A NEW ART GALLERY ALGORITHM FOR SENSOR LOCATION

Andrea Bottino, Aldo Laurentini
Dipartimento di Automatica e Informatica, Politecnico di Torino, Corso Duca degli Abruzzi 24, Torino, Italy

Keywords: Art gallery, sensor positioning, planning, surveillance.

Abstract: Locating sensors in2D can be often modeled as an Art Gallery problem. Unfortunately, this problem is NP-hard, and no finite algorithm, even exponential, is known for its solution. Algorithms able to closely approximate the optimal solution and computationally feasible in the worst case are unlikely to exist. However this is an important problem and algorithms with "good" performance in practical cases are sorely needed. After reviewing the available algorithms, we propose a new sensors location incremental technique. The technique converges toward the optimal solution. It locally refines a starting approximation provided by an integer covering algorithm, where each edge is observed entirely by at least one sensor. A lower bound for the number of sensor, specific of the polygon considered, is used for halting the algorithm, and a set of rules are provided to simplify the problem.

1 INTRODUCTION

A number of computer vision tasks, as inspection, surveillance, image based rendering, constructing environment models, require multiple sensor locations, or the displacement of a sensor in multiple positions.

Sensor placement is an active area of research. A recent survey (Scott and Roth, 1992) refers in particular to tasks as reconstruction and inspection. Several other tasks and techniques were considered in the more seasoned surveys (Tarabanis et al., 1995, Newman and Jain, 1995). Practical sensor planning problems require considering a number of constraints, such as image resolution, field of view of the sensors, feature visibility, lighting, etc. Visibility is clearly the fundamental constraint for any kind of task and optical sensor. The sensor is usually modeled as a point and referred to as a "viewpoint". A feature of an object is said to be visible, or not occluded, from the viewpoint if any segment joining a point of the feature and the viewpoint does not intersects the environment or the object itself.

Although in general the problem is three-dimensional, in some cases it can be restricted to 2D. This is for instance the case of buildings, which can be modeled as objects obtained by extrusion.

For omni directional or rotating sensors, the 2D visibility can be modeled as an Art Gallery problem, which asks to position a minimum set of "guards" in a polygon. Unfortunately, the problem, as well as several of its variations, is NP-hard, and no finite algorithm, not even exponential, for locating a minimum set of guards is known. In addition, algorithms computationally feasible in the worst case and able to closely approximate the optimal solution are unlikely to exist (Eidenbenz et al., 2001). However, this is an important practical problem and approximate algorithms computationally feasible and supplying satisfactory solutions in practical cases are sorely needed.

In this paper, after discussing the available algorithms, we present a new approximate sensor positioning techniques. The algorithm is incremental and converges toward the optimal solution. It makes use a lower bound, specific of the polygon considered, for the number of sensors, and of rules for locally refining a starting approximate solution. This solution is provided by an integer covering algorithm, where each edge must be observed entirely by one sensor at least. The proposed technique can also take into account constraints as range and incidence.

Bottino A. and Laurentini A. (2005).
A NEW ART GALLERY ALGORITHM FOR SENSOR LOCATION.
In *Proceedings of the Second International Conference on Informatics in Control, Automation and Robotics - Robotics and Automation*, pages 242-249
DOI: 10.5220/0001174002420249

2 ART GALLERY PROBLEMS AND EDGE COVERING

The original problem, stated in 1975, refers to the surveillance, or "cover" of polygonal areas. The famous Art Gallery Theorem stated the upper tight bound $\lfloor n/3 \rfloor$ for the minimum number of "guards" (omni directional sensors) for covering any polygon with n edges, metaphorically the interior of an art gallery. The upper tight bound $\lfloor (n+h)/3 \rfloor$ holds for polygons with n edges and h holes. Many variations of the problem have been considered, and much work has been done for finding bounds in these cases. The decision problems related to the original problem (are k guards sufficient for covering a given polygon?), as well as those related to several similar problems, has been found to be NP-hard (Danner and Kavraki, 2002). No exact finite algorithm for locating a minimum set of sensors is known. For further details, the reader is referred to the monograph of O'Rourke (1987), and to the surveys of Shermer (1992) and Urrutia (2000).

