Altuğ, Ostrowki, Mahony, 2002, Hamel, Mahony, Lozano, Ostrowski, 2002, Pounds, Mahony, Gresham, 2004, Bouabdallah, Siegwart, 2005, Fantoni, Antoni, Lozano, Nazenc, 2001, Praly, Ortega, Kaliora, 2000). The purpose of this study is to present a modelling and a new hybrid control method based on LQ for the linear part and backstepping for the non linear part of the control, for an autonomous four-rotor helicopter. The application has been realized on a quadrotor named QuadriXflyer (figure 1) and composed by a frame with four carbon rods setting up a straight cross. An actuator is fasten at the end of each rob and is constituted by a propeller, a reducer and a direct current electric motor. Each propeller is driven in rotation by its motor via a reducer. The QuadriXflyer has two propellers with a right thread and two propellers with a left thread. This is useful for the yaw stabilization. The aim of this quadrotor is to evolve autonomously between waypoints given by an operator before flight.

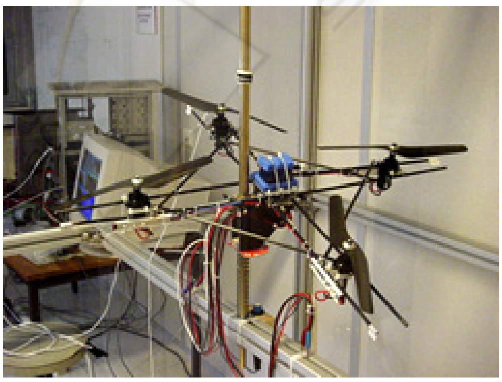


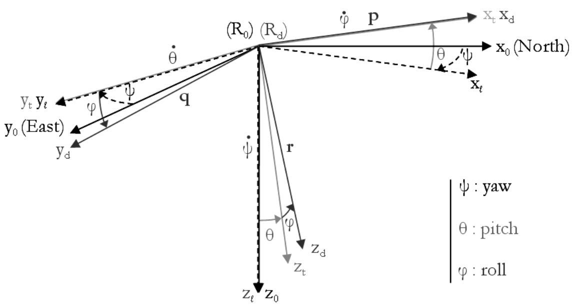
Figure 1: The QuadriXflyer

2 MODELLING

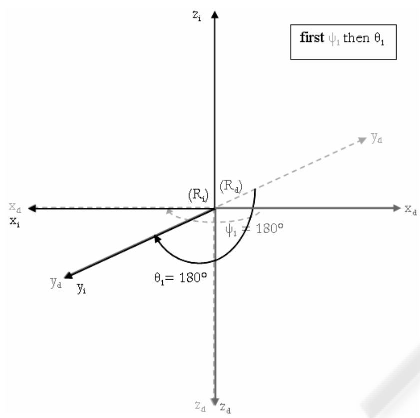
To validate the control laws, a non-linear dynamic simulation model was created using Simulink. This model takes into account the QuadriXflyer as a rigid frame mechanical system, actuators (propellers + motors + reducers), gyroscopic and thrusts efforts. The other assumptions are that the only aerodynamics effects are propellers thrusts and drag torques. Previous publications (Bouabdallah, Murrieri, Siegwart, 2004, Hamel, Mahony, Lozano, Ostrowski, 2002, Pounds, Mahony, Gresham, 2004) have presented models where propellers axes and z QuadriXflyer axe were parallel.

Belin S., Carton M. and Macaire F. (2005).
MODELLING AND LQ-BACKSTEPPING CONTROL FOR A QUADROTOR.
In *Proceedings of the Second International Conference on Informatics in Control, Automation and Robotics - Robotics and Automation*, pages 134-139
DOI: 10.5220/0001177501340139

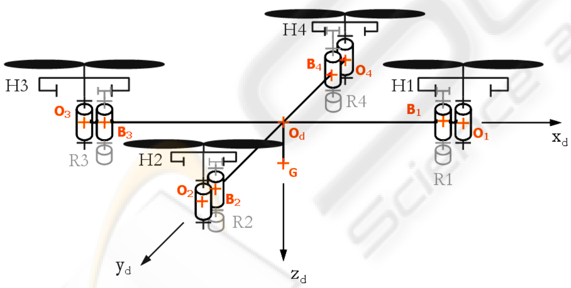
Euler angles ψ, θ, φ, define the rotations between Earth frame (Ro) and QuadriXflyer frame (Rd):



Two angles define the rotations between each actuator own frame (Ri) and frame (Rd).