Sensors positioning problems usually deals with observing, or covering, the boundary of objects and environment. Then in 2D we are content with observing the edges of a polygonal environment. We call this the Edge Covering (EC) problem, and the classic problem the Interior Covering (IC) problem. The EC problem and its relation with IC have been analyzed in Laurentini, 1999. For both EC and IC, the worst-case number of guards for polygons with and without holes is the same, but an optimum set of IC guards is not in general an optimum set of EC guards and vice-versa, and no simple rule, as adding or deleting guards, seems to exists for transforming an optimal solution of one problem in an optimal solution of the other. Also the decision problem associated to EC is NP-hard, since the classic proofs for polygons with and without holes also hold for edge covering (Laurentini, 1999). As for IC, at present no finite exact algorithm is known for locating a minimum set of EC guards in a given polygon.

In addition, recent result (Eidenbenz et al., 2001) shows that no worst-case computationally feasible approximate algorithm able to find solution close to the optimum is likely to exist. These results apply to both EC and IC, as well as to others problems in the area.

Finally, let us observe that the apparently continuous nature of EC (and IC) prevents putting the problems in the class NP. On this point, see also O'Rourke and Supowitz, 1983.

In the following section we will discuss the approximate algorithms existing for EC.

3 EXISTING APPROXIMATED EDGE COVERING ALGORITHMS

Some approximate seasoned algorithms for IC are reported in Shermer, 1992. All these algorithms are polynomial. It can be easily seen that their performance in relation with the optimal solution can be as bad as possible (O(n) guards, where n is the number of edges, when O(1) are sufficient). These algorithms have not been implemented, and no experimental results comparing the average performances of these algorithms with the optimal solution have been presented. Anyway, we have seen that in general the optimal EC and IC covers are different.

More recently, some attempt has been made for constructing *practical* sensor positioning algorithms. Kazakakis and Argyros (2002) have proposed a heuristic that divides the polygon into a number of convex polygons, each of which can be inspected by a guard with visibility range restriction. The algorithm has been implemented and some experimental results have been reported. Time performances are good, but the authors do not discuss how far are the solutions from optimum.

The randomized approach (Danner and Kavraki, 2002, Gonzales-Banos and Latombe, 1998 and 2001), appears the main practical technique available. We discuss here the most recent (not implemented) randomized algorithm of Gonzales-Banos and Latombe (2001), which also takes into account range and incidence constraints. The algorithm is as follows. First, the authors observe that, given an optimal solution for locating the guards, perturbing the positions of the guards into sufficiently small areas does not affect optimality. This leads to a randomized approach, where a number of viewpoints are located at random in the polygon, hoping of locating some points sufficiently near the points of an optimal solution. The next step consists in dividing the polygon boundaries into cells such that each viewpoint sees exactly a set of these cells. Selecting a minimal set of points among the random points is equivalent to solve an NP-complete set-covering problem. It is known that a greedy solution is polynomial, but has an approximation ratio bounded by (1+lg$p$), where p is the cardinality of the largest subset. It is clear that this factor is not a strong guarantee of good approximation for polygons with many edges and holes, since p is $O(m(n+h))$, where m is the number of random point, which is assumed to be rather large and in any case much larger than n. However, a clever exploitation of the particular set structure makes it possible to reduce the problem within an approximation ratio of $(1+ q)$, where q is $O(\lg(n+h)\cdot\lg(c\lg(n+h))$ and c is the optimal size. The advantage is that q does not depend on the presumably large number of random points.

Concluding, the authors obtain an algorithm polynomial in the worst-case, at the expense of the closeness to the optimal solution. In fact, also the improved approximation factor can be large for polygons with hundreds of edges and some tens of holes. In addition, it is clear that a main problem with this algorithm is the density of the sampling, "a choice that is perhaps more a craft than a science" according to the authors themselves. An incremental algorithm could be developed adding new random points, but we have no idea of the closeness to optimum of the solution, and we do not know where is more convenient to add new samples.

4 WHAT AN APPROXIMATE PRACTICAL ALGORITHM SHOULD DO

The performance of a "good" or "practical" Art Gallery algorithm should be analyzed in terms of both running times and closeness to the optimal solution of the approximation obtained. Since we deal with a problem that we are not even able to put in NP, we cannot be too exigent regarding the worst case behavior, being satisfied with algorithms capable of running in practical cases, in reasonable times. Clearly, running time depends on the number of edges and on the shape of the polygon. We can assume that a reasonable choice to model many practical environments or 2D objects is to consider polygons with several tens, or some hundreds, of edges. As for shape, the algorithm should be tested with examples of real environments.

Another desirable feature of a practical algorithm is the possibility of approaching the optimal solution by refining an initial approximation, at the expense of computation time. For obtaining a balanced trade-off between precision and time, we should have:

- some information about the quality of the approximation obtained at each step, which can be used to decide whether to stop the algorithm
- an incremental algorithm, able to refine locally the solution of the previous step.

The purpose of this paper is to present an algorithm that, to some degree, fulfills the previous requirements of a "good" or "practical" algorithm. Even if we are not able to discretize an apparently continuous problem, we propose an incremental algorithm, able to refine locally the solution, and we also provide a lower bound, specific of the case considered, that can be used to decide when to stop the incremental process. The algorithm starts with an initial approximation supplied by an integer edge covering algorithm.

5 INTEGER EDGE COVERING

The Integer Edge Covering (IEC) problem is a useful restriction of EC, requiring each edge to be entirely covered by at least one guard. For some practical task, observing entirely a feature of an object, usually modeled as an edge in 2D, could be preferable. In addition, for several not particularly tricky polygons the optimal unrestricted solution appears not too far from an optimal integer cover. For these reasons, integer covering is the starting approximation of our unrestricted edge-covering algorithm. A simple example showing the difference between EC and IEC is shown in Fig.1

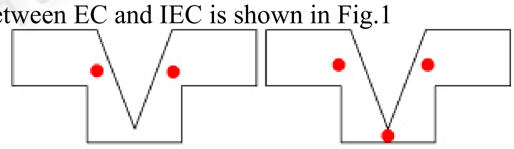


Figure 1: Two EC sensors cover the interior boundary of the polygon, but three IEC sensors are required

Bounds for IEC have been discussed in Laurentini, 1999. Regarding complexity, it is easy to see that the reductions of 3-SAT (O'Rourke, 1987) for polygons with and without holes also hold for IEC, since both reductions construct polygons where the edges are observed entirely by at least one guard. Thus, also the decision problem associated to IEC is NP-hard. The difference is that the restriction allows putting the problem in NP, so that it becomes NP-complete and finite algorithms are possible. In fact, the polygon can be divided into a set of non-overlapping zones, such that each point in each zone covers entirely the same set of edges. Then an hypothesis of solution consists in a finite string of characters that specifies a particular subset of these zones.

An algorithm for finding a set of zones where locating a minimum set of guards is reported in Laurentini, 1999. A modified version of this algorithm has been implemented in Bottino and Laurentini, 2004. The algorithm works for any polygonal environment (external coverage of multiple polygons, internal coverage of polygons with or without holes). Referring the readers to the original paper for details, the main steps of the algorithm are as follows:

IEC Algorithm

Step 1. Compute a partition Π of the viewing space into regions Z_i such that:

1. The same set $\boldsymbol{E}_i$ of edges is entirely visible from each point of Z_i, $\forall i$. Actually, each region is also labeled with the partially visible edges and the number of occlusions.
2. The regions Z_i are maximum regions, that is $\boldsymbol{E}_i \not\subset \boldsymbol{E}_j$ where Z_j is any region contiguous to Z_i

Step 2. Select the *dominant* regions and the *essential* regions. A region Z_i is dominant if there is no other region Z_j such that $\boldsymbol{E}_i \subset \boldsymbol{E}_j$. An essential zone is a dominant zone that covers an edge not covered by any other dominant zone.