The figure below shows the general QuadriXflyer cinematic diagram:



Our generic model accepts propellers tilt to simulate lacks of precision or to develop news control methods using four tilted propellers running in the same direction. This new control method (not developed in this paper) will allow to control the yaw with tangential forces created by propellers tilt instead of use the propellers drag torques differences.

The simulator consists in two main Simulink blocks: one calculates the efforts provided by the actuators to the main body and the other integrates these efforts to calculate linear positions of the center of gravity X, Y, Z and Euler angles ψ, θ, φ.

For a propeller with angular velocity w_i, thrust and drag torque are modeled by:

$$F_i = \frac{1}{2}.\rho.\,P_z.(w_i)^2 \qquad C_i = \frac{1}{2}.\rho.\,M_z.(w_i)^2,$$

Where P_z and M_z are non-sized thrust and drag torque coefficients.

The first Simulink block contains a direct current electrical motor model based on the equations below:

$$U(t) = RI(t) + L\frac{dI(t)}{dt} + E_m(t) \quad E_m(t) = K_m w_i(t)$$

$$E_m(t)I(t) = C_{em}(t)w_i(t) \qquad J(t)\frac{dw_{ri}(t)}{dt} = C_{em}(t)$$

The torque applied to the main body, depends on motor, reducer, and propeller inertias. Therefore, this torque value is calculated for each actuator using rotor and propeller dynamic moments and C_i propeller's drag torque:

$$\vec{M}_{Oi}(a_i \to d) = C_i.\vec{z}_i - \vec{\delta}(B_i, R_i/0) - \vec{\delta}(O_i, H_i/0)$$

The second Simulink block calculates the whole torques applied to the main body at QuadriXflyer center of gravity. Using the results above, and applying the dynamic fundamental principle. Linear accelerations, p, q, r angular velocities and ψ, θ, φ are calculable:

$$m\begin{bmatrix}\ddot{X}\\ \ddot{Y}\\ \ddot{Z}\end{bmatrix} = \begin{bmatrix}0\\ 0\\ m.g\end{bmatrix} + M_{do}\cdot\begin{bmatrix}\sum_{i=1}^{4} F_{ix}\\ \sum_{i=1}^{4} F_{iy}\\ \sum_{i=1}^{4} F_{iz}\end{bmatrix},$$

where $M_{do} = M_{od}^{-1} = M_{od}^{T}$ and :

$$M_{od} = \begin{bmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{bmatrix}$$

$$\begin{bmatrix} I_x\dot{p}+qr(I_z-I_y) \\ I_y\dot{q}+pr(I_x-I_z) \\ I_z\dot{r}+pq(I_y-I_x) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{4} M_{Gix} \\ \sum_{i=1}^{4} M_{Giy} \\ \sum_{i=1}^{4} M_{Giz} \end{bmatrix}$$

With

$$\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{\sin(\varphi)}{\cos(\theta)} & \dfrac{\cos(\varphi)}{\cos(\theta)} \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 1 & \tan(\theta).\sin(\varphi) & \tan(\theta).\cos(\varphi) \end{bmatrix} . \begin{bmatrix} p \\ q \\ r \end{bmatrix}_{(Rd)}$$

3 CONTROL DESIGN

Previous publications (Bouabdallah, Murrieri, Siegwart, 2004, Altuğ, Ostrowki, Mahony, 2002, Hamel, Mahony, Lozano, Ostrowski, 2002, Pounds, Mahony, Gresham, 2004) have presented backstepping methods that take into account nonlinear dynamics of the quadrotor (as gyroscopic effects and gravity).

Firstly, a nonlinear controller is designed with a backstepping method to control the 3-degree of freedom in rotation and 1-degree of freedom in translation (height).

The synthesis model is the following:

$$\begin{cases} \ddot{z} = g - \dfrac{(\cos\theta.\cos\varphi)}{m}(F_1+F_2+F_3+F_4) \\ \dot{p} = \dfrac{d}{I_X}(F_2-F_4)+q.r\left(\dfrac{I_Y-I_Z}{I_X}\right) \\ \dot{q} = \dfrac{d}{I_Y}(F_3-F_1)+p.r\left(\dfrac{I_Z-I_X}{I_Y}\right) \\ \dot{r} = \dfrac{1}{I_Z}(C_1-C_2+C_3-C_4)+p.q\left(\dfrac{I_X-I_Y}{I_Z}\right) \end{cases} \quad (3.1)$$

with d the distance between a rotor and the center of gravity.

Gyroscopic moments from rotors are not considered due to the missing on the quadrotor actuators of rotation speed sensor. However we have identified their dynamics and we consider their static gain in synthesis with the formula:

$$\begin{cases} F_i = a_i.u_i \\ C_i = b_i.u_i \end{cases}$$

where u_i are the control inputs, a_i and b_i the static gains. It is possible to consider different static gain for each actuator.