Step 3. Find an optimum set of zones covering all the edges. This is a set covering problem restricted to the set of edges not covered by the essential zones, and to the subsets corresponding to the dominant zones that: a) are not essential; b) cover some edge not already covered by the essential zones.

Steps 1 and 2 are polynomial (Laurentini, 1999); Step 3 is exponential in the worst case. However, in many cases the simplifications introduced by dominant and essential zones strongly reduce the complexity of the algorithm. Observe that the algorithm does not provide in general all the possible minimal sets of zones, since there could be solutions including non-dominant zones. However, this does not seems a serious drawback, since each non-dominant zone can be replaced by a dominant zone that covers some more edges, and multiple coverage in several practical cases is preferable, for instance in case of sensor failure.

6 THE SENSOR POSITIONING ALGORITHM

Integer edge covering provides a starting approximation, which can be incrementally refined for approaching the optimal unrestricted solution. For deciding if a solution is acceptable, we compute a *lower bound* LB(P) for the optimum number of sensors, specific of the polygon P considered. If the starting solution is far from the lower bound, we can improve this solution by splitting some of the edges and re-applying the IEC algorithm. It is clear that this procedure converges toward the optimal solution, if computationally feasible. For reducing the computational burden, we also supply rules that tell us which edges must not be divided in order to reach the optimal solution. Summarizing, our sensor positioning algorithm works as follows.

Given a polygon P, compute the lower bound LB(P) with the LBA algorithm described in section 6.2

Apply the IEC algorithm and find an approximate covering. Compare the cardinality CA of the approximate covering with the lower bound. If they are equal, the solution is optimal also for the unrestricted problem. If $(CA/LB(P))<1+\varepsilon$, where ε is some predefined threshold, stop; otherwise:

Apply the algorithm INDIVA, described in section 6.3 for finding the edges that must not be divided, split the others and return to 2

Actually, it could be necessary to stop the algorithm before reaching a satisfactory cover due to the computation time. In the rest of this section we describe the algorithms INDIVA and LBA.

6.1 Integer and weak visibility polygons

Both algorithms LBA and INDIVA make use of the concept of *weak* and *integer visibility polygons* of the edges (J. O'Rourke, 2002). Let us briefly recall their definitions:

the *Weak visibility polygon* $W(e_i)$ of an edge e_i is the polygon whose points see some points of e_i

the *Integer visibility polygon* $I(e_i)$ is the polygon whose points see entirely e_i. An example is shown in Fig.2

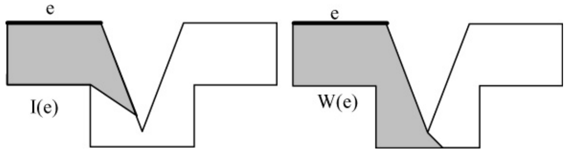


Figure 2: The integer and weak visibility polygons of the edge e

Polynomial algorithms for computing weak and integer visibility polygons of an edge are described in the literature (Sack and Suri, 1990). In our case however, both polygons can be computed as a byproduct of the IEC algorithm. If there are *p* zones in the partition Π, computing the visibility polygons for all edges is O(*pn*).

6.2 LBA, the Algorithm for Finding a Lower Bound

LBA Algorithm finds a maximum subset of disjoint (not intersecting) weak visibility polygons W(e_i). The cardinality of this set is LB(P).

Since each weak visibility polygon must contain at least one sensor, no arrangement of EC sensors can have fewer sensors than LB(P). A simple example is shown in Fig.3, where the lower bound is 2.

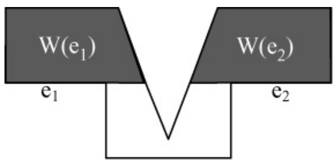


Figure 3: A maximum set of non-intersecting weak visibility polygons

Computing the lower bound requires solving the *maximum independent set problem* for a graph G where each node represents the weak visibility polygon of an edge of P, and each edge of G connects nodes corresponding to intersecting weak visibility polygons. The problem is equivalent to the *maximum clique problem* for the *complement* graph G' (the graph obtained by joining those pairs of vertices that are not adjacent in G). It is well known that these are NP-complete problems.

However, we stress that *exact* branch-and-bound algorithms for these problems have been presented and extensively tested (Woods, 1997, Oestergard, 2002), showing more than acceptable performance for graphs with hundreds of nodes. Then, we assume that computing LB(P) is computationally feasible for the practical cases considered.

6.3 INDIVA, the Algorithm for Finding Indivisible Edges

In this section we describe a set of rules for finding the *indivisible* edges of P. An edge is called indivisible if optimal sets of sensors exist such that edge is entirely observed by at least one sensor.

Clearly, for approaching these optimal solutions we do not need to split these edges. The first two rules are as follows:

Rule1. If $W(e_i) = I(e_i)$, e_i is indivisible.

Rule2. If $W(e_i) \subseteq I(e_j)$, e_j is indivisible.

Both rules follows from the fact that *at least one sensor of any minimum set* must be located in each weak visibility polygon, and then also in the integer visibility polygon satisfying one of the above rules.

A simple example will show how to apply these rules, and that they can be powerful tools for simplifying the problem. Let us consider the polygon in fig. 2(a).

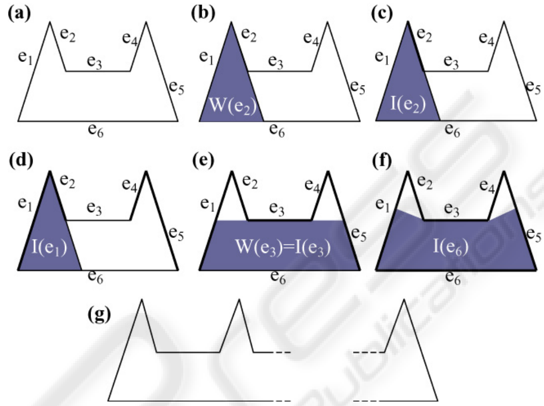


Figure 4: In this case, the optimum EC and IEC covers are equal

The weak visibility polygon of e_2 is shown in (b). It is coincident (c) with the integer visibility polygon I(e_2), and then for *Rule 1* e_2 is indivisible, and marked with a thicker line. In (d) it shown that I(e_1)= W(e_2), and then for *Rule 2* also e_1 is indivisible, as well as, for similar reasons, e_4 and e_5. As for e_3, it is indivisible, since W(e_3)= I(e_3) (Fig.1 (e). Finally, in (f) it is shown I(e_6). Since I(e_6)⊃W(e_3), also e_6 is indivisible.

Concluding, in this example the unrestricted minimal set of guards is that provided by the integer-covering algorithm. The same result could have been obtained by computing the IEC solution, whose cardinality is equal to the lower bound LB(P). This also happens for any polygon of the family shown in (g), which is used for showing that the bound supplied by the Art Gallery theorem is tight.

We stress that if one of previous rules can be applied to an edge, this edge is entirely observed by a sensor for *any* optimal solution.

We have found three other rules for determining other edges indivisible for *some* optimal solution. They are based on the idea that for some edge *e* we can discard some parts of W(*e*), having lesser visibility of the boundary of P than other parts.

Let us recall that the algorithm IEC divides P into a set of regions whose points share the same visibility condition of the boundary of P. In particular, each zone is labeled with: a) the edges seen entirely by each point of the region; b) the edges seen partially with the number of occlusions. Let E(p) denote the points of the boundary of P seen by a point p, *excluding points belonging to indivisible edges*, which are fully observed by some other viewpoint. We say that a region R of W(e_i) is the *best region* of W(e_i) if for any point $p \in$R and any point $q \in$(W(e_i)- R) it is E(p)$\supseteq$ E(q). It is clear that, if such region exists, any optimal solution with one viewpoint in W(e_i)-R can be substituted by an optimal solution with a viewpoint in R. It follows that all the edges fully observed by R are indivisible. Then we get the following rule:

Rule3: edges which are completely seen by any point belonging to the best region of W(e) (if it exists) are indivisible.