To simplify the problem, we use a one to one transformation on the inputs (Bouabdallah, Murrieri, Siegwart, 2004):

$$\begin{cases} W_1 = F_1+F_2+F_3+F_4 \\ W_2 = F_2-F_4 \\ W_3 = F_3-F_1 \\ W_4 = C_1+C_2+C_3+C_4 \end{cases}$$

Backstepping control is based on the research of a Lyapunov function. The following function is used in order to stabilize height and attitude and to regulate vertical and rotational speeds:

$$U(X) = \left((z-z_d)^2+\dot{z}^2+\left(\int p-\int p_d\right)^2+p^2\right)\Big/2 + \left(\left(\int q-\int q_d\right)^2+q^2+\left(\int r-\int r_d\right)^2+r^2\right)\Big/2 > 0$$

The objective is to find a formulation of the inputs W_i that leads to $\dot{U}(X)<0$ and stabilize the system (3.1).

Following inputs are suitable:

$$\begin{cases} W_1 = \dfrac{m}{\cos\theta.cos\varphi}\left[(z-z_d)+k_1.\dot{z}+g\right] \\ W_2 = \dfrac{I_X}{d}\left[-\left(\int p-\int p_d\right)-k_2p-q.r\left(\dfrac{I_Y-I_Z}{I_X}\right)\right] \\ W_3 = \dfrac{I_Y}{d}\left[-\left(\int q-\int q_d\right)-k_3q-p.r\left(\dfrac{I_Z-I_X}{I_Y}\right)\right] \\ W_4 = \dfrac{I_Z}{d}\left[-\left(\int r-\int r_d\right)-k_4r-p.q\left(\dfrac{I_X-I_Y}{I_Z}\right)\right] \end{cases}$$

where k_i are tuning parameters.

We notice that these inputs have a specific structure. They are compound with linear parts similar to the equation (3.2) and nonlinear ones that compensate gravity, tilt and gyroscopic effects.

$$u = k_p(x-x_d)+k_d\dot{x} \quad (3.2)$$

Simulations have been made with this controller. Two essential improvements have emerged:

- When we command Euler angles (not the integrals of p, q, r), it appears steady-state errors due to the non linear transformation between the VTOL (Vertical Take Off and Landing) and the earth frames (cf. §2).
- Tuning parameters k introduced in the Lyapunov function are not optimized.

These remarks have led to the following considerations:

- There must be an integral action to cancel the steady-state error.
- An LQ synthesis could be used to optimize k parameters.
- We should keep terms from backstepping that compensate the nonlinear effects.

Thus we choose the architecture presented in Figure 2. As the nonlinearity of the VTOL is compensated by the nonlinear feedback, G(s) appears to be linear (theoretically). It justifies the use of a linear quadratic command applied on G(s).

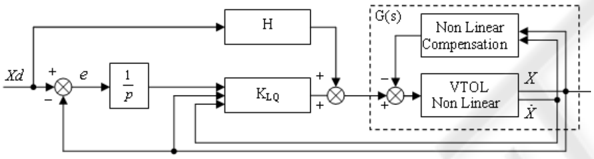


Figure 2: Final control design

The terms coming from backstepping synthesis and used for nonlinear compensation are as follows:

$$\begin{cases} W_1 = \dfrac{mg}{\cos\theta . cos\varphi} \\ W_2 = -q.r\dfrac{\left(I_Y - I_Z\right)}{d} \\ W_3 = -p.r\dfrac{\left(I_Z - I_X\right)}{d} \\ W_4 = -p.q\dfrac{\left(I_X - I_Y\right)}{d} \end{cases}$$

This nonlinear feedback compensates the projection of gravity and the gyroscopic effects. For example, when the quadrotor is tilting, the effect on weight is directly compensated.

In order to stabilize and to obtain good time-domain performances, we have chosen to apply an augmented Linear Quadratic control method as presented in Figure 3 on the linearized model from the nonlinear one (3.1).

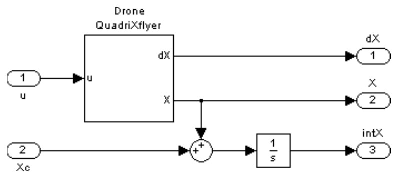


Figure 3: Synthesis LQ model

Four control inputs u_i for each electric motor and twelve states are used.

$$\begin{cases} \left(\dot{z}, p, q, r, z, \int p, \int q, \int r\right) & (3.3) \\ \left(\int \Delta z, \int \Delta_{\int p}, \int \Delta_{\int q}, \int \Delta_{\int r}\right) & (3.4) \end{cases}$$

The eight initial states (3.3) of the quadrotor has been augmented by four integrators states (3.4) corresponding to the integrator of the height, yaw angle, pitch angle and roll angle errors in order to obtain null steady-state errors in response to step inputs.