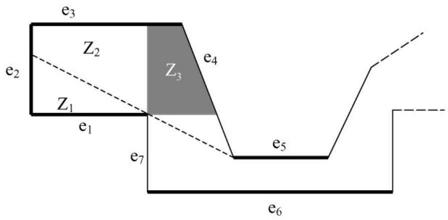


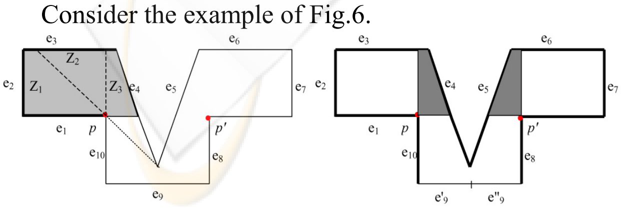
Figure 5: Z_3 is the best region of W(e_1)

As an example, consider the polygon of Fig. 5. By applying Rules 1) and 2) we find that edges e_1, e_2, e_3, e_5, e_6 are indivisible. Consider W(e_1). Region Z_1 sees fully edges e_1, e_2, e_3 and partially e_4. Region Z_2 sees fully edges e_1, e_2, e_3, e_4 and partially e_6, which must not be considered since it has been already found to be indivisible. Then Z_3 is the best region, since any point in it sees fully edges e_1, e_2, e_3, e_4 and e_7. Concluding, we have found that also e_4 and e_7 are indivisible.

The best point of W(e) is defined (if it exist) as the point p_o of W(e) such that E(p_o)$\supseteq$E(q) (excluding indivisible edges) for any other point $q \in$W(e_i). If such a point exists, in any optimal solution a viewpoint belonging to W(e_i) can be substituted by p_o. Then we get the following rule:

Rule 4: An edge is indivisible if it is observed by the best point of W(e)

Consider the example of Fig.6.



(a) (b)

Figure 6: Point p is the best point of W(e_1) (a). The zones where two optimal viewpoints can be located (b)

We assume that a viewpoint lying on a vertex of P is able to see the edges converging at the vertex. This is justified by the fact that, excluding exceptional alignment conditions, p can be displaced in a region nearby without affecting the optimality. Using rules 1 and 2 e_1, e_2, e_3 are easily found to be indivisible. It is clear that p, which sees entirely e_1, e_2, e_3, e_4, e_{10}, and a part of e_9 largest or equal to that seen by any other point is the best point of W(e_1). Then also e_4 and e_{10} are indivisible.

The full algorithm for this polygon works as follows. LB(P) is 2, as shown in Fig.3. Applying the IC algorithm we obtain 3 viewpoints. Since this is different from the lower bound, we apply rules 1, 2 and 4 and find that only e_9 could be divided. After splitting in two e_9 we apply again the IC algorithm, and find two areas, containing the two best points p and p' where the viewpoints can be independently located (Fig. 6(b)). The cardinality of the solution is equal to LB(P), and then the solution is optimal.

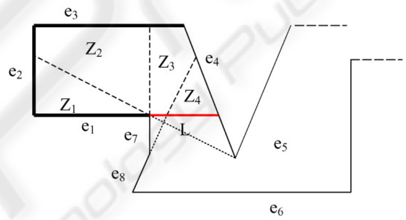


Figure 7: The line L contains the best boundary points

Let us observe that the definition of best point of a region W(e_i) can be extended, excluding from E(p) not only other indivisible edges, but also edges or parts of edges covered by other best points. This is an effective extension, as it will be shown by the second example in section 7.

Finally, consider W(e_1) in the case shown in Fig.7. Applying rules (1) and (2) we find that e_1, e_2, e_3 are indivisible. However, we are not able to apply rules (3) or (4), since in W(e_1) there is no best point or region according to the definition given. This depends on the fact that there is no point or region that sees a part of e_6 larger then any other point. However, we observe that, for each point $p \in$W(e_1), there is a point of the boundary line L that sees the same set of integer edges, and a larger part of e_6. This means that any minimal solution with a viewpoint point inside W(e_1) can be transformed in a minimal solution with a viewpoint lying on L. In these minimal solution the set of edges entirely observed is the set entirely observed by any point of L, in this case e_1, e_2, e_3, e_4, e_7. Then also e_4 and e_7 are indivisible. We will call a line L the best boundary lines of W(e) if, for any point $q \in$W(e) there is a point p$\in$L such that E(p)$\supseteq$ E(q). We obtain:

Rule 5: An edge is indivisible if it is entirely observed by any point of the best boundary line (if it exists)

7 TWO EXAMPLES

First, let us apply our algorithm to a polygon taken from the paper (Gonzales-Banos and Latombe, 2001) of Danner and Kavraki.

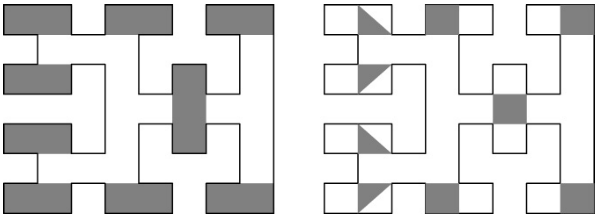


Figure 8: LB(P) (left) and an optimal solution (right)

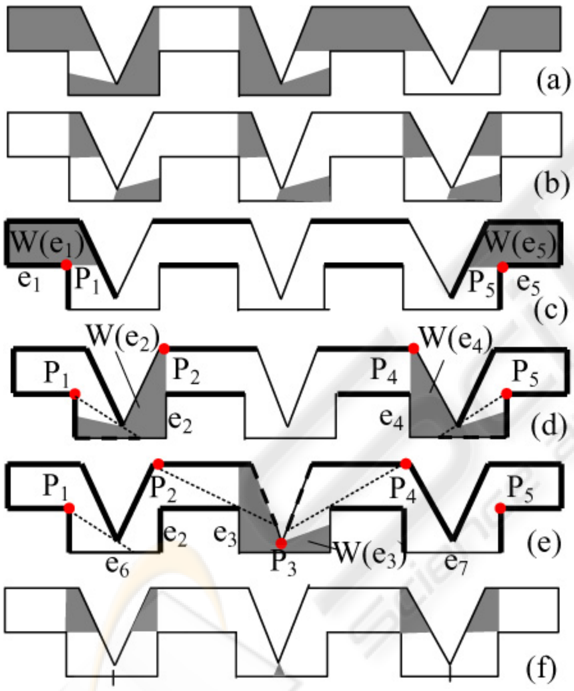


Figure 9: In this case, the algorithm reaches in a few steps the optimal solution

The cardinality of the maximum set of non-intersecting weak visibility regions (shown on the left in Fig.8) is 9. Then LB(P) = 9.

One of the minimum solutions provided by the IC algorithm is shown in Fig 8 on the right. Since the number of zones where to independently locate a minimum set of integer edge guards is 9, this is also an optimal solution of the unrestricted problem.

In the second example (Fig.9), LB(P) is 5 (Fig.9(a) and the cardinality of the IC approximation is 7(in Fig.7(b) one optimal integer cover is shown). Rules (1), (2) and (4) determine the indivisible edges shown in Fig.9(c), and two best points of W(e_1) and W(e_5). Applying again rule (4) taking into account the edges and parts of edges (hatched in the figure) observed by the two best points, two other best points of W(e_2) and W(e_4) are found(Fig. 9(d)). Another application of rule (4) supplies the best point of W(e_3) (Fig.9(e)). Concluding, all edges are indivisible, excluding e_6 and e_7. Splitting in two these edges and applying again the IEC algorithm, the solution shown in Fig.9(f) is found. Five sensors can be located anywhere in the five regions shown, and therefore solution is optimal.