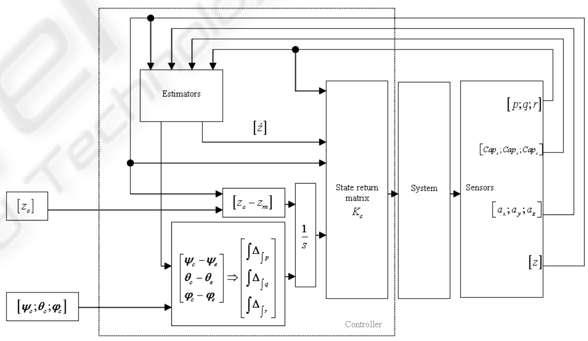


Figure 4: LQ control design

Gyrometers and ultrasonic sound sensor giving respectively the states (p,q,r,z) and $(\int p, \int q, \int r)$ by integration, the states $(\dot{z})$ and (3.4) are estimated need to Kalman filters, with accelerometers and heading measurements. The Euler angles have been chosen to control the QuadriXflyer attitude; the last four states are obtained by integration of the formulas (3.5) results.

This architecture is displayed in Figure 4. Moreover, to avoid oscillations on the control inputs coming from measurement perturbations, a low-pass frequency filter with a cut-off pulsation of 20 Hz has been added.

$$\begin{cases} \Delta z = z_d - z \\ \Delta \int p = (\varphi_d - \varphi) - \sin(\theta).(\psi_d - \psi) \\ \Delta \int q = \cos(\varphi).(\theta_d - \theta) + \sin(\varphi).\cos(\theta).(\psi_d - \psi) \\ \Delta \int r = -\sin(\varphi).(\theta_d - \theta) + \cos(\varphi).\cos(\theta).(\psi_d - \psi) \end{cases} \quad (3.5)$$

The LQ controller tuning was carried out in order to trade-off performance against consumption via Q_x and R matrix of a quadratic criterion (3.6).

$$J = \int \left(x^T.Q_x.x + u^T.R.u \right).dt \quad (3.6)$$

The R matrix is chosen as the identity matrix and the four diagonal terms of Q_x are tuned to master independently the four dynamics (height, yaw, pitch and roll) of the quadrotor, we have regulated these dynamics independently. Due to slow dynamics in height and law compared to the pitch and roll dynamics, these weightings have been augmented and chose to respect the QuadriXflyer requirements (stability, temporal performances like overshoot, time delay, oscillations).
Finally, we observe that the synthesized controller (Table 1) will act in a natural way on the system.

Table 1: Synthesized controller

	z	$\int p$	$\int q$	$\int r$	$\dot{z}$	p	q	r	$\int \Delta_z$	$\int \Delta_{\int p}$	$\int \Delta_{\int q}$	$\int \Delta_{\int r}$
u_1	–6.1	0	1.1	21.8	–4.5	0	0.7	10.2	4.2	0	–0.9	–23.4
u_2	–6.1	–1.1	0	–21.8	–4.5	–0.7	0	–10.2	4.2	0.9	0	23.4
u_3	–6.1	0	–1.1	21.8	–4.5	0	–0.7	10.2	4.2	0	0.9	–23.4
u_4	–6.1	1.1	0	–21.8	–4.5	0.7	0	–10.2	4.2	–0.9	0	23.4

We add a static matrix of feedforward controller H that allows pre-compensating slow poles of actuators by introducing zeros in the closed loop.