8 CONCLUSIONS AND FUTURE WORK

We have presented a new art gallery sensor location algorithms. The algorithm starts with an approximate solution provided by an integer edge covering algorithm, since in many practical cases this appears not too far from the optimal solution. A lower bound for the optimal number of sensor is computed. If the cardinality of the solution is equal to the lower bound, the solution is optimal; if not, an optimal solution can be approached by a local refinement of the integer edge covering solution. A set of rules is provided for determining which edges could be split to approach the optimal solution. Some examples show how the algorithm works.

Is the algorithm a “good” algorithm for practical cases, or, in other words, does it supplies in reasonable time’s solutions close to the optimum? For assessing its performance, we are currently implementing the full algorithm and looking for further rules for finding indivisible edges, as well as rules for splitting divisible edges. Then we will apply the algorithm to a number of 2D maps taken from real environments.

A final remark. Our algorithm can easily take into account range and incidence constraints. For each edge *e* the constraints define a region C(*e*) of P where the viewpoint can be located. Applying the IEC algorithm, we must consider as candidates to cover an edge *e* only the regions, or their parts, belonging to C(*e*).

REFERENCES

A. Bottino, A. Laurentini "Optimal positioning of sensors in 2D". Proc. 9th Iberoamerican Congress on Pattern Recognition, Puebla (Mexico), October 26-29, 2004

J.C. Culberson and R.A. Reckhow," Covering polygons is hard," *J Algorithms* , Vol.17, pp. 2–44,1994

T. Danner and L.E. Kavraki," Randomized planning for short inspections paths," *IEEE Int. Conf. On Robotics and Automation,*April 2002, pp. 971-976

S.Eidenbenz, C. Stamm and P. Widmayer," Inapproximability results for guarding polygons and terrains," *Algorithmica,* Vol.31, pp.79-113, 2001

H. Gonzales-Banos and J-C. Latombe," A randomised art gallery algorithm for sensor placement," *ACM SCG'01,* June3-5,2001

H. Gonzales-Banos and J-C. Latombe."Planning robot motion for range-image acquisition and automatic 3D model construction," *Proc. AAAI Fall Symposyum Series,* 1998

G. D. Kazakakis, and A.A. Argyros, "Fast positioning of limited visibility guards for inspection of 2D workspaces," *Proc. 2002 IEEE/RSJ Intl. Conf. On Intelligent Robots and Systems,* pp.2843-2848, October 2002

A.Laurentini,"Guarding the walls of an art gallery," *The Visual Computer*, vol.15, pp.265-278, 1999

T.S. Newman and A.K. Jain, "A survay of automated visual inspection," *Comput. Vis. Image Understand. ,* vol.61, no.2, pp.231-262, 1995

J. O'Rourke and K.J. Supowit, "Some NP-hard polygon decomposition problems,"*IEEE Trans IT,* Vol.29, no.2, pp.181-190, March 1983

J. O'Rourke , *Art gallery theorems and algorithms*, Oxford University Press, New York,1987

P. Oestergard, "A fast algorithm for the maximum clique problem,"*Discrete Applied Mathematics,*Vol.120, pp.197-207, 2002

J.R.Sack and S.Suri,"An optimal algorithm for detecting weak visibility of a polygon," *IEEE Trans. on Computers,*Vol.39, pp.1213-1219, 1990

T. Shermer, "Recent results in art galleries," *IEEE Proc*. Vol. 80, pp.1384-1399, 1992

W.R.Scott and G.Roth,"View planning for Automated Three-Dimensional Object Reconstruction and Inspection," *ACM Comput. Surveys,*Vol.35, No.1, March 2003, pp.64-96

K.A. Tarabanis, P.K. Allen, and R. Y. Tsai, " A survey of sensor planning in computer vision," ," *IEEE Trans. Robot. and Automat*., vol. 11, no. 1 , pp. 86 –104, 1995

J. Urrutia, "Art Gallery and Illumination Problems" *Handbook on Computational Geometry*, Elsevier Science Publishers, J.R. Sack and J. Urrutia eds. pp. 973-1026, 2000.

D.R. Woods," An algorithm for finding a maximum clique in a graph," *Operations Research Letters,*Vol.21, pp. 211-217,1997