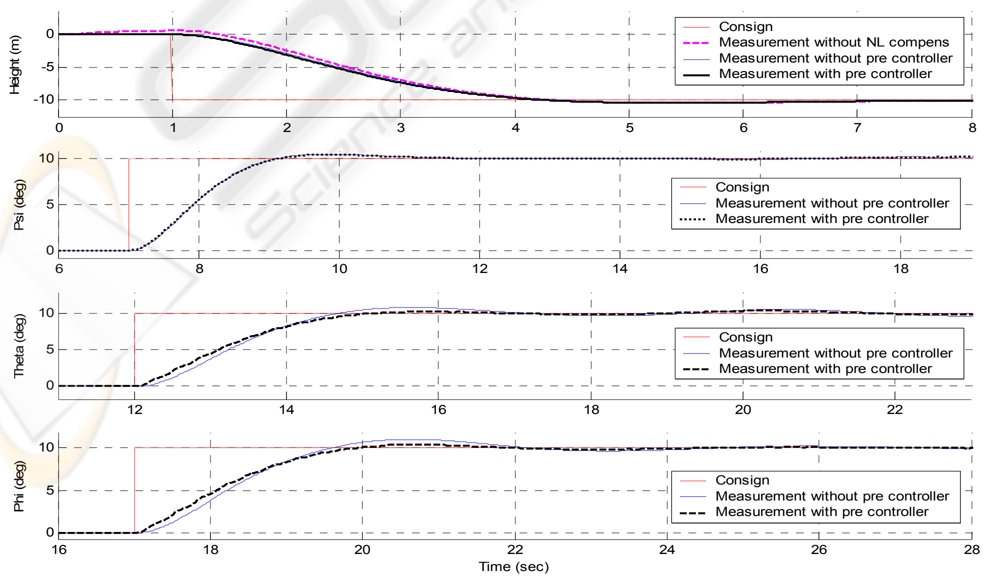
Physical considerations suggest the following structure for H:

$$H = \begin{bmatrix} -h_z & 0 & h_\theta & h_\psi \\ -h_z & -h_\varphi & 0 & -h_\psi \\ -h_z & 0 & -h_\theta & h_\psi \\ -h_z & h_\varphi & 0 & -h_\psi \end{bmatrix}$$

Each column corresponds to a dynamic (height, roll…) and each row corresponds to an actuator. If the actuators were perfect, each column would have parameters with same absolute values. However it is possible to consider differences between rotors dynamics by tuning these parameters.

The simulation 1 allows the effect of the feedforward controller and the terms coming from backstepping to be comparing.

The nonlinear compensation effect appears at the simulation beginning to maintain the quadrotor height. The feedforward controller accelerates the quadrotor response after a roll or pitch demand.



Simulation 1: Comparison between the effect of the precontroller and the terms coming from backstepping

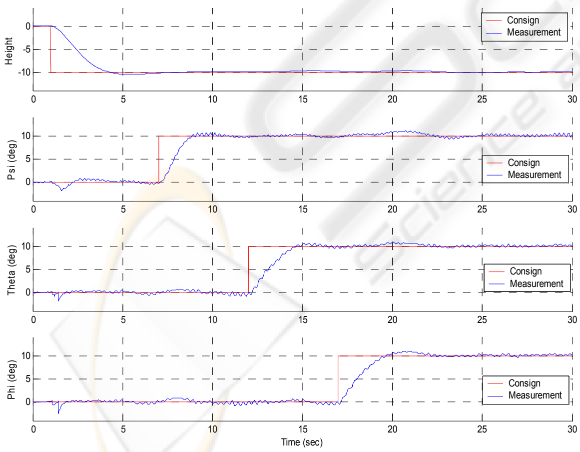
4 ROBUSTNESS STUDIES

Since the QuadriXflyer model is nonlinear, gain, phase and delay margins are not representative. To ensure stability and the respect the time-domain performances when the quadrotor is subjected to exogenous disturbances (wind), internal disturbances (noises on measurements) or uncertainties (inertia) on the synthesis model, some robustness studies have been performed.

These studies take into account:

- Bias and variance on the gyrometric measurements.
- Variance on the accelerometer measurements.
- Cap measurement perturbations to simulate a disturbing element like a magnetic material near the sensor.
- Command perturbations to simulate a wind gust, an air pocket.
- Height measurement perturbations to simulate the fast passage of a disturbing element like a tree between the quadrotor and the ground.
- Inertia errors in the model synthesis due to a false approximation in his theoretical calculation.
- Lack of precision on the parallelism of the propellers.

Example: Robustness study with variance on the accelerometer and gyrometric measurements:



Simulation 2: Robustness study

The QuadriXflyer is stable even if we can observe (Simulation 2) low oscillations on the pitch, roll and yaw dynamics.

5 CONCLUSION

In this paper we have proposed a modelling and a new hybrid LQ-backstepping control method for a quadrotor. This method combines nonlinear and linear controls and makes it possible to compensate in particular the nonlinear effect of gravity while preserving the performances of an LQ controller. The introduction of a feedforward controller allows the compensation of slow poles of the system, in particular those of the actuators without system stability deterioration.

ACKNOWLEGMENTS

We would like to thank Arnaud Mainsant and Rachid El Mafkouk for their participation to this work. We also thank Roland Bouttes, Daniel Alazard, Pierre Apkarian and Philippe Mouyon for their encouragements.

